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Automated Generation and Optimization of Ballistic Lunar Capture Transfer Trajectories

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Automated Generation and Optimization of Ballistic Lunar Capture Transfer Trajectories

by

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Dedication

This dissertation is dedicated to my wife and daughter, who have been a constant source of love and laughter in my life. You keep my dreams alive and give me the courage to follow them.

Acknowledgements

I would like to thank those who have provided me the ability to complete this work. First, my wife deserves recognition for her constant support and devotion. Her encouragement and sacrifice on my behalf is the reason that this project was possible.

Next, my advisor, Dr. Cesar Ocampo, deserves special recognition for planting the seeds of this research and for inspiring me through his infectious love for engineering. He always challenged me to be my best, and allowed me the flexibility to explore the problems that interested me. I would also like to thank the other professors on my dissertation committee. Dr. David Hull provided me with a terrific foundation in the fields of optimal control theory and numerical methods in optimization. His classes were the most organized and well presented that I have experienced. Dr. Wallace Fowler was an inspiration early in my graduate education, and provided me with valuable skills in mission design. I would like to thank Dr. Belinda Marchand for the scope that she gave my work, and for valuable comments and revisions to this paper. Finally, I would also like to thank Dr. Edward Belbruno, first for his innovative approach to the lunar transfer problem, and also for his willingness to go out of his way to serve on my dissertation committee.

A final mention is made to my loving parents. Their gifts to me have been countless. This work is a tribute to their lifelong support of my education.

Automated Generation and Optimization of Ballistic Lunar Capture Transfer Trajectories

Publication No._____________

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The successful completion of the Hiten mission in 1991 provided real-world validation of a class of trajectories defined as ballistic lunar capture transfers. This class of transfers is often considered for missions to the Moon and for tours of the moons of other planets. In this study, the dynamics of the three and four body problems are examined to better explain the mechanisms of low energy transfers in the Earth-Moon system, and to determine their optimality. Families of periodic orbits in the restricted Earth-Sun-spacecraft three body problem are shown to be generating families for low energy transfers between orbits of the Earth. The low energy orbit-to-orbit transfers are shown to require less fuel than optimal direct transfers between the same orbits in the Earth-Sun-spacecraft circular restricted three body problem. The low energy transfers are categorized based on their generating family and the number of flybys in the reference three body trajectory.

The practical application of these generating families to spacecraft mission design is demonstrated through a robust nonlinear targeting algorithm for finding Sun-Earth-Moon-spacecraft four body transfers based on startup transfers indentified in the Earth-Sun three body problem. The local optimality of the transfers is examined through use of Lawden's primer vector theory, and new conditions of optimality for single-impulse-to-capture lunar transfers are established.

Table of Contents

List of Tables

List of Figures

List of Symbols and Abbreviations

Symbol	*Description*
BLCT	Ballistic Lunar Capture Transfer
WSB	Weak Stability Boundary
CRTBP	Circular Restricted Three Body Problem
Δv	Magnitude of an impulsive spacecraft maneuver
μ	Mass parameter
μ_s	Mass parameter of the secondary body
μ_p	Mass parameter of the primary body
$\mathbf{r}$	Position vector
$\mathbf{v}$	Velocity vector
r_x , r_y , r_z	Component of the position vector in the x, y, and z directions
t	Time
v	Magnitude of the spacecraft's velocity
C	Jacobi constant of the CRTBP
T	Period of a transfer
μ_e, μ_s, μ_m	Mass parameter of the Earth, Sun, and Moon
J	Performance index
θ	Angle between the primer vector and velocity vector at the final time
$\delta(t - t_0)$	Dirac delta function
T	Thrust acceleration
m	Mass of a spacecraft
τ	Time-like variable associated with a specified orbit
$\boldsymbol{\theta}$	Constraint vector at the initial time

$\boldsymbol{\psi}$	Constraint vector at the final time
$\mathbf{u}$	Control vector
$\mathbf{l}$	Steering vector for the engine thrust
$\mathbf{C}$	Control constraint vector
$\mathbf{g}$	Gravitational acceleration vector
G	Endpoint function
H	Hamiltonian function
$\boldsymbol{\lambda}$	Costate vector
$\hat{H}$	Extended Hamiltonian function
$\boldsymbol{\mu}$	Vector of constant Lagrange multipliers associated with the control constraint
$\mathbf{p}$	Primer vector
$\Phi(t,t_0)$	State transition matrix
BCRFBP	Bi-Circular Restricted Four Body Problem
ω	Angular frequency
f	True anomaly
E	Keplerian energy
Δt	Transfer time
α,β,γ	Orientation angles of the low Earth orbit
υ	Lagrange multiplier associated with the constraint vector at the initial time
$\boldsymbol{\xi}$	Vector of Lagrange multipliers associated with the constraint at the final time
a,e,i,ω,Ω	Orbital elements
$\mathbf{a}$	Parameter vector
$\mathbf{c}$	Constraint vector

Common Subscripts

Subscript	***Description***
s	In relation to the secondary mass in the CRTBP
p	In relation to the primary mass in the CRTBP
1	Associated with the initial impulse
2	Associated with the final impulse
k	Associated with the intermediate impulse
0	Associated with the intial time
f	Associated with the final time
c	Associated with an unknown time
r	Associated with the position vector
v	Associated with the velocity vector
m	Associated with the mass of the spacecraft
em	Associated with the Earth-Moon frame
e	With respect to the Earth
m	With respect to the Moon
s	With respect to the Sun
$(em)s$	Associated with the (Earth-Moon combined mass)-Sun CRTBP
sc	With respect to the spacecraft
RTBP	Restricted three body problem
RFBP	Restricted four body problem

1. Introduction

1.1 Definition of a Ballistic Lunar Capture Transfer

The efficient design of lunar transfers in a multi-body regime has been a relevant topic for many years. Complex lunar transfers in the Earth-Moon system are available that take advantage of multi-body dynamics and chaotic motions.[1] A particular class of these transfers, ballistic lunar capture trajectories (BLCTs), has been identified as potentially offering fuel savings in the transfer between Earth orbit and lunar orbit when compared to traditional two-body methods based on the Hohmann transfer.[2] The practical design and optimization of BLCTs is the topic being considered in this research.

BLCTs are characterized by a spacecraft transitioning from a hyperbolic lunar orbit into an elliptical lunar orbit without the need of a maneuver at the time of transition.[1] The BLCTs of current interest achieve this transition in the neighborhood of the Earth-Moon collinear exterior Lagrange point, L_2. With these types of trajectories, it is possible to transfer a satellite from low Earth obit to lunar orbit with a single impulsive maneuver; however, the disadvantage is that the transfer time is typically on the order of ninety days. Figure 1 shows the characteristic geometry of the BLCT in an Earth-centered coordinate system that rotates with the Earth-Sun line. A BLCT is also called a low energy transfer due to the similarity in energy states of the spacecraft and the target state as the spacecraft approaches the termination of the transfer. In this dissertation, these two terms will be used interchangeably when discussing lunar transfers.

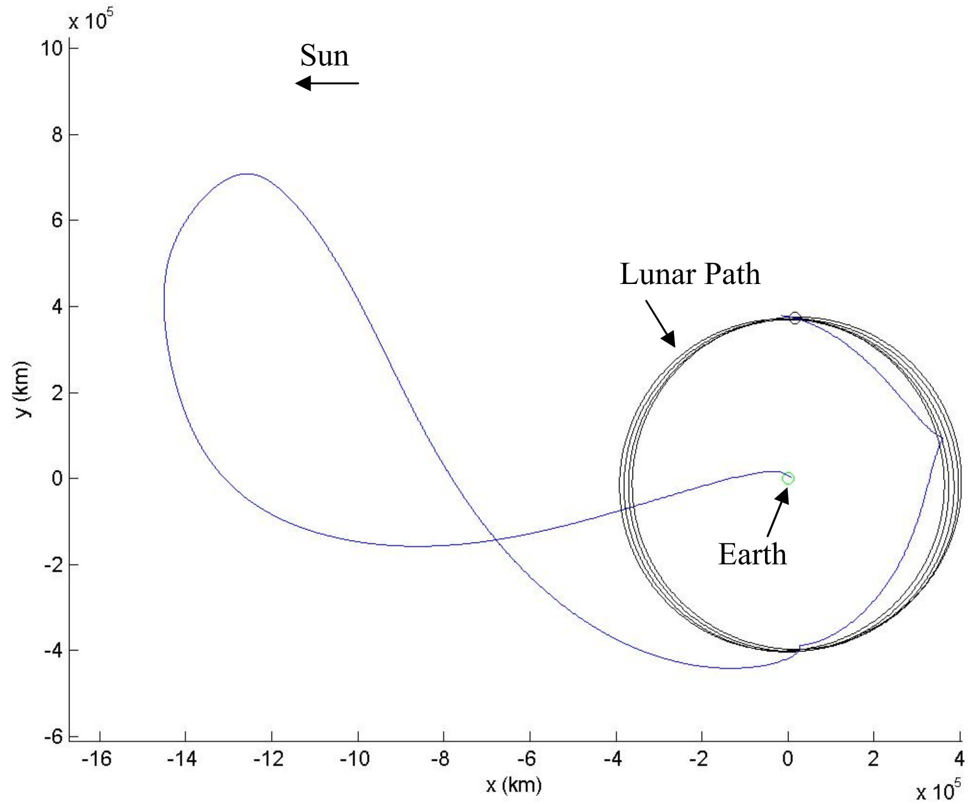

Figure 1. A BLCT in Earth-Sun rotating coordinates

1.2 Previous Work

The work that documents the current theory for designing and optimizing low energy transfers is summarized here. The list of works is not comprehensive, but covers the major publications that have established the state of the art for analyzing the trajectories.

1.2.1 The Weak Stability Boundary

In 1968, Conley was the first to construct low energy transfers to lunar orbit.[3] Belbruno continued the study of ballistic capture and escape orbits through the study of the Weak Stability Boundary (WSB), a chaotic region of phase space that exists around masses in multi-body problems.[4] Low energy transfers exploit these chaotic regions to allow a trajectory to transition from an uncaptured state to a captured state without a maneuver. Trajectories that utilized the WSB were first reported by Belbruno in 1987.[1] The trajectories described in the initial work were ballistic segments of a low thrust trajectory that transitioned to a captured orbit in the neighborhood of the Earth-Moon L_1 Lagrange point.

Belbruno and Miller[2] were later able to use the lunar WSB in combination with the Sun's gravitational influence to create a trajectory that rescued a stranded satellite in orbit around the Earth and place it in lunar orbit using a very small fuel budget. The transfer, dubbed the Hiten mission, completed its mission successfully, and was the first mission to implement WSB concepts. The transfer in the Hiten mission approached the Moon through the neighborhood of the Earth-Moon L_2 Lagrange point. It is these types of exterior transfers that are the subject of this dissertation, as they are possible to achieve with a single impulsive maneuver from low Earth orbit. Belbruno presents the theory of the WSB and the chaotic dynamics involved in it in his book, which serves as a very comprehensive source for both the analysis of WSB dynamics and its applications to spacecraft mission design.[4] More recently, other authors have offered a new mathematical definition of the WSB and have associated it with the manifolds of periodic orbits in the circular restricted three body problem (CRTBP).[5]

1.2.2 Dynamical Systems Theory

One tool that has proven useful in the study of BLCTs is dynamical systems theory. In the context of dynamical systems theory, BLCTs are equivalent to a heteroclinic connection between two periodic orbits, one around the Earth and the other around the Moon.[6] The notion of heteroclinic connections between periodic orbits in dynamical systems dates back at least to 1937.[7] Recent studies by Koon, et al.,[8,9] Gomez and Masdemont,[10] and Canalias and Masdemont,[11] consider the use of heteroclinic connections in the circular restricted three body problem for the design of interplanetary transfers and of tours of planet-moon systems. These connections are established by numerically identifying connections between stable and unstable manifolds of periodic orbits in the CRTBP,[9] and methods for their computation have been provided by the authors.

When analyzed in the context of dynamical systems theory, BLCTs are shown to follow manifolds of halo orbits in the Earth-Sun and Earth-Moon CRTBPs. The manifolds were exploited for trajectory design by Howell, Barden and Lo in the context of missions to libration point orbits.[12] In these mission designs, linear targeting algorithms were used to design missions from a specified orbit to a stable manifold of the desired trajectory. The spacecraft follows the manifold and ballistically enters into the libration point orbit. The method has been applied to libration point missions such as Genesis.[13]

Several authors have applied similar theory to the analysis and targeting of low energy transfers to the Moon.[14-17] The problem becomes more complicated for the lunar transfers due to the need for the simultaneous calculation of manifolds of the Earth-Moon libration points and the Earth-Sun libration points. Algorithms exist that utilize the invariant manifolds associated with libration point orbits in both the Sun-Earth and Earth-Moon CRTBPs. For example, Koon, et al.[14] targeted BLCTs by finding intersections of these invariant manifolds. Similarly, Yamato and Spencer[15] approximated the invariant manifolds in a perturbed CRTBP, yielding transit orbits and lunar capture. These transfers are typically found with manifolds of halo orbits around the libration points. Folta[16] formulates the problem with Lissajous orbits around the libration point serving as the periodic orbit from which the manifolds are computed. These transfers are computed in the context of formation flight in WSB regions.

Parker and Lo[17] have categorized families of BLCTs found from invariant manifolds with the intention of allowing a mission planner to choose appropriate trajectories for specific missions. This work has produced catalogs of BLCTs that are parameterized by various mission requirements. This work is the most complete analysis of the design of ballistic lunar capture trajectories using the manifold techniques.

1.2.3 Bi-elliptic transfer comparison

Another methodology for analyzing BLCTs is to compare them to a bi-elliptic transfer from low Earth orbit to an circular orbit of lunar radius in the two body problem. Ivashkin,[18] for example, has compared them to bi-elliptic transfers in a central body gravity field. In this comparison, the solar perturbation of the Earth-centered two body problem provides the intermediate Δv of the bi-elliptic transfer that increases the periapse distance of the orbit. From Lidov,[19] an estimation of the effect of this perturbation on the periapse distance can be made.

1.2.4 Indirect optimization and primer vector theory

The problem of minimizing the fuel consumption associated with impulsive orbit-to-orbit transfers has received a great amount of attention. Formalized by Lawden in 1963,[20] the necessary conditions for an optimal impulsive transfer are imposed on the *primer vector*, the reflection of the vector of adjoint variables in the optimal control problem that correspond to the velocity of the spacecraft. Lawden's necessary conditions specify the magnitude and orientation of the primer vector at the times of impulse. Specifically, the primer vector must align with the direction of the impulse and have a uniform magnitude at each impulse. The magnitude has been traditionally chosen to be unity at each impulse, but the designated value is arbitrary. Lion and Handelsman[21] extended Lawden's work to determine when an additional impulse would improve on the fuel cost of the transfer. Jezewski[22] later gives a summary of primer vector theory, and introduces an estimation of the magnitude of an intermediate impulse that would optimize the transfer.

Primer vector theory has been used to evaluate the optimality of several different types of transfers. Optimal transfers in an inverse square gravitational field have been analyzed for various types of missions. Rendezvous in the vicinity of a circular orbit has been explored by Prussing,[23] direct ascent rendezvous by Gross and Prussing,[24] and rendezvous between circular orbits by Prussing and Chiu.[25] Jezewski[22] has studied the problem of multiple impulse transfers and transfers with inequality constraints.

Primer vector theory has been applied to low energy transfers is certain situations. The primer vector in the three body problem was studied by Hiday-Johnston and Howell[26] to evaluate the optimality of transfers between libration point orbits, by D'Amario and Edelbaum[27] in the transfer from a collinear libration point to a lunar obit, and by Ocampo[28] in the problem of insertion into distant retrograde orbits.

The use of primer vector theory in the four body problem, which is required for Lunar WSB transfers, has been less comprehensive. In one example, Pu and Edelbaum[29] created a numerical method to compute 2-impulse and 3-impulse optimal transfers in the four body problem. Once the trajectories were numerically optimized, the primer vector history of the transfers was evaluated to verify their optimality.

1.3 Motivation

The goal of this research is to develop tools to aid in the construction and optimization of BLCTs. Though the work in this dissertation is limited to the Earth-Moon system, it applies to any Planet-Moon transfer. Moon-Moon transfers are not considered due to the presence of another body, creating a 5-body system and placing the topic outside the scope of this research. The goal of the research into construction of BLCTs is to design algorithms that are fast, robust, and do not require the overhead of manifold computation. It is desired to use the more traditional methodology of using a simplified model to produce reference trajectories and then applying numerical targeting algorithms to converge to solutions in the full model. For the purposes of BLCT's the Earth-Sun CRTBP is considered a simplified model, and low energy transfers in the CRTBP are sought as reference trajectories for the full four body model. Furthermore, the optimality of these transfers is to be considered through the use of primer vector theory, a field that has to this point not been applied to ballistic capture trajectories.

Due to their complex nature, traditional methods for trajectory design and evaluation are problematic in their application to BLCTs. One such problem is the inability to use a two body transfer, such as the Hohmann transfer, as a reference orbit when designing a mission.[12] In a traditional mission design, these simple transfers are incrementally adjusted in more complicated systems to create real-world transfers. The complicated dynamics of the four body problem, however, present problems when applying this approach to the BLCT. The simple transfers do not contain the complexity needed to converge to a solution in the more complex system.

A second problem is that the traditional tool for evaluating the optimality of transfers, primer vector theory, has never been applied to a problem where a spacecraft inserts into a captured orbit ballistically. New conditions must be derived from primer vector theory to analyze the BLCT.

The BLCTs designed by Belbruno and Miller require the dynamics of a four-body system.[2] The Earth, the Moon and the Sun each significantly affect the transfer, and are required for the observation of a ballistic capture. One consequence of working in a four body problem is the existence of chaotic dynamics in the system. In fact, BLCTs necessarily travel through chaotic regions in phase space. The presence of chaos complicates numerical analysis of BLCTs due to the unpredictability of the consequences of slight alterations to the spacecraft's state. Convergence to any target becomes difficult to achieve, and highly accurate initial guesses are required for any numerical targeting routine.

In order to deal with the issues related to the four body problem, this dissertation proposes a new method of observing and analyzing low energy transfers in a three body problem. Periodic orbits around the Earth in the Earth-Sun restricted three body problem are shown to display similar characteristics to the BLCT. These orbits are used to create low energy transfers in the three body problem that offer savings potential compared to traditional direct transfers. The simplification of the force model provides a system where convergence is easier to achieve and that can be used to base the analysis of BLCTs.

The low energy transfers in the three body problem are shown to produce very good initial guesses for targeting algorithms in the four body problem.[29] A method is described for a robust automated nonlinear targeting algorithm for real-world transfers in the four body problem. The nonlinear targeting algorithm overcomes the obstacles of the chaotic dynamics by employing accurate initial guesses based on the three body solutions and also by using precise derivatives based on the integration of the spacecraft's state transition matrix.

The second uncommon feature of a BLCT is that they contain a single impulse, but still terminate in an orbit about a different body. This feature produces a unique situation when the optimality of the transfer is being evaluated. A tool that has been traditionally used to verify the local optimality of a spacecraft transfer, primer vector theory, has never been applied to such a situation. New necessary conditions for an optimal transfer are derived and presented here using primer vector theory to account for the unique transfer architecture. The necessary conditions are then applied to examples of BLCTs and the local optimality verified.

1.4 CONTRIBUTIONS

The analysis contained in this dissertation is based in part on two previous papers[30,31] which discuss the targeting and optimization of BLCTs. Additionally, it expands on their conclusions and presents original theory not considered in the papers. The contributions from the papers are found in two chapters. The first paper (Reference 30) is summarized in chapter four, and the second paper (Reference 31) contains significant portions of the analysis presented in chapter five. A third paper will be submitted for review and publication based on the contents of chapter six. Summaries of the two previously published papers are presented below.

Reference 30 presents the first documentation of the application of periodic orbits that are centered on the Earth in the Earth-Sun three body problem to BLCTs. A single family from the catalog of periodic orbits given by Markellos[32] is shown to contain arcs that mimic the behavior of BLCTs. These arcs are exploited in a nonlinear targeting algorithm to produce BLCTs in an automated way for an arbitrary launch date. The results of the nonlinear targeting algorithm are quantified by supplying random launch dates to the algorithm and judging the usefulness of the resulting trajectories.

Theory is presented in Reference 31 for the application of primer vector theory to ballistically captured transfer trajectories. Necessary conditions are derived from the Euler-Lagrange necessary conditions in optimal control theory. The necessary conditions are applied to BLCTs, and an example trajectory is shown to meet the necessary conditions of optimality. A conclusion from this work is the understanding that for certain BLCTs the cost cannot be reduced by applying an intermediate impulse.

1.5 Dissertation Organization

This dissertation is organized into seven chapters and three appendices. The first chapter provides a definition of the topic and motivation for the research. Previous research of other authors is summarized, and the contributions to this dissertation are discussed. The second chapter introduces a new method for approximating BLCTs in a simplified model. Previously documented periodic orbits in the Earth-Sun system are shown to contain arcs that display some of the same characteristics as a BLCT. These periodic orbits are used to create optimal low energy orbit-to-orbit transfers in the Earth-Sun three body problem, which are shown to be less costly when compared to optimal direct transfers under certain conditions. The third chapter provides analysis that demonstrates the relationship between the three body low energy transfers and BLCTs in the four body problem. The fourth chapter exploits this relationship to create a robust nonlinear targeting algorithm for BLCTs that would be practical for use in real-world missions. The fifth chapter begins the analysis of the optimality of transfers that are ballistically captured at the transfer's termination. Primer vector theory is applied to the problem and new necessary conditions for optimality are given. The sixth chapter applies the necessary conditions of optimality to BLCTs with various fixed transfer times. The analysis results in a quantitative relationship between transfer time and fuel cost. The final chapter summarizes the results of the research and offers conclusions.

The first appendix provides the derivatives that were used in all of the numerical targeting and optimization routines. The second appendix tabulates the initial conditions of the three body low energy orbit-to-orbit transfers and their direct transfer counterparts that were used in the analysis in chapter two. The third appendix is a documentation of the optimal BLCTs that are presented in chapter six.

2. Dynamics of Three Body Low Energy Transfers

2.1 Introduction

The BLCT relies on the chaotic dynamics of the four body problem.[4] The WSB itself is a chaotic region in phase space where small changes in the state of a spacecraft produce unpredictable changes in the trajectory of the spacecraft. The transfer from low Earth orbit to lunar orbit via the WSB requires the spacecraft to interact with both the Earth-Sun WSB and Earth-Moon WSB. This requirement poses problems in the targeting and analysis of BLCTs because it excludes the customary practice of analyzing spacecraft trajectories in simple models that are relatively well understood and then using iterative techniques to adjust the orbits as more elements of the real-world force field are added.

Simpler models may not contain the necessary features to produce BLCTs, but they can be helpful in understanding the nature of the cost savings provided by BLCTs. In fact, with an appropriate understanding of the simplified dynamics, coarse initial guesses for ballistic lunar capture nonlinear targeting algorithms may be produced in the Earth-Sun three body problem. This chapter will introduce the analysis in the three body problem that achieves both a greater understanding of the nature of the savings generated from WSB transfers and a method of generating initial guesses for targeting the transfers in the four body problem.

The Earth-Sun CRTBP is useful in the analysis of BLCTs for two important reasons. First, the CRTBP is a relatively well studied dynamical system. Szebehely, Broucke, Hénon, Markellos, and many others have produced multiple works on the numerical study of the CRTBP.[33,34,35,36,37] Included in these studies are catalogs of periodic orbits in the problem, both in the vicinity of the primary body and in the vicinity of the secondary body. Orbits within these catalogs have been shown to display many features of the CRTBP. Second, the Δv savings in the BLCT is, as will be shown, primarily due to the presence of the solar gravitational force. Markellos's catalog includes periodic orbits that demonstrate the solar effect on orbits containing arcs which are similar to BLCTs. By studying these periodic orbits and the arcs contained within them, an initial analysis of the savings one can achieve through a BLCT can be made.

2.2 THE CIRCULAR RESTRICTED THREE BODY PROBLEM

The analysis of BLCTs begins in the CRTBP.[33] This problem describes the motion of an infinitesimally small mass through space. Its motion is perturbed by the gravitational attraction of two other bodies: a primary point mass and a secondary point mass that is less massive than or equally massive to the mass of the primary. The primary mass and secondary mass are in unperturbed circular orbits around their common center of mass.

A coordinate system is established with an origin collocated with the secondary body. It has a rotation rate that fixes the x-axis to the line connecting the primary mass and the secondary mass. The units of the coordinate system are scaled such that the distance between the primary and secondary masses is unitary and the rotation rate of the coordinate system is one rotation per time unit. Finally, the gravitational parameter is scaled such that the mass parameter of the secondary body is equal to μ and the mass parameter of the primary body is 1-μ. Under this scaling convention, μ is defined to be

$$\mu = \frac{\mu_s}{\mu_p + \mu_s}, \tag{2.2.1}$$

where μ_s and μ_p are the unscaled mass parameters of the secondary body and the primary body, respectively. The motion of the small particle through this system is governed by the following equation,

$$\ddot{\mathbf{r}} = -\frac{1-\mu}{r_1^3}\begin{pmatrix} r_x+1 \\ r_y \\ r_z \end{pmatrix} - \frac{\mu}{r_2^3}\begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix} + 2\begin{pmatrix} \dot{r}_y \\ -\dot{r}_x \\ 0 \end{pmatrix} + \begin{pmatrix} r_x+1-\mu \\ r_y \\ 0 \end{pmatrix}, \tag{2.2.2}$$

where the vector $\mathbf{r}$ is the position vector in the CRTBP coordinates described above. The CRTBP has a well known integral of motion, the Jacobi integral, shown in Eq. (2.2.3).

$$c = \left(r_x+1-\mu\right)^2 + r_y^{\ 2} + 2\frac{1-\mu}{r_1} + 2\frac{\mu}{r_2} - \left(\dot{r}_x^{\ 2} + \dot{r}_y^{\ 2} + \dot{r}_z^{\ 2}\right) \tag{2.2.3}$$

The gravitational parameters and scaling of the CRTBP in this document pertain to the Earth-Sun system. The intent of the analysis of the three body problem is to provide insights into the four body problem that includes the Moon. To gain as much accuracy as possible in the three body simplification of the four body problem, the gravitational parameter of the secondary body in Eq. (2.2.1) is modified to include the mass of the Moon as well as the Earth. The location of this combined mass is approximated to be a constant one astronomical unit from the location of the Sun. The period of the rotation of the frame is calculated using Eq. (2.2.4).

$$T = \sqrt{\frac{1\ \mathrm{au}^3}{\mu_e + \mu_s + \mu_m}} \tag{2.2.4}$$

The mass parameters used in the numerical analyses are shown in Table 1.

Table 1. Mass Parameter Values

Mass Parameter	Value (km^3/sec^2)
μ_s	1.32715×10^{11}
μ_e	3.986004×10^{5}
μ_m	4.9029×10^{3}

2.3 PERIAPSE RAISING PERIODIC ORBITS

Unlike the two body problem, there is no general solution to the equations of motion (Eq.(2.2.2)) in the CRTBP.[33] In past years, a great deal of research has focused on developing tools for overcoming the lack of an explicit solution and gaining an understanding the dynamics of the problem. Examples of the tools and theory developed for understanding the CRTBP include mapping and cataloging periodic orbits,[32,34-37] mapping zero velocity surfaces of the Jacobi constant to define constraints on the small particle's motion,[33] and applying dynamical systems theory to define additional constraints on the motion of a particle that travels through the neighborhood of periodic orbits about the libration points.[12-17]

The dynamical systems theory approach that has often been used when analyzing BLCTs differs from the classical method of using periodic orbits to analyze the CRTBP. According to the conjecture by Poincaré,[38] the population of periodic orbits in the CRTBP is dense, ensuring that for every orbit there is a periodic orbit located an infinitesimally small distance away in phase space. Thus, understanding the behavior of periodic orbits leads to a better understanding of the CRTBP. Using this classical approach, the following analysis contributes to the study of BLCTs by analyzing a specific set of related periodic orbits in the CRTBP.

The study of periodic orbits is helpful in the study of the BLCTs because of their similar characteristics. In a BLCT a spacecraft departs a low Earth orbit on a transfer trajectory that has an apogee well beyond the Moon. As this transfer trajectory returns to the Earth for its next periapse, the periapse radius has been significantly increased without the expense of a second maneuver. Much of the cost savings in the transfer can be attributed to this free increase in periapse radius, which is provided by the solar gravitational force. Similar periapse-raising transfers are observed in many different periodic orbits in the Sun-Earth CRTBP.

Of particular interest are the periodic orbits with close flybys of the secondary body that also contain periapses that have been raised to a magnitude that is on the same order as the lunar orbital radius about the Earth. The catalog of Markellos[32] has documented several of these types of orbits in the Sun-Jupiter CRTBP. Examples of the orbits are reproduced in the Earth-Sun system in Figure 3-Figure 8. Arcs of these periodic orbits produce orbit-to-orbit transfers in the three body problem that are advantageous in terms of fuel costs when compared to direct transfers. These arcs lay the foundation for the construction of BLCTs in the four body problem and demonstrate their capacity for fuel savings.

2.3.1 Generating Families from Markellos

The periodic orbits in the CRTBP used in this document will be referred to by their family name according to the catalog of Markellos. Markellos's naming convention consists of three separate parts. The first element of the name is a letter that refers to the geometry of the generating orbit. The family *f* is of interest in this study, as this family was generated with orbits around the secondary body of the system. The second element of the name is a number that refers to the count of rotations of an elliptic orbit in the two body problem which is used to generate the periodic orbit in the three body problem. The third and final element of the name is a number that refers to the order of the orbit, and is closely correlated to the number of periapses in the orbit before it closes and repeats.

One orbit listed below breaks from this naming convention, the orbit *f*3. This orbit occurs at the intersection point of other families in Markellos's naming convention, and was designated *f*3 to signify its critical location.

The orbits of interest in this study are listed in Table 2 through Table 7. Each orbit has been translated from the Sun-Jupiter system that Markellos used in his catalog into the Earth-Sun system described in Eqns. (2.2.2) - (2.2.4). For each orbit family, an example orbit is shown along with a table documenting the non-dimensional orbital periods, periapse radii, and Jacobi constants of a range of family members. The common feature of all of the orbits is the existence of a close approach to the secondary body followed by a return to that body at a much larger periapse. Also, note that the periapse is raised as a spacecraft moves on an arc through the second quadrant. The geometry of the orbit becomes important in the selection of arcs for initial guesses in the four body problem.

For each family, a related family can be formed that is reflected on the y-axis. The members of the related family are not exact reflections of their counterparts, but are geometrically similar and can be computed easily by refining a reflection of the family. This relationship is designated by using a prime in the naming convention for the reflected family. For example, the reflection of family *f*14 will be referred to as *f'*14. Figure 2 shows the relationship between these two families.

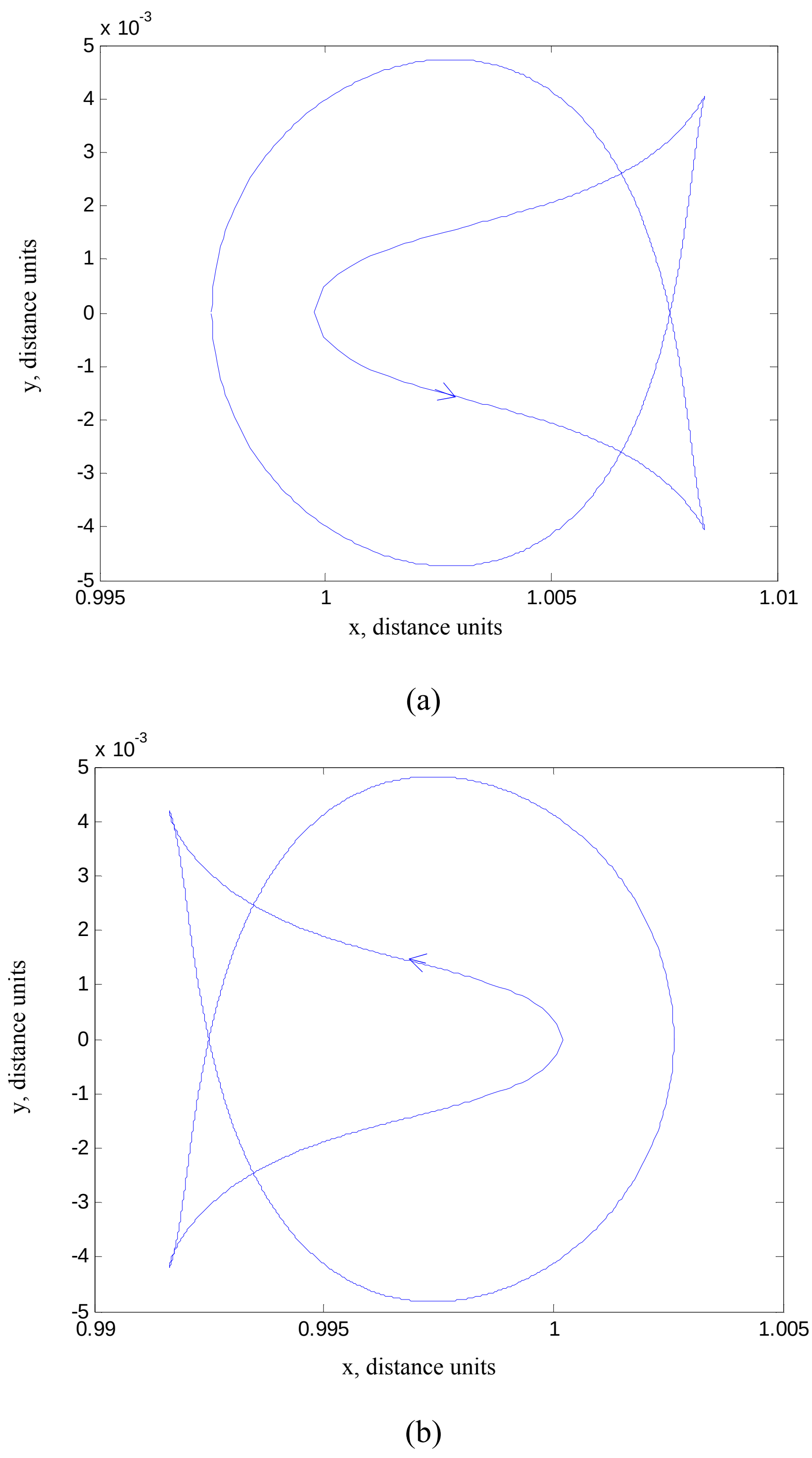

Figure 2. Members of family (a) *f'*14 and (b) *f*14 in rotating coordinates[37]

The orbits in the following tables are completely defined by the Jacobi constant and their closest periapse. They can be reproduced by placing the spacecraft on the x-axis at a distance from the secondary body equal to the closest periapse and calculating the velocity in the positive y direction based on the Jacobi constant.

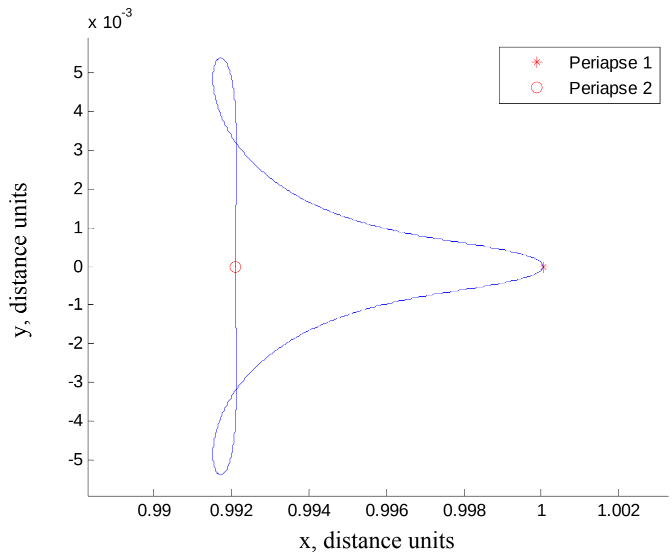

Figure 3. A member of family *f*3 shown in rotating coordinates centered on the primary body[37]

Table 2. Family *f*3

Family f3			
Jacobi constant (nondimensional)	Perigee radii (km)		Period (days)
	1	2	
3.00079515	3427	1160516	197.05
3.00079656	3766	1162956	196.80
3.00079802	4139	1165503	196.53
3.00079954	4548	1168160	196.26
3.00080112	4998	1170932	195.96
3.00080277	5492	1173823	195.66
3.00080447	6036	1176836	195.33
3.00080624	6633	1179975	194.99
3.00080981	7945	1186350	194.28
3.00081160	8660	1189569	193.92
3.00081344	9439	1192906	193.53
3.00081534	10288	1196362	193.13
3.00081730	11214	1199941	192.70
3.00081932	12224	1203643	192.25
3.00082139	13324	1207471	191.77
3.00082351	14523	1211425	191.27
3.00082569	15830	1215506	190.73
3.00082793	17255	1219714	190.16
3.00083022	18808	1224047	189.56
3.00083255	20500	1228505	188.92
3.00083494	22345	1233082	188.24
3.00083737	24356	1237776	187.50
3.00083984	26548	1242578	186.72
3.00084234	28938	1247481	185.88
3.00084488	31542	1252472	184.97

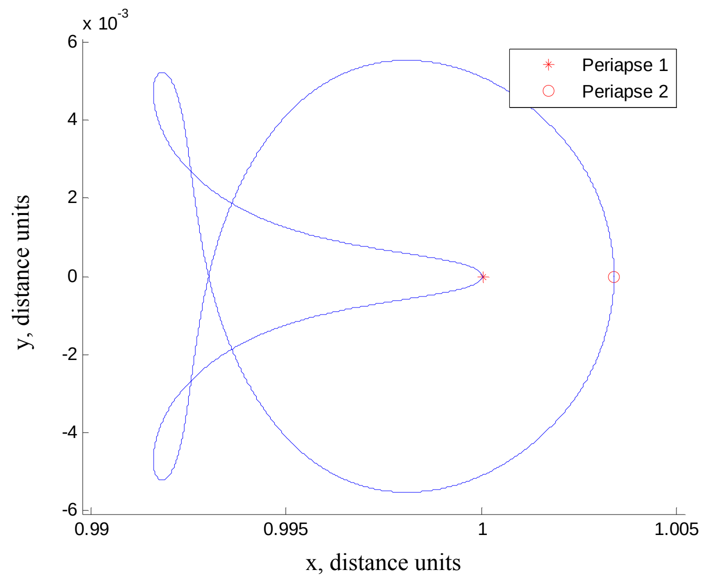

Figure 4. A member of family *f*14 shown in rotating coordinates centered on the primary body.[37]

Table 3. Family *f*14

Jacobi constant (nondimensional)	Perigee radii (km)		Period (days)
	1	2	
3.00079951	3427	546384	302.01
3.00080093	3766	542791	301.30
3.00080241	4139	539000	300.57
3.00080395	4548	534998	299.82
3.00080555	4998	530776	299.04
3.00080720	5492	526323	298.24
3.00080892	6036	521628	297.42
3.00081071	6633	516678	296.57
3.00081430	7945	506458	294.88
3.00081610	8660	501212	294.04
3.00081796	9439	495717	293.19
3.00081988	10288	489965	292.32
3.00082185	11214	483948	291.43
3.00082388	12224	477656	290.53
3.00082596	13324	471083	289.61
3.00082810	14523	464221	288.68
3.00083029	15830	457062	287.73
3.00083254	17255	449600	286.78
3.00083483	18808	441830	285.81
3.00083718	20500	433745	284.84
3.00083957	22345	425342	283.86
3.00084200	24356	416617	282.88
3.00084447	26548	407568	281.89
3.00084698	28938	398193	280.91
3.00084952	31542	388492	279.93

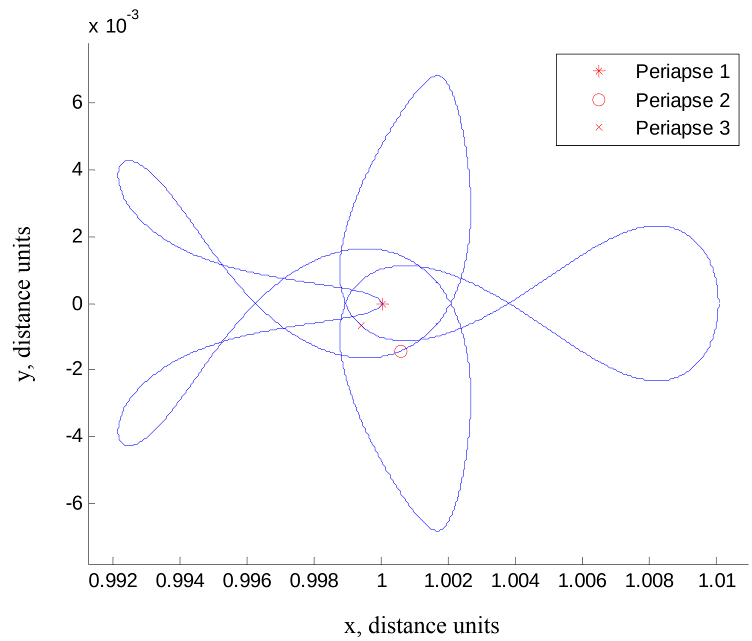

Figure 5. A member of family *f*16 shown in rotating coordinates centered on the primary body.[37]

Table 4. Family *f*16

Family f16				
Jacobi constant (nondimensional)	Perigee radii (km)			Period (days)
	1	2	3	
3.00084872	5492	218915	121724	374.00
3.00084985	6036	222987	125379	375.51
3.00085101	6633	227313	129300	377.16
3.00085217	7289	231912	133511	378.96
3.00085332	7945	236373	137637	380.73
3.00085446	8660	241100	142056	382.67
3.00085562	9439	246115	146796	384.77
3.00085679	10288	251442	151890	387.06
3.00085798	11214	257109	157377	389.57
3.00085917	12224	263148	163302	392.33
3.00086038	13324	269596	169720	395.37
3.00086158	14523	276497	176696	398.74
3.00086277	15830	283906	184309	402.50
3.00086394	17255	291891	192661	406.71
3.00086509	18808	300538	201881	411.47
3.00086618	20500	309964	212144	416.89
3.00086720	22345	320334	223693	423.14
3.00086812	24356	331900	236891	430.47
3.00086887	26548	345079	252335	439.25
3.00086936	28938	360684	271153	450.22
3.00086935	31542	380768	296134	465.09

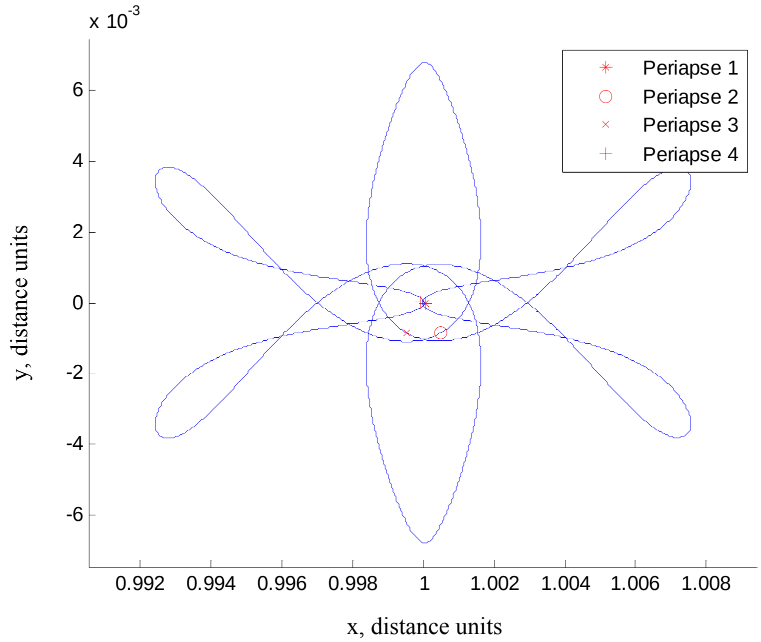

Figure 6. A member of family *f*17 shown in rotating coordinates centered on the primary body.[37]

Table 5. Family *f*17

Family f17					
Jacobi constant (nondimensional)	Perigee radii (km)				Period (days)
	1	2	3	4	
3.00087601	7230	147508	145040	8190	372.04
3.00087686	7880	151036	148540	8894	373.06
3.00087772	8589	154780	152255	9661	374.16
3.00087859	9362	158754	156199	10496	375.31
3.00087947	10205	162980	160393	11406	376.58
3.00088035	11124	167477	164856	12396	377.93
3.00088123	12125	172269	169614	13474	379.38
3.00088210	13216	177384	174693	14649	380.95
3.00088297	14405	182854	180124	15930	382.64
3.00088382	15702	188714	185944	17325	384.48
3.00088465	17115	195009	192195	18847	386.49
3.00088545	18655	201788	198929	20506	388.68
3.00088621	20334	209117	206209	22316	391.09
3.00088693	22164	217074	214112	24291	393.74
3.00088758	24159	225760	222740	26448	396.70
3.00088816	26333	235311	232226	28805	400.00
3.00088863	28703	245916	242756	31383	403.73
3.00088897	31287	257857	254609	34207	407.99
3.00088915	34103	271586	268231	37307	412.95
3.00088909	37172	287941	284444	40723	418.83
3.00088870	40517	308846	305142	44519	426.07
3.00088779	44164	342336	338225	48864	435.50
3.00088581	48139	364807	374896	57810	449.63
3.00088102	52471	355688	386379	70551	475.86

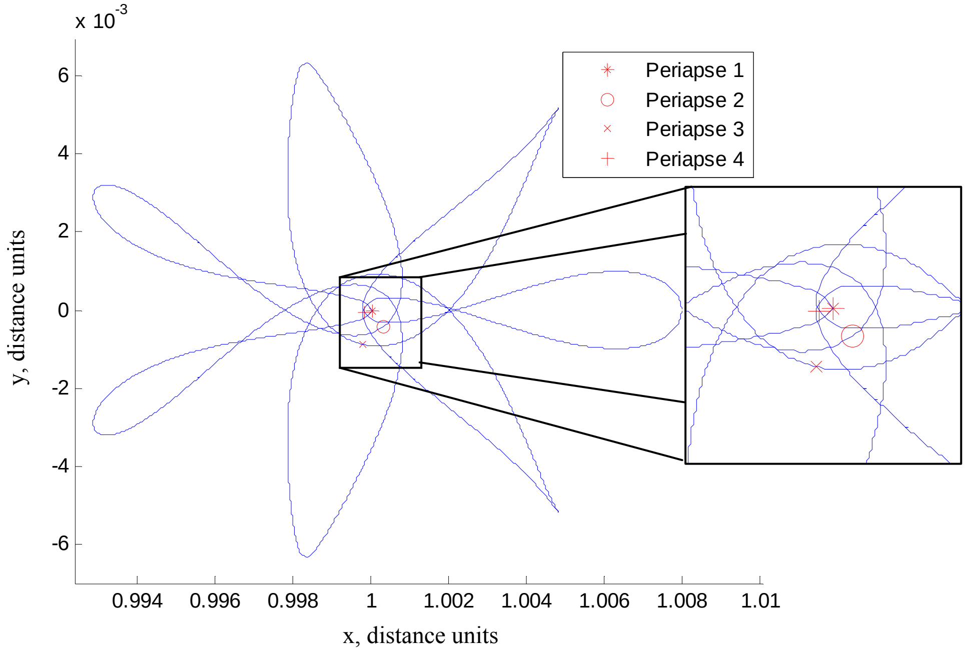

Figure 7. A member of family *f*18 shown in rotating coordinates centered on the primary body with a zoom near the primary body[37]

Table 6. Family *f*18

Family f18					
Jacobi constant (nondimensional)	Perigee radii (km)				Period (days)
	1	2	3	4	
3.00091721	7945	83017	138947	26042	368.66
3.00091815	8660	85465	142228	27379	369.44
3.00091910	9439	88058	145694	28811	370.28
3.00092007	10288	90806	149358	30345	371.18
3.00092104	11214	93720	153234	31991	372.14
3.00092202	12224	96813	157336	33756	373.17
3.00092301	13324	100096	161680	35650	374.28
3.00092400	14523	103585	166282	37685	375.47
3.00092498	15830	107293	171164	39871	376.76
3.00092595	17255	111239	176344	42223	378.15
3.00092691	18808	115440	181847	44752	379.66
3.00092784	20500	119917	187698	47477	381.30
3.00092874	22345	124694	193928	50412	383.08
3.00092959	24356	129795	200567	53578	385.03
3.00093039	26548	135251	207655	56996	387.16
3.00093111	28938	141094	215234	60689	389.51
3.00093174	31542	147363	223355	64686	392.10
3.00093227	34381	154103	232080	69017	394.97
3.00093266	37475	161370	241480	73718	398.19
3.00093287	40848	169230	251647	78833	401.80
3.00093288	44524	177767	262698	84415	405.90
3.00093263	48532	187096	274787	90529	410.62
3.00093238	52899	197371	288133	97265	416.11

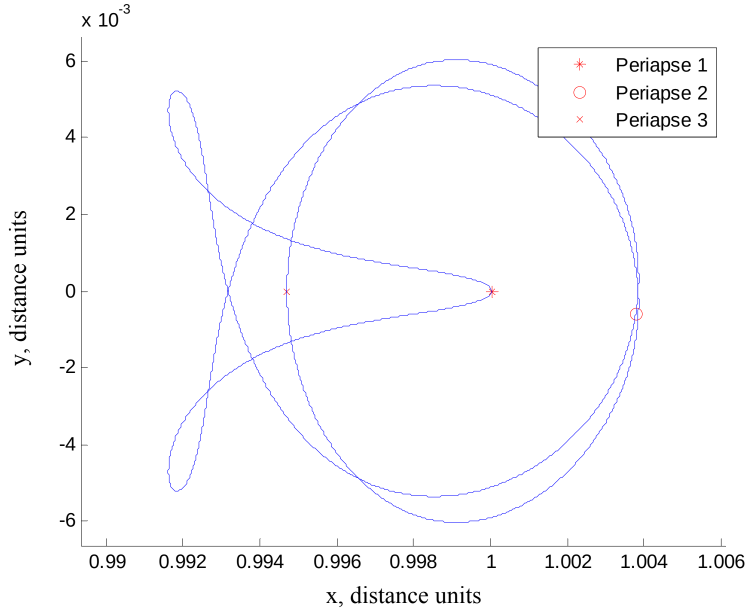

Figure 8. A member of family *f*25 shown in rotating coordinates centered on the primary body.[37]

Table 7. Family *f*25

Family f25				
Jacobi constant (nondimensional)	Perigee radii (km)			Period (days)
	1	2	3	
3.00085722	7653	571860	795944	395.10
3.00085872	8036	569165	797537	394.20
3.00086022	8437	566385	799196	393.29
3.00086172	8859	563520	800926	392.36
3.00086321	9302	560566	802728	391.43
3.00086470	9767	557522	804607	390.48
3.00086617	10256	554385	806565	389.52
3.00086764	10769	551154	808606	388.56
3.00086909	11307	547825	810733	387.58
3.00087052	11872	544399	812951	386.59
3.00087194	12466	540871	815263	385.60
3.00087333	13089	537241	817672	384.59
3.00087469	13744	533506	820183	383.58
3.00087602	14431	529664	822801	382.56
3.00087732	15152	525714	825528	381.54
3.00087857	15910	521654	828370	380.50
3.00087978	16706	517483	831332	379.47
3.00088094	17541	513198	834417	378.43
3.00088204	18418	508799	837630	377.39
3.00088309	19339	504284	840976	376.34
3.00088406	20306	499652	844461	375.30
3.00088496	21321	494902	848089	374.25
3.00088577	22387	490032	851865	373.21
3.00088650	23506	485043	855794	372.17
3.00088712	24682	479932	859883	371.13
3.00088763	25916	474700	864136	370.10
3.00088803	27212	469347	868558	369.07
3.00088829	28572	463872	873157	368.06
3.00088841	30001	458274	877937	367.05
3.00088837	31501	452554	882905	366.05
3.00088817	33076	446712	888066	365.07
3.00088790	34730	440748	893427	364.10
3.00088865	36466	434663	898995	363.16

Table 8. (continued)

Family f25				
Jacobi constant (nondimensional)	perigee radii (km)			period (days)
	1	2	3	
3.00084256	7653	571860	795944	395.10
3.00084398	8036	569165	797537	394.20
3.00084542	8437	566385	799196	393.29
3.00084686	8859	563520	800926	392.36
3.00084832	9302	560566	802728	391.43
3.00084978	9767	557522	804607	390.48
3.00085126	10256	554385	806565	389.52
3.00085274	10769	551154	808606	388.56
3.00085423	11307	547825	810733	387.58
3.00085572	11872	544399	812951	386.59
3.00085722	12466	540871	815263	385.60
3.00085872	13089	537241	817672	384.59
3.00086022	13744	533506	820183	383.58
3.00086172	14431	529664	822801	382.56
3.00086321	15152	525714	825528	381.54
3.00086470	15910	521654	828370	380.50
3.00086617	16706	517483	831332	379.47
3.00086764	17541	513198	834417	378.43
3.00086909	18418	508799	837630	377.39
3.00087052	19339	504284	840976	376.34
3.00087194	20306	499652	844461	375.30
3.00087333	21321	494902	848089	374.25
3.00087469	22387	490032	851865	373.21

2.4 OPTIMAL ORBIT-TO-ORBIT TRANSFER IN THE THREE BODY PROBLEM

The free increase in the periapse radii that is contained in the periodic orbits shown above provides a potential for savings when transferring a spacecraft from one circular orbit to another. The change in the orbital parameters on account of the solar effect has been studied and approximately quantified by previous authors.[19] The solar effect has also been used to construct optimal transfers that include plane changes.[39]

The observation of a free increase in perigee along a ballistic arc motivates the construction of an orbit-to-orbit transfer that exploits this increase. Such a transfer can be created by inserting a spacecraft from a circular parking orbit into a periodic transfer orbit which has a periapse coinciding with the circular orbit. The spacecraft then follows the periodic orbit until it arrives at another periapse, at which point a second maneuver is applied that circularizes the orbit and concludes the transfer. The effect of the solar influence is that the second maneuver is considerably smaller than it would have been had the spacecraft been transferred along a direct orbit. This result is consistent with low energy transfers observed in other problems, for example the transfer of a spacecraft between libration points.[3] In this dissertation, a transfer constructed in this manner will be referred to as a low energy orbit-to-orbit transfer in the three body problem.

The optimality of these low energy orbit-to-orbit transfers can be evaluated in two ways. First, the local optimality of the transfers will be tested by applying primer vector theory to the problem. Necessary conditions for locally optimal orbit-to-orbit impulsive transfers will be applied, and if the conditions can be satisfied, then their local optimality will be validated. Secondly, the Δv required for the transfers will be compared to optimal direct transfers computed based on Hohmann transfers. This comparison will allow a threshold to be established that will indicate the point at which low energy transfers become cost-effective relative to direct transfers.

In Section 2.4, the analysis for the comparison begins with the introduction of the necessary conditions of optimality for a two impulse orbit-to-orbit transfer. The methods of constructing optimal low energy transfers and direct transfers are then described. Finally, a quantitative comparison of optimal low energy transfers and direct transfers is made through numerical analysis of a catalog of low energy transfers and their direct transfer counterparts.

2.4.1 Lawden's Conditions for Orbit-to-orbit Transfer

Consider the orbit-to-orbit transfer problem for an impulsively maneuvering spacecraft in a general force field. The derivation of the necessary conditions for an optimal control of this spacecraft is based on Hull's formulation of the optimal control problem, and has equivalent results to those of Lawden, Jezewski, and Lion and Handelsman.[20-22] The derivation of the necessary conditions is presented here in order to provide a basis for comparison to other optimal control problems in subsequent chapters.

2.4.1.1 Cost function

An optimal control will minimize the total mass consumed in the two impulses allowed to the spacecraft, represented by the following performance index,

$$J = \int_{t_1^-}^{t_2^+} \frac{I_1}{c}\delta(t-t_1) + \frac{I_2}{c}\delta(t-t_2)dt\,, \tag{2.4.1}$$

where Δv_1 and Δv_2 are the magnitudes of the initial and final maneuvers, respectively, and $\delta(t-t_0)$ is an impulse function, in which the impulse occurs at time $t = t_0$. Impulse functions are used here as a way to represent a finite change in the velocity of the spacecraft due to an infinite acceleration. The function itself is a limit function defined by the relationship

$$\delta(t-t_c) = \lim_{n\to\infty} \delta_n(t-t_c) \tag{2.4.2}$$

where $\delta_n(t-t_c)$ is a delta sequence that may be defined by the following piecewise continuous function,[40]

$$\delta_n(t-t_c) = \begin{cases} 0 \text{ for } t < t_c - \dfrac{1}{2n} \\ n \text{ for } t_c - \dfrac{1}{2n} < t < t_c + \dfrac{1}{2n} \\ 0 \text{ for } t_c + \dfrac{1}{2n} < t \end{cases}. \tag{2.4.3}$$

The delta function is used to represent an impulsive maneuver as follows. The integral of the delta function has the property

$$\int_{-\infty}^{\infty} f(t)\delta(t-t_c)dt = f(t_c), \tag{2.4.4}$$

When the delta function is used to represent the impulsive maneuver, the resulting velocity change due to the maneuver can be determined explicitly using Eq. (2.4.5).

$$\begin{aligned} \Delta\mathbf{v}_c &= \int_{t_c^-}^{t_c^+} \Delta v_c \delta(t-t_c)\mathbf{l}_c dt \\ &= \int_{-\infty}^{\infty} \Delta v_c \delta(t-t_c)\mathbf{l}_c dt \\ &= \Delta v_c \mathbf{l}_c \end{aligned} \tag{2.4.5}$$

where $\mathbf{l}_c$ is a unit vector that specifies the direction of the impulse. In this way, Eq. (2.4.1) represents the combined magnitudes of the two $\Delta\mathbf{v}$'s.

2.4.1.2 Controls and constraints

The boundary conditions of the problem constrain the initial and final state of the spacecraft to lie on two pre-established orbits. Following the methodology of Ocampo,[28] two time-like variables, τ_1 and τ_2, are introduced into the problem to define a plane in phase space that contains the initial and final orbits. These variables are defined such that $\mathbf{r}(\tau_1)$ and $\mathbf{v}(\tau_1)$ lie on the initial orbit for any τ_1, and $\mathbf{r}(\tau_2)$ and $\mathbf{v}(\tau_2)$ lie on the final orbit for any τ_2. In this way, τ_1 and τ_2 are similar to the true anomaly of an orbit, or any other variable that defines a spacecrafts orientation on a particular orbit. The boundary conditions that constrain the initial state, $\boldsymbol{\theta}$, and the final state, $\boldsymbol{\Psi}$, have the following form:

$$\boldsymbol{\theta} = \begin{pmatrix} \mathbf{r}(t_1) - \mathbf{r}(\tau_1) \\ \mathbf{v}(t_1) - \mathbf{v}(\tau_1) \end{pmatrix} = \mathbf{0} \tag{2.4.6}$$

$$\boldsymbol{\psi} = \begin{pmatrix} \mathbf{r}(t_2) - \mathbf{r}(\tau_2) \\ \mathbf{v}(t_2) - \mathbf{v}(\tau_2) \end{pmatrix} = \mathbf{0}\,. \tag{2.4.7}$$

The controls in the problem, shown in Eq. (2.4.8), include the magnitude and direction of the transfer insertion maneuver at time t_1 and the orbit insertion maneuver at time t_2. Also included in the control vector are τ_1 and τ_2. Additionally, both t_1 and t_2 are free parameters.

$$\mathbf{u} = \begin{pmatrix} \Delta v_1 & \mathbf{l}_1^{\mathrm{T}} & \tau_1 & \Delta v_2 & \mathbf{l}_2^{\mathrm{T}} & \tau_2 \end{pmatrix}^{\mathrm{T}}. \tag{2.4.8}$$

A control constraint exists on the thrust direction vectors $\mathbf{l}_1$ and $\mathbf{l}_2$ so that their magnitudes are equal to one,

$$\mathbf{C} = \begin{pmatrix} \mathbf{l}_1^{\mathrm{T}}\mathbf{l}_1 - 1 \\ \mathbf{l}_2^{\mathrm{T}}\mathbf{l}_2 - 1 \end{pmatrix} = \mathbf{0}\,. \tag{2.4.9}$$

The differential constraint for the problem is the state equation

$$\dot{\mathbf{x}} = \begin{pmatrix} \dot{\mathbf{r}} \\ \dot{\mathbf{v}} \end{pmatrix} = \begin{pmatrix} \mathbf{v} \\ \mathbf{g} + \delta(t - t_1)\Delta v_1 \mathbf{l}_1 + \delta(t - t_2)\Delta v_2 \mathbf{l}_2 \end{pmatrix} \tag{2.4.10}$$

where $\mathbf{r}$ and $\mathbf{v}$ are the position vector and the velocity vector of the spacecraft, respectively. The gravitational acceleration vector $\mathbf{g}$ represents a general force field, and no assumptions about the composition of the gravity field will be made. The acceleration vector for the spacecraft consists of the summation of the gravitational acceleration vector and the acceleration vector of the impulsive maneuvers.

The endpoint function, G, and the Hamiltonian, H, of the problem are

$$G = \boldsymbol{\upsilon}^T \boldsymbol{\psi} + \boldsymbol{\xi}^T \boldsymbol{\theta} \tag{2.4.11}$$

$$H = \boldsymbol{\lambda}_r{}^T \mathbf{v} + \boldsymbol{\lambda}_v{}^T \left(\mathbf{g} + \Delta v_1 \delta(t - t_1) \mathbf{l}_1 + \Delta v_2 \delta(t - t_2) \mathbf{l}_2 \right) + \Delta v_1 \delta(t - t_1) + \Delta v_2 \delta(t - t_2) \tag{2.4.12}$$

where $\boldsymbol{\upsilon}$ and $\boldsymbol{\psi}$ are vectors of constant Lagrange multipliers that are adjoined to the endpoint constraints. The Hamiltonian is constant if the gravitational acceleration vector is not a function of time. For the general force field, however, the Hamiltonian is not an integral of the problem.

Because a control constraint is involved in the problem, an extended Hamiltonian, $\hat{H}$, is formed by using a vector of Lagrange multipliers, $\boldsymbol{\mu}$, to adjoin the control constraint vector to the Hamiltonian

$$\begin{aligned} \hat{H} = {} & \boldsymbol{\lambda}_r{}^T \mathbf{v} + \boldsymbol{\lambda}_v{}^T \left(\mathbf{g} + \Delta v_1 \delta(t - t_1) \mathbf{l}_1 + \Delta v_2 \delta(t - t_2) \mathbf{l}_2 \right) \\ & + \Delta v_1 \delta(t - t_1) + \Delta v_2 \delta(t - t_2) + \boldsymbol{\mu}^T \mathbf{C} \end{aligned} \tag{2.4.13}$$

The performance index and constraints described above form a time-free optimal control problem. The necessary conditions from the first differential are given by the Euler-Lagrange equations,[41]

$$\dot{\mathbf{x}} = \mathbf{f} \tag{2.4.14}$$

$$\dot{\boldsymbol{\lambda}} = -\hat{H}_{\mathbf{x}}{}^T \tag{2.4.15}$$

$$\mathbf{0} = \hat{H}_{\mathbf{u}}{}^T \tag{2.4.16}$$

and the natural boundary conditions,

$$\begin{aligned} \hat{H}_2 &= -G_{t_2} \\ \hat{H}_1 &= G_{t_1} \\ \boldsymbol{\lambda}_2 &= G_{\mathbf{x}_2}^T \\ \boldsymbol{\lambda}_1 &= -G_{\mathbf{x}_1}^T \end{aligned} \quad . \tag{2.4.17}$$

2.4.1.3 Necessary conditions

From Eq. (2.4.16), the partial derivatives of the extended Hamiltonian with respect to each of the controls in the control vector **u** must be equal to zero for an optimal control. Taking the partial derivative with respect to the unit thrust vector $\mathbf{l}_1$ gives

$$\hat{H}_{\mathbf{l}_1} = \Delta v_1 \delta(t - t_1) \boldsymbol{\lambda}_v^T + 2\mu_1 \mathbf{l}_1^T = 0 \tag{2.4.18}$$

In order for their sum to be zero, Eq. (2.4.18) dictates that for the time that the delta function is evaluated, the Lagrange multiplier vector for the velocity vector and the unit thrust vector are parallel, or

$$\mathbf{l}_1 = \pm \frac{\boldsymbol{\lambda}_{v_1}}{\left|\boldsymbol{\lambda}_{v_1}\right|} . \tag{2.4.19}$$

This result is the classical result of primer vector theory, where the reflection of the vector of Lagrange multipliers associated with the velocity has been dubbed the primer vector. Similar results can be found for the second impulse. For periods when the delta function is evaluated to be zero, the primer vector is indeterminate.

The magnitude of the primer vector at the impulse can be found by taking the partial derivative of the Hamiltonian with respect to each Δv.

$$\hat{H}_{\Delta v_1} = \delta(t - t_1) \boldsymbol{\lambda}_v^T \mathbf{l}_1 + \delta(t - t_1) = 0 \tag{2.4.20}$$

Combining Eq. (2.4.19) and Eq. (2.4.20) gives

$$\pm\left|\boldsymbol{\lambda}_v\right| \delta(t - t_1) + \delta(t - t_1) = 0 . \tag{2.4.21}$$

When this equation is evaluated at the time of the maneuver, it can be reduced to a form that gives the magnitude of the primer vector.

$$\pm\left|\boldsymbol{\lambda}_{v_1}\right| = -1 , \tag{2.4.22}$$

or

$$\mp|\mathbf{p}_1| = -1 \tag{2.4.23}$$

Clearly, the negative value should be chosen in this equation, meaning that the vector of Lagrange multipliers is oriented in the opposite direction from the steering vector, with a unit magnitude at the time of the impulse. Similarly, the magnitude of the primer vector at the second burn is

$$-\left|\boldsymbol{\lambda}_{v_2}\right| = -1\,. \tag{2.4.24}$$

or

$$|\mathbf{p}_1| = |\mathbf{p}_2| = 1\,. \tag{2.4.25}$$

Therefore, at each impulse, the magnitudes of the primer vector at the impulse must be equal and aligned with the direction of the respective impulses.

Equation (2.4.25) dictates that the primer vector points in the same direction as the impulse. The orientation can be verified through the application of the Weierstrass condition.[41] This condition states that a minimal control must minimize the Hamiltonian when compared to acceptable neighboring controls. When the direction of the steering vector is considered with the Weierstrass condition, the following equation must hold,

$$\hat{\mathrm{H}}^* - \hat{\mathrm{H}} = \Delta v \delta(t - t_0)\left(\boldsymbol{\lambda}_v{}^T \mathbf{l}^* - \boldsymbol{\lambda}_v{}^T \mathbf{l}\right) > 0\,, \tag{2.4.26}$$

where $\mathbf{l}^*$ is a non-optimal control, and $\hat{\mathrm{H}}^*$ is the corresponding non-optimal Hamiltonian. When evaluated at the impulse, this equation reduces to

$$\left(\boldsymbol{\lambda}_v{}^T \mathbf{l}^* - \boldsymbol{\lambda}_v{}^T \mathbf{l}\right) > 0 \tag{2.4.27}$$

The control that minimizes satisfies this condition is

$$\mathbf{l} = -\frac{\boldsymbol{\lambda}_v}{|\boldsymbol{\lambda}_v|} \tag{2.4.28}$$

Therefore, the primer vector is defined to be the negative of the adjoint variable associated with the velocity to preserve the traditional result that the primer vector points in the same direction of the maneuver,

$$\mathbf{p} = -\boldsymbol{\lambda}_{\mathrm{v}} \tag{2.4.29}$$

Continuing the derivation of the necessary conditions, consider the endpoint conditions on the Hamiltonian in Eq. (2.4.17). When evaluated at time t_1 the Hamiltonian of an optimal control has the value

$$H_1 = \boldsymbol{\lambda}_{r_1}{}^{T}\mathbf{v}_1 + \boldsymbol{\lambda}_{v_1}{}^{T}\mathbf{g}_1 - \left|\boldsymbol{\lambda}_{v_1}\right|\Delta v_1 \delta(t - t_1) + \Delta v_1 \delta(t - t_1) = 0 \tag{2.4.30}$$

Combining Eq. (2.4.30) with Eq. (2.4.25) gives

$$\boldsymbol{\lambda}_{r_1}{}^{T}\mathbf{v}_1 + \boldsymbol{\lambda}_{v_1}{}^{T}\mathbf{g}_1 = 0 \tag{2.4.31}$$

Eq. (2.4.31) must remain true as you take its limit from each side of t_1. Of the variables in Eq. (2.4.31), only $\mathbf{v}_1$ is not continuous across the impulse. As a result, to maintain the continuity of the Hamiltonian, the following relationship must be satisfied:

$$\boldsymbol{\lambda}_{r_1}{}^{T}\left(\mathbf{v}_1{}^{+} - \mathbf{v}_1{}^{-}\right) = 0\,. \tag{2.4.32}$$

The primer vector has been established to be the unit vector in the negative direction of the impulse. Equation (2.4.32) becomes

$$\Delta v_1 \boldsymbol{\lambda}_{r_1}{}^{T}\boldsymbol{\lambda}_{v_1} = 0\,. \tag{2.4.33}$$

From Eq. (2.4.15), $-\boldsymbol{\lambda}_{r}$ is known to be the time derivative of the primer vector. Consequently, Eq. (2.4.33) dictates that, at the impulse, the time rate of change of the magnitude of the primer vector must be zero. Similar results can be found for the second impulse such that

$$\Delta v_2 \boldsymbol{\lambda}_{r_2}{}^{T}\boldsymbol{\lambda}_{v_2} = 0\,. \tag{2.4.34}$$

2.5 Definition of Low Energy Orbit-to-orbit Transfer in the CRTBP

The periapse-raising periodic orbits detailed in the preceding sections can be used to construct efficient, two impulse, low energy orbit-to-orbit transfers. In this section transfers from one circular orbit to another in the CRTBP are considered. The initial circular orbit is required to coincide with the periapse of the periodic orbit that lies closest to the secondary mass. The final circular orbit coincides with one of the subsequent periapses along the periodic orbit. The Δv required by these transfers are compared to that of optimal direct two impulse transfers constructed based on the Hohmann transfer.

2.5.1 Construction of Orbit-to-orbit Transfers along a Periodic Orbit

In order to define an orbit-to-orbit transfer, the definition of a circular orbit in the three body problem must be considered. Both the initial and final orbits are considered to be circular if the instantaneous eccentricity of the spacecraft's orbit about the secondary mass is zero. It is noted that this two-body condition does not necessarily give a circular orbit in the three body problem, or even an orbit that is periodic; however, at low orbital radii it provides a good approximation of circular orbits and a concrete way of comparing the two transfers.

The construction of a transfer of this type begins with the selection of the radii of the two circular orbits to be transferred between. The radii are deliberately chosen to coincide with two periapses of the periodic orbits. The circular velocities of the initial and final orbits are calculated using the two-body formula,

$$v_{circ} = \sqrt{\frac{\mu_e}{r}} . \tag{2.5.1}$$

The velocity is then scaled and rotated into the CRTBP coordinates, and the magnitudes of the two impulses are calculated by subtracting the magnitude of v_{circ} from the magnitude of the velocity of the periodic orbit at the selected periapse. The impulse occurs in the direction of the velocity vector of the spacecraft at the point where the periapse of the periodic orbit coincides with the circular orbit. The transfer time is selected such that the transfer terminates at the desired periapse of the periodic orbit, and is therefore a property of the periodic orbit.

In order to maximize the savings of the low energy transfer, a numerical optimization routine is applied to it, and its optimality is then verified through the analysis of the primer vector of the transfer. The numerical optimization problem uses a sequential quadratic programming routine[42] to achieve an optimal solution. The parameters of the routine are defined here.

A transfer is sought that minimizes the fuel cost required to move a satellite from an orbit that instantaneously satisfies the parameters of a specified two body circular orbit to a second orbit that instantaneously satisfies the parameters of a second specified circular orbit. Let **a** be the vector of free parameters of the transfer, and **c** be the vector of equality constraints that an optimal transfer must satisfy. For this problem, the parameter vector has nine dimensions,

$$\mathbf{a} = \left(\mathbf{\Delta v}_1 \quad \mathbf{\Delta v}_2 \quad \mathrm{t}_2 \quad \tau_1 \quad \tau_2\right)^{\mathrm{T}}, \tag{2.5.2}$$

where $\mathbf{\Delta v}_1$ and $\mathbf{\Delta v}_2$ are the magnitude and direction of the initial and final impulses, respectively, t_2 is the flight time of the transfer, and τ_1 and τ_2 are time-like variables that signify the departure point on the initial orbit and the arrival point on the final orbit, respectively. The constraint vector specifies that the final state lies on the targeted orbit, and is six-dimensional,

$$\mathbf{c} = \begin{pmatrix} \mathbf{r}(\tau_2) - \mathbf{r}(\mathrm{t}_2) \\ \mathbf{v}(\tau_2) - \mathbf{v}(\mathrm{t}_2) \end{pmatrix}. \tag{2.5.3}$$

A performance index *f* is defined such that the variation of the parameters in **a** minimizes *J* in such a way that **c** is satisfied. In this problem the performance index minimizes the cost of the transfer,

$$J = |\boldsymbol{\Delta v}_1| + |\boldsymbol{\Delta v}_2| . \tag{2.5.4}$$

The optimization problem defined in Eqns. (2.5.2) to (2.5.4) is numerically solved in a sequential quadratic programming algorithm.[42] The results of the numerical analyses are provided in Appendix A. The convergence of the numerical algorithm is assisted by providing precise derivatives to the routine. Partial derivatives based on the state transition matrix have been shown to be accurate to a higher order than the standard central difference derivatives, and also speed the convergence of the algorithm.[43]

The state transition matrix of the three body problem is found by numerically integrating the relationship in Eq. (2.5.5), where *F* is the time dependent state propagation matrix for the desired dynamical model, shown in Eq. (2.5.6), where the state vector **X** is defined in Eq. (2.5.7).

$$\begin{aligned} \dot{\Phi}(t,t_0) &= F(t)\Phi(t,t_0) \\ \Phi(t_0,t_0) &= I \end{aligned} \tag{2.5.5}$$

$$F(t) = \frac{\partial \dot{\mathbf{X}}(t)}{\partial \mathbf{X}(t)} \tag{2.5.6}$$

$$\mathbf{X} = \begin{pmatrix} \mathbf{r} \\ \mathbf{v} \end{pmatrix} \tag{2.5.7}$$

The state propagation matrix for the CRTBP is given in Eq. (2.5.8).

$$F(t) = \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ \left(3\frac{1-\mu}{r_1^5}\mathbf{r}_1\mathbf{r}^{\mathbf{T}} - \frac{1-\mu}{r_1^3}\mathbf{I} + \frac{\mu}{r_2^5}\mathbf{r}_2\mathbf{r}^{\mathbf{T}} - \frac{\mu}{r_2^3}\mathbf{I} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right) & \begin{pmatrix} 0 & 2 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{pmatrix} \tag{2.5.8}$$

The derivatives required by the numerical targeting routine are given in Eqns. (2.5.9) and (2.5.10).

$$\frac{\partial \mathbf{c}}{\partial \mathbf{a}} = \begin{pmatrix} -\frac{\partial \mathbf{r}_2}{\partial \mathbf{X}_2}\Phi(t_2,t_1)\frac{\partial \mathbf{X}_1}{\partial \Delta \mathbf{v}_1} & \mathbf{0} & -\dot{\mathbf{r}}_2 & -\frac{\partial \mathbf{r}_2}{\partial \mathbf{X}_2}\Phi(t_2,t_1)\frac{\partial \mathbf{X}_1}{\partial \tau_1} & \frac{\partial \mathbf{r}(\tau_2)}{\partial \tau_2} \\ -\frac{\partial \mathbf{v}_2}{\partial \mathbf{X}_2}\Phi(t_2,t_1)\frac{\partial \mathbf{X}_1}{\partial \Delta \mathbf{v}_1} & -\mathbf{I} & -\dot{\mathbf{v}}_2 & -\frac{\partial \mathbf{v}_2}{\partial \mathbf{X}_2}\Phi(t_2,t_1)\frac{\partial \mathbf{X}_1}{\partial \tau_1} & \frac{\partial \mathbf{v}(\tau_2)}{\partial \tau_2} \end{pmatrix} \tag{2.5.9}$$

$$\frac{\partial J}{\partial \mathbf{a}} = \begin{pmatrix} -\frac{1}{\Delta v_1^{\,3}}\boldsymbol{\Delta}\mathbf{v}_1^{\,T} & -\frac{1}{\Delta v_2^{\,3}}\boldsymbol{\Delta}\mathbf{v}_2^{\,T} & 0 & 0 & 0 \end{pmatrix} \tag{2.5.10}$$

where,

$$\begin{aligned}
\frac{\partial \mathbf{r}_2}{\partial \mathbf{X}_2} &= \begin{pmatrix} \mathbf{I} & \mathbf{0} \end{pmatrix}, \\
\frac{\partial \mathbf{X}_1}{\partial \Delta \mathbf{v}_1} &= \begin{pmatrix} \mathbf{0} \\ \mathbf{I} \end{pmatrix}, \\
\frac{\partial \mathbf{X}_1}{\partial \tau_1} &= \begin{pmatrix} r_1(-\sin\tau_1) \\ r_1(\cos\tau_1) \\ 0 \\ -v_1(\cos\tau_1) \\ -v_1(\sin\tau_1) \\ 0 \end{pmatrix}, \\
\frac{\partial \mathbf{v}_2}{\partial \mathbf{X}_2} &= \begin{pmatrix} \mathbf{0} & \mathbf{I} \end{pmatrix}, \\
\frac{\partial \mathbf{r}(\tau_2)}{\partial \tau_2} &= \begin{pmatrix} r_1(-\sin\tau_1) \\ r_1(\cos\tau_1) \\ 0 \end{pmatrix}, \\
\frac{\partial \mathbf{v}(\tau_2)}{\partial \tau_2} &= \begin{pmatrix} -v_2(\cos\tau_2) \\ -v_2(\sin\tau_2) \\ 0 \end{pmatrix}.
\end{aligned} \tag{2.5.11}$$

The primer vector history of an orbit to orbit transfer confirms the local optimality of the transfer if it can be shown to satisfy the necessary conditions described above. Once the trajectory has been numerically optimized with a sequential quadratic programming algorithm, its primer vector history can be calculated through the integration of the costate vector. At the time of the initial maneuver, T1, the initial value of the primer vector is set to have a magnitude of one and be oriented in the opposite direction of the impulse. The initial value of its time derivative is found by specifying the value of the primer vector at the final time and using the state transition matrix to determine the initial derivative.[28]

$$\dot{p}_1 = \Phi_{12}\left(t_2, t_1\right)^{-1}\left(p_2 - \Phi_{11}\left(t_2, t_1\right)p_1\right) \tag{2.5.12}$$

where $\Phi\left(t_2, t_1\right)$ is the state transition matrix of the CRTBP (defined in Appendix A), and

$$\Phi\left(t_2, t_1\right) = \begin{pmatrix} \Phi_{11}\left(t_2, t_1\right) & \Phi_{12}\left(t_2, t_1\right) \\ \Phi_{21}\left(t_2, t_1\right) & \Phi_{22}\left(t_2, t_1\right) \end{pmatrix}. \tag{2.5.13}$$

The primer vector automatically satisfies the conditions in Eq. (2.4.22) and Eq. (2.4.24) in the definition of this problem. A transfer can be said to satisfy the necessary conditions of an optimal control if Eqns. (2.4.33) and (2.4.34) are also satisfied. Specifically, the time derivative of the magnitude of the primer vector must be zero at the initial and final times.

2.6 Numerical Examples of Low Energy Orbit-to-orbit Transfers

Numerical results are presented here for each family of periodic orbits documented above. The results can be viewed in two separate groups. The first group, transfers in families f16, f17, and f18, leads to optimal orbit to orbit transfers. The primer vector histories have unit magnitude at the initial and final times, the time derivatives of the magnitude of the primer vector at those times is zero, and the magnitude of the primer vector never surpasses unit value. The second group, comprised of transfers from families f14, f3 and f25, is comprised of optimal two impulse transfers, but their primer vector histories reveal them to be capable of improvement with additional impulses. The necessary conditions of optimality are satisfied at the initial and final times, but the magnitude of the primer vector exceeds unit value at some point along the trajectory. As Jezeweski and others have pointed out, additional impulses can lower transfer costs in this situation.[22]

An example of orbit-to-orbit transfers for each family is provided here. Transfers to perigees in the second half of the periodic orbit are omitted due to the symmetry of the problem. Transfers of this latter type would have properties that are similar to the shown transfers, but with longer times of flight.

Each transfer below is shown in rotating coordinates, along with a figure displaying the history of the primer vector magnitude of the transfer, and a table with the parameters that define the transfer. The magnitudes of the impulses are provided along with the time of flight and the radii of the circular orbits transferred between. The angle θ shown in the table is the angle between the primer vector and the vector of its time derivative at the initial time. This angle is provided to clarify the optimality condition at the initial time because the scale of the primer vector figure is such that the slope of the curve at the initial time is deceiving.

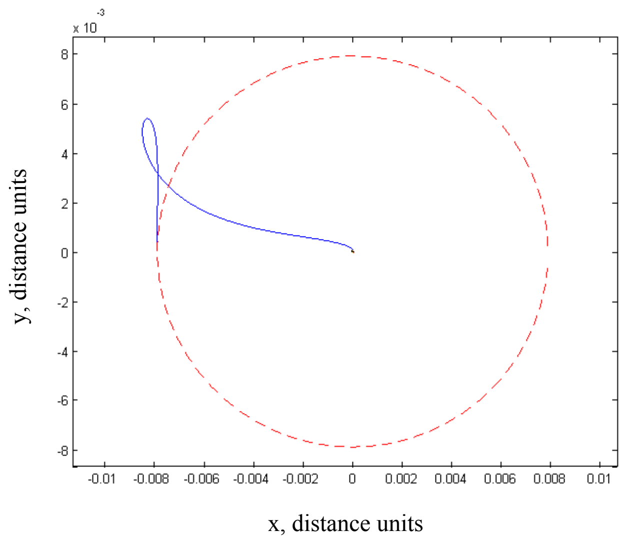

Figure 9. A *f*3p1 transfer shown in rotating coordinates centered on the secondary body

Table 8. *f*3p1 transfer details

f3p1 details					
Δv_1	Δv_2	transfer time	r_1	r_2	θ
1.025008E-01	1.260632E-05	1.667055E+00	4.859040E-05	7.888298E-03	1.569062

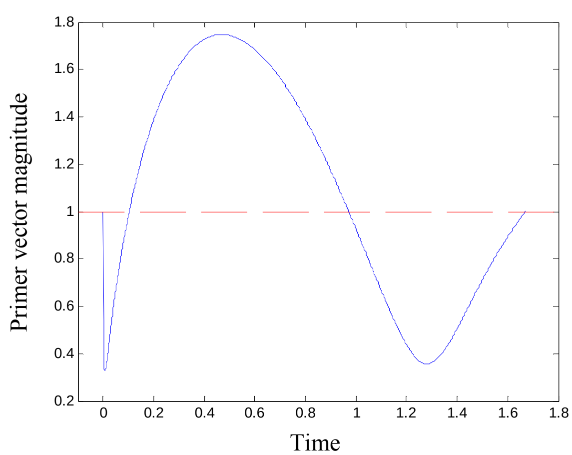

Figure 10. Primer vector magnitude of the *f*3p1 transfer

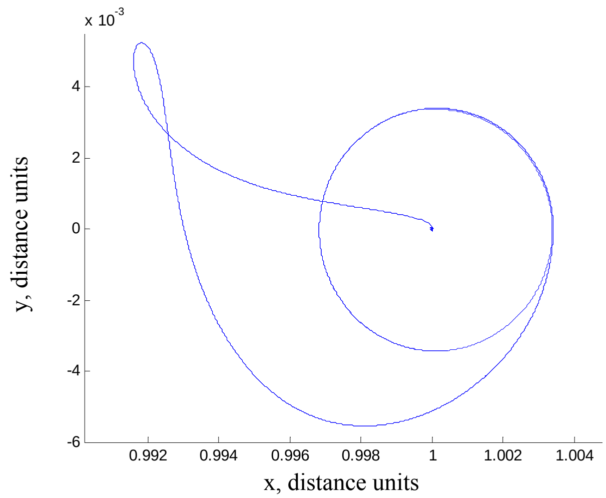

Figure 11. A *f*14p1 transfer shown in rotating coordinates centered on the secondary body

Table 9. *f*14p1 transfer properties

f14p1 details					
Δv_1	Δv_2	transfer time	r_1	r_2	θ
1.024945E-01	5.066696E-03	2.532167E+00	4.859040E-05	3.409689E-03	1.5685

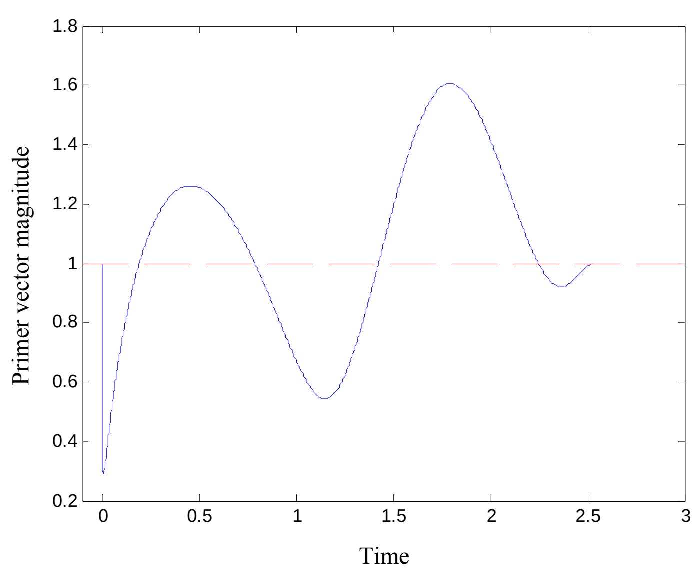

Figure 12. Primer vector magnitude of the *f*14p1 transfer

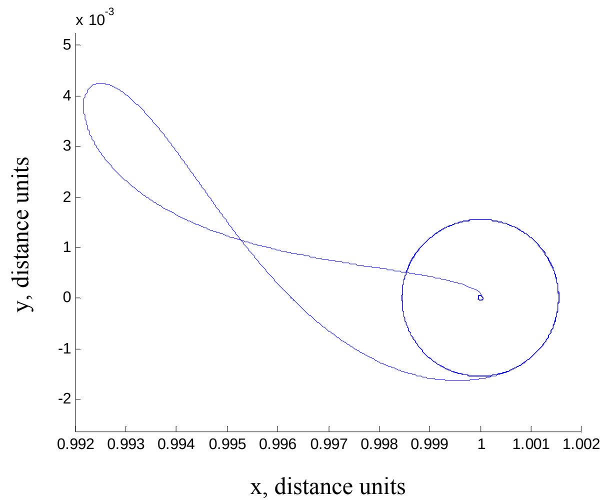

Figure 13. A *f*16p1 transfer shown in rotating coordinates centered on the secondary body

Table 10. *f*16p1 transfer properties

f16p1 details					
Δv_1	Δv_2	transfer time	r_1	r_2	θ_1 (rad)
1.024383E-01	1.259534E-02	1.340640E+00	4.859040E-05	1.547387E-03	-7.70E-06

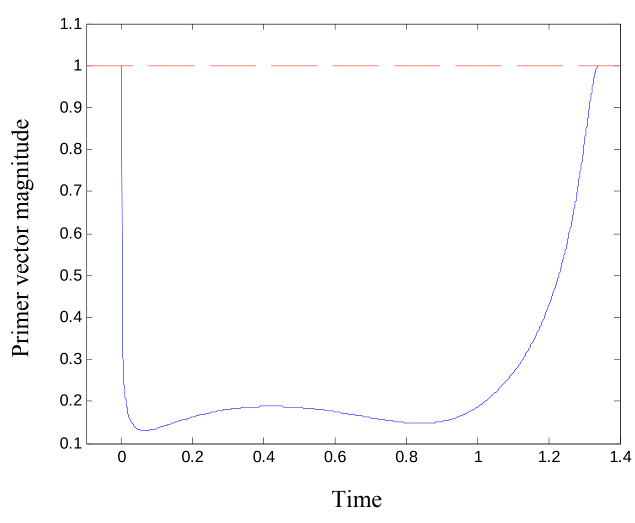

Figure 14. Primer vector magnitude of the *f*16p1 transfer

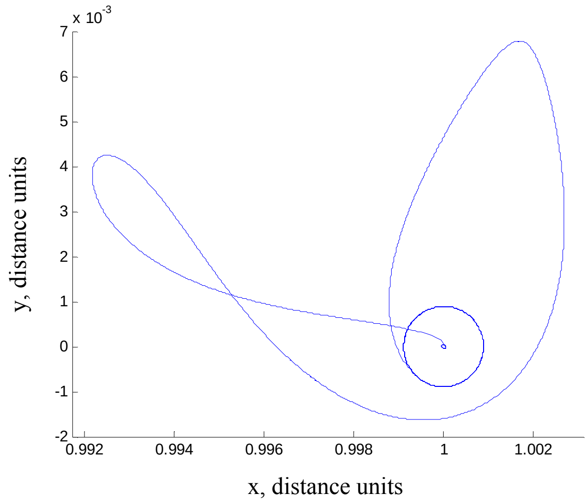

Figure 15. A *f*16p2 transfer shown in rotating coordinates centered on the secondary body

Table 11. *f*16p2 transfer properties

f16p2 details					
Δv_1	Δv_2	transfer time	r_1	r_2	θ_1 (rad)
1.024383E-01	1.968534E-02	2.237143E+00	4.859040E-05	8.910178E-04	-7.70E-06

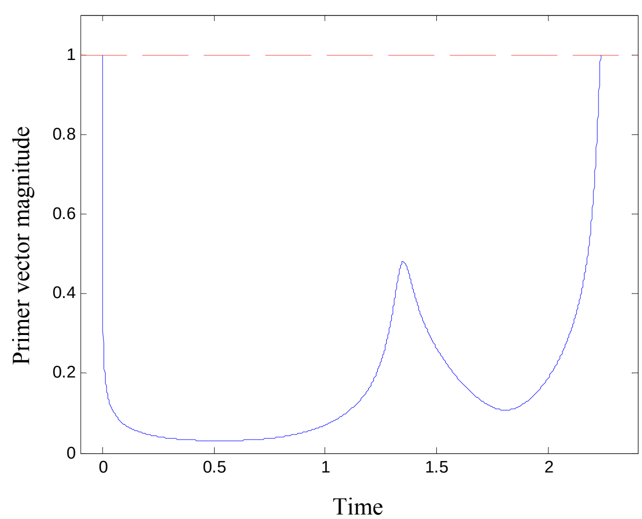

Figure 16. Primer vector magnitude of the *f*16p2 transfer

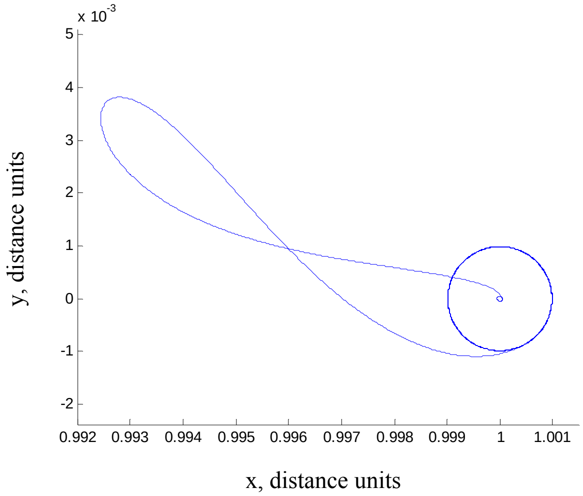

Figure 17. A *f*17p1 transfer shown in a rotating frame centered on the secondary body

Table 12. *f*17p1 transfer properties

f17p1 details					
Δv_1	Δv_2	transfer time	r_1	r_2	θ
1.024057E-01	1.812006E-02	1.180418E+00	4.859040E-05	9.862153E-04	1.5709

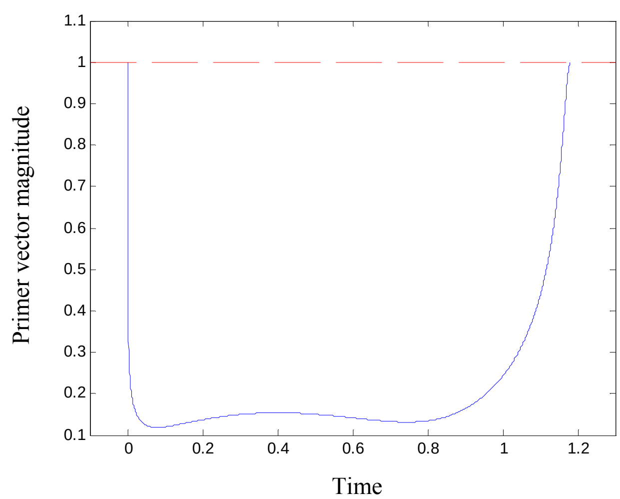

Figure 18. Primer vector magnitude of the *f*17p1 transfer

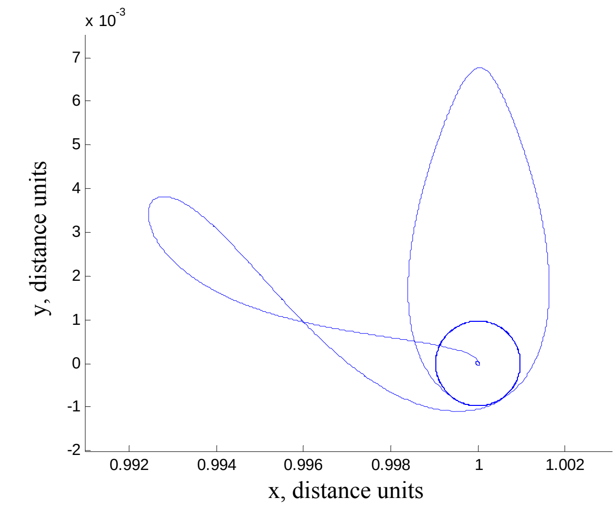

Figure 19. A *f*17p2 transfer shown in rotating coordinates centered on the secondary body

Table 13. *f*17p2 transfer properties

f17p2 details					
Δv_1	Δv_2	transfer time	r_1	r_2	θ
1.024057E-01	1.834966E-02	2.002422E+00	4.859040E-05	9.698469E-04	1.5693

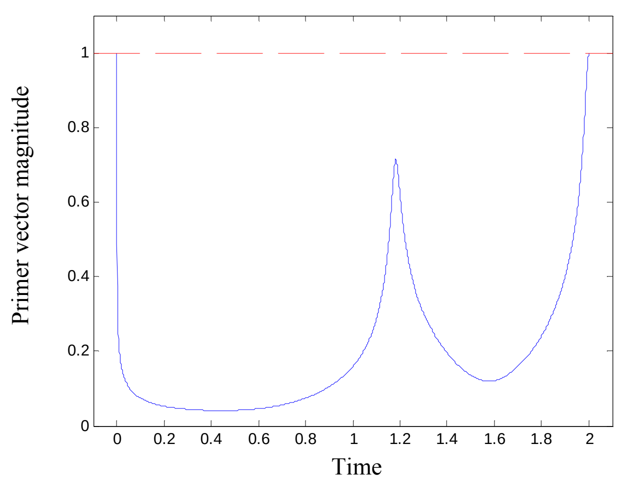

Figure 20. Primer vector magnitude of the *f*17p2 transfer

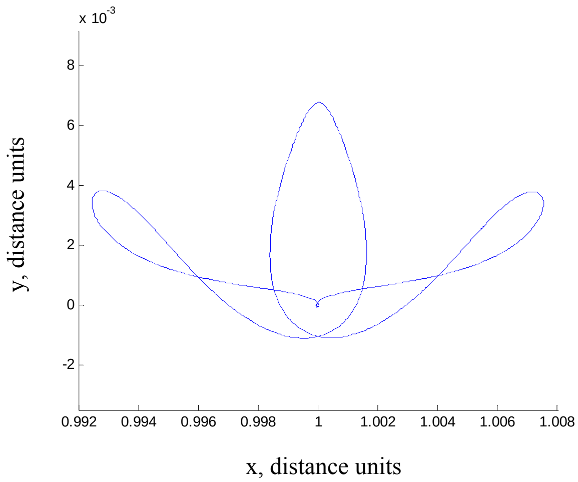

Figure 21. A *f*17p3 transfer shown in a rotating frame centered on the secondary body

Table 14. *f*17p3 transfer properties

f17p3 details					
Δv_1	Δv_2	transfer time	r_1	r_2	θ
1.024057E-01	9.583300E-02	3.178998E+00	4.859040E-05	9.698469E-04	1.5702

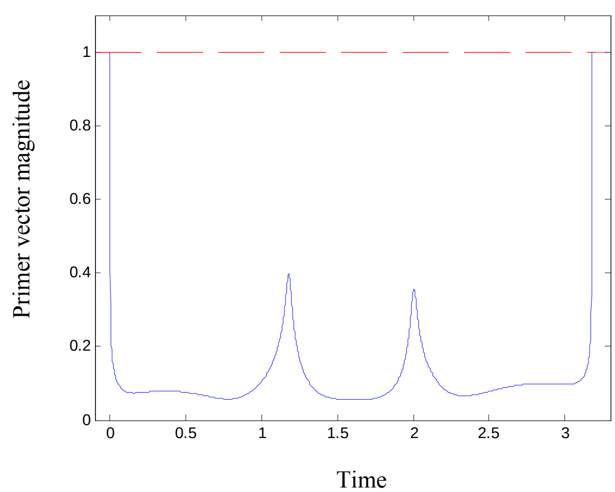

Figure 22. Primer vector magnitude of the *f*17p3 transfer

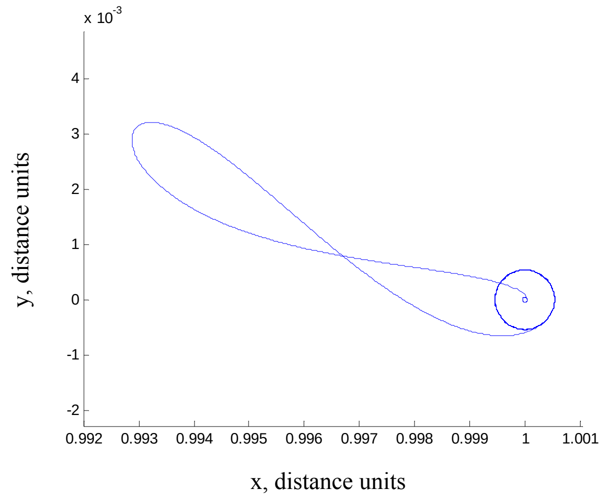

Figure 23. A *f*18p1 transfer shown in rotating coordinates centered on the secondary body

Table 15. *f*18p1 transfer properties

f18p1 details					
Δv_1	Δv_2	transfer time	r_1	r_2	θ
1.023474E-01	2.720737E-02	9.977199E-01	4.859040E-05	5.383304E-04	1.570684

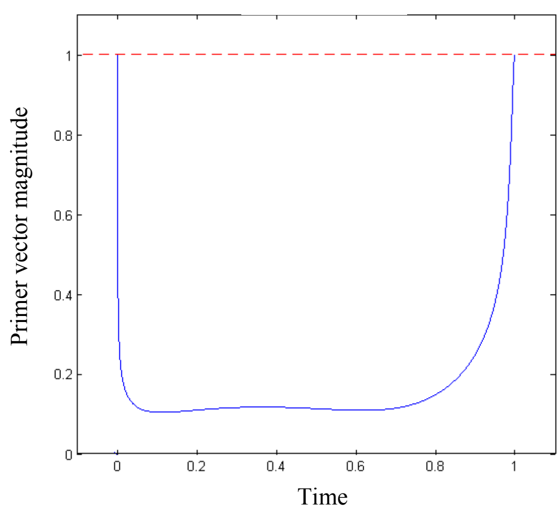

Figure 24. Primer vector magnitude of the *f*18p1 transfer

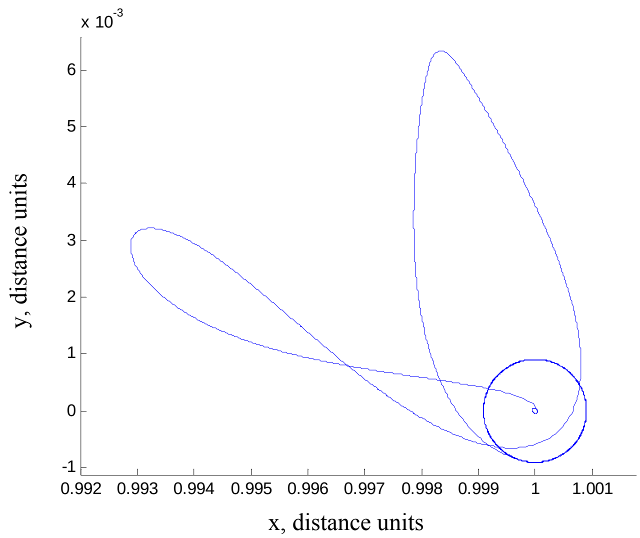

Figure 25. A *f*18p2 transfer shown in rotating coordinates centered on the secondary body

Table 16. *f*18p2 transfer properties

f18p2 details					
Δv_1	Δv_2	transfer time	r_1	r_2	θ
1.023474E-01	1.902173E-02	1.760327E+00	4.859040E-05	9.061915E-04	1.570272

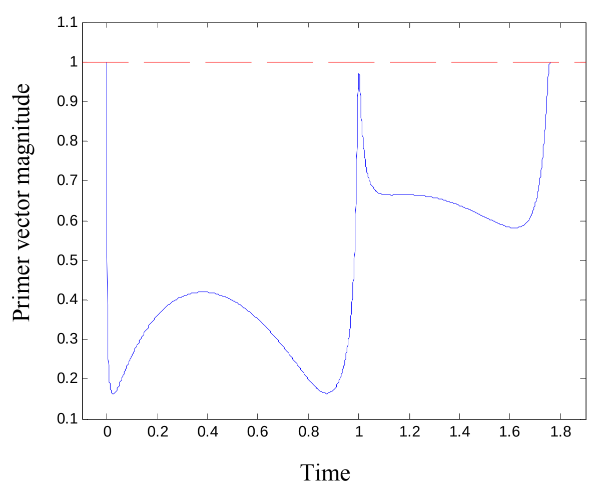

Figure 26. Primer vector magnitude of the *f*18p2 transfer

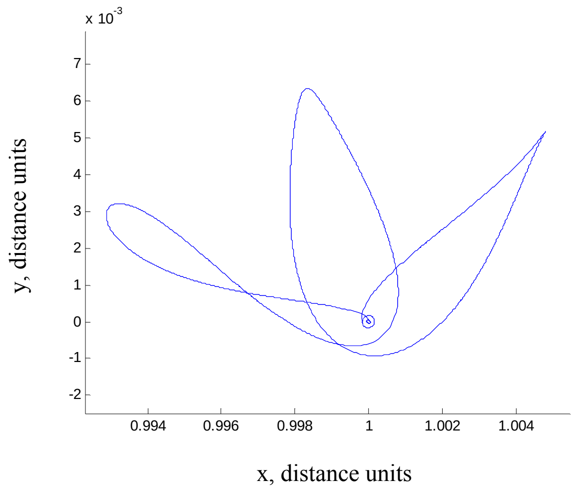

Figure 27. A *f*18p3 transfer shown in rotating coordinates centered on the secondary body

Table 17. *f*18p3 transfer properties

f18p3 details

Δv_1	Δv_2	transfer time	r_1	r_2	θ
1.023474E-01	5.384386E-02	2.616799E+00	4.859040E-05	1.656536E-04	1.568848

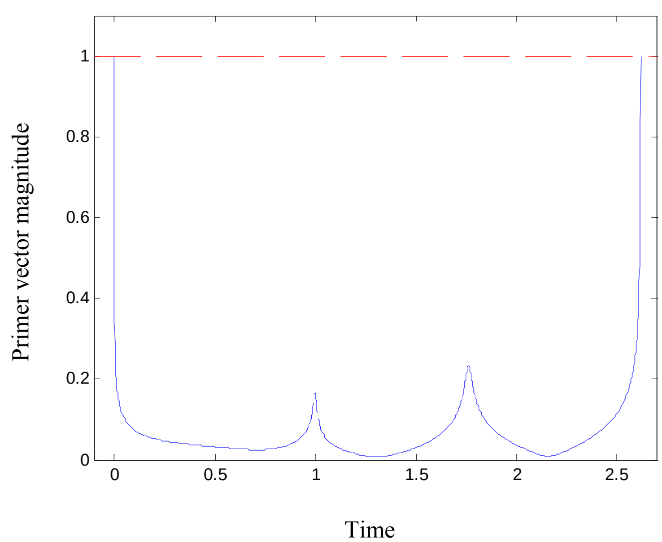

Figure 28. Primer vector magnitude of the *f*18p3 transfer

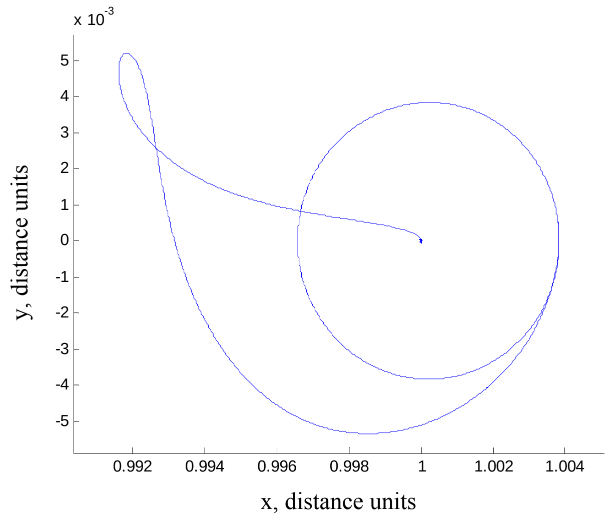

Figure 29. A *f*25p1 transfer shown in rotating coordinates centered on the secondary body

Table 18. *f*25p1 transfer properties

f25p1 details					
Δv_1	Δv_2	transfer time	r_1	r_2	θ
1.024932E-01	4.024216E-03	2.445303E+00	4.859040E-05	3.829824E-03	1.567198

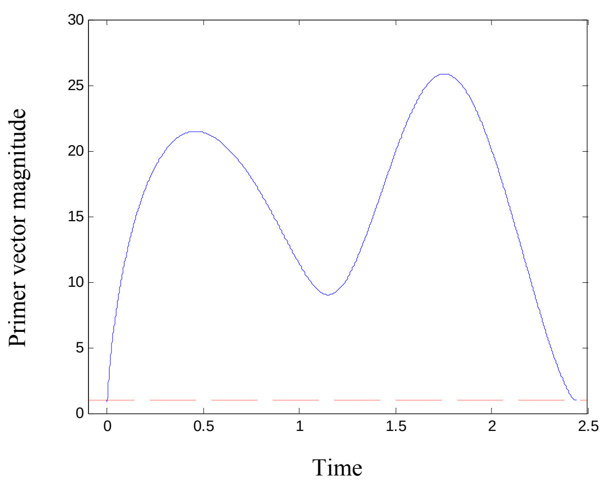

Figure 30. Primer vector magnitude of the *f*25p1 transfer

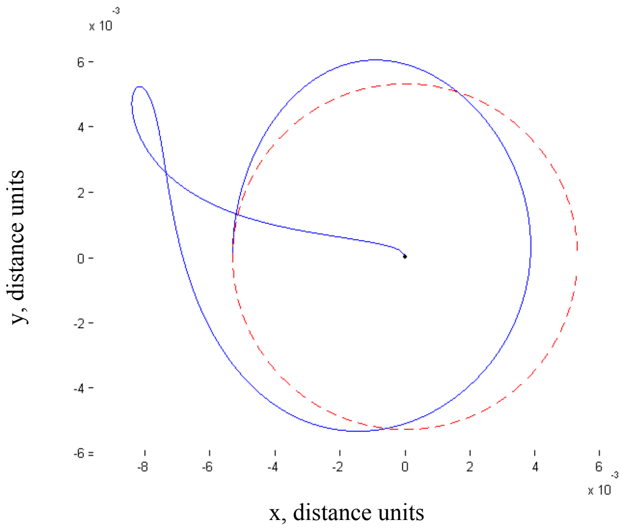

Figure 31. A *f*25p2 transfer shown in rotating coordinates centered on the secondary body

Table 19. *f*25p2 transfer properties

f25p2 details					
Δv_1	Δv_2	transfer time	r_1	r_2	θ
1.024932E-01	1.508461E-03	3.391673E+00	4.859040E-05	5.296104E-03	1.569396

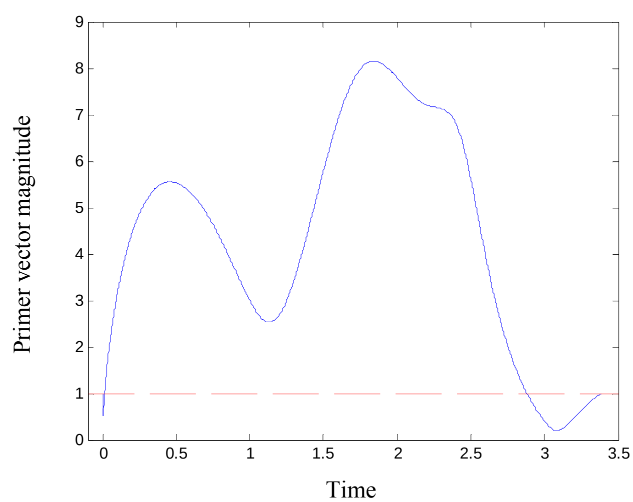

Figure 32. Primer vector magnitude of the *f*25p2 transfer

2.7 NUMERICAL THRESHOLD FOR SAVINGS VIA LOW ENERGY TRANSFERS IN THE THREE BODY PROBLEM

The local optimality of a low energy transfer does not guarantee cost savings versus a direct transfer. Through numerical analysis, however, a threshold can be established where it becomes favorable to transfer a satellite using low energy methods as opposed to direct methods. To this end, a survey is conducted of low energy transfers between orbits of different radii in the CRTBP. The orbits documented in the previous section serve as reference transfers for the survey. In addition to these reference transfers, the survey is expanded by relaxing the definition of low energy transfers and including transfers that no longer follow the periodic orbits between perigees, but rather transfer between circular orbits of specified radii. By comparing the survey of optimal low energy transfers to their direct transfer counterparts, the point at which it becomes preferable to transfer via low energy methods is defined.

2.7.1 Definition of the Low Energy Transfer Survey

In addition to transfers that follow the periodic orbits from periapse to periapse, a transfer can be formed between circular orbits that neighbor the initial and final orbits of the periodic orbit transfers. Although they do not necessarily follow arcs of the periodic orbits, they neighbor these periodic orbits and also receive the same periapse raising solar effect, so they are also considered to be low energy transfers.

For each family of transfers documented above, an analysis is performed on them and orbits derived based on them. First, each family of orbits is expanded so that the nearest periapse of each orbit in the family covers a specified range of radii. The radii of the smaller orbits are chosen to span the region between the Earth's surface and the point at which the periodic orbit family dissolves. Orbit to orbit transfers are formed from each member of each family using the method established above. Second, additional orbits are found that do not necessarily transfer from periapse to periapse along the periodic orbit. The reference orbits are used as an initial guess to generate these orbits, and they are optimized using numerical methods.

2.7.2 Targeting Neighboring Low Energy Transfers

Forming orbit-to-orbit transfers from periapse-to-periapse transfers along arcs of the cataloged periodic orbits above does not allow freedom to specify the radii of the initial and final circular orbits. The radius of the second periapse is a function of the family of periodic orbits chosen and the initial radius. In this section, low energy transfers are sought that expand the envelope of possible final radii. These transfers do not necessarily transfer the satellite along an arc of a periodic orbit and do not necessarily begin and end at a state of periapse. They do, however, meet the necessary conditions of optimality for a two-impulse orbit-to-orbit transfer in the CRTBP.

Low energy transfers between two fixed circular orbits are found and added to the catalog following the process outlined here. First, a periapse raising periodic orbit is chosen from one of the families documented above. This periodic orbit serves as a reference orbit for the targeting process. A transfer between periapses of the periodic orbit is constructed. From this reference transfer, a new transfer is targeted originating at the same circular orbit and ending at a circular orbit of a specified radius a small increment away from the terminal orbit of the orbit. This transfer is targeted and optimized.

2.7.3 Trend Lines in Orbit-to-orbit Transfer

The survey of low energy transfers can be compared to direct transfers between the same orbits. In forming direct transfers for this comparison, the targeting parameters and constraints are identical. The targeting routine converges on different solutions based solely on the initial conditions provided to it. This framework allows for a direct comparison between low energy transfers and Hohmann transfer-based transfers.

2.7.3.1 Construction of optimal direct orbit-to-orbit transfers in the CRTBP

The targeting of an optimal orbit-to-orbit transfer in the CRTBP begins with a classical Hohmann transfer in the two body problem.[44] The magnitudes of two impulses of the transfer are computed by comparing the velocities of the circular orbits and a transfer ellipse that has a periapse that coincides with the smaller circular orbit and an apogee that coincides with the larger circular orbit:

$$\Delta v_1 = \sqrt{\frac{2\mu_e}{r_1} - \frac{2\mu_e}{r_1 + r_2}} - \sqrt{\frac{\mu_e}{r_1}} \tag{2.7.1}$$

$$\Delta v_2 = \sqrt{\frac{\mu_e}{r_2}} - \sqrt{\frac{2\mu_e}{r_2} - \frac{2\mu_e}{r_1 + r_2}} \tag{2.7.2}$$

The period of the transfer is also well known:

$$T_{hohmann} = \pi\sqrt{\frac{\left(\frac{r_1 + r_2}{2}\right)^3}{\mu_e}} \tag{2.7.3}$$

The impulses are applied in the direction of the spacecraft's velocity at time $t_0 = 0$ and $t_f = T_{hohmann}$.

The transfer defined in Eqns. (2.7.1) through (2.7.3) is used as an initial guess in a numerical optimization routine. The performance index, parameters and constraints of the optimization are identical to those defined for the low energy transfer in section 3.5. The primer vector history is also computed in the same way for the direct transfer as the low energy transfer. Each direct transfer is verified to be primer optimal. The results of the numerical generation of direct transfers are available in Appendix A.

The following figures display the results of the comparison between low energy transfers and direct transfers for selected transfer families. Families of transfers to first and second perigees are considered. The plots track the savings achieved via low energy transfer between two circular obits versus the ratio of the radius of the final circular orbit to thee radius of the initial circular orbit. Positive values signify an advantage to the low energy transfer, and negative values signify that the direct transfer is less expensive. Each point in the figures is a comparison between a specific low energy transfer to an optimal direct transfer between the same orbits. The parameters that define each orbit used in the comparison can be found in Appendix A.

The plots show similar results for transfers belonging to each family but significant differences in transfers to different perigees. For each plot for the transfers to the first perigees, the savings has a maximum value of around fifteen percent, and savings begin to be achieved when the ratio of initial to final orbital radii is approximately six. The transfers to the second perigees are not as beneficial as the transfers to the first. The maximum savings is approximately ten percent in each survey of the second perigee transfers. These savings deteriorate as the ratio of orbital radii increase. The ratio also must be higher, around eight, for savings to be possible.

The contrast between the p1 transfers and p2 transfers is to be expected based on the periodic orbit geometry. The second perigee after the nearest Earth flyby is nearer the Earth than the first perigee in every member of every family studied in this survey. Because the free perigee increase is not as large, less savings should be expected from these transfers.

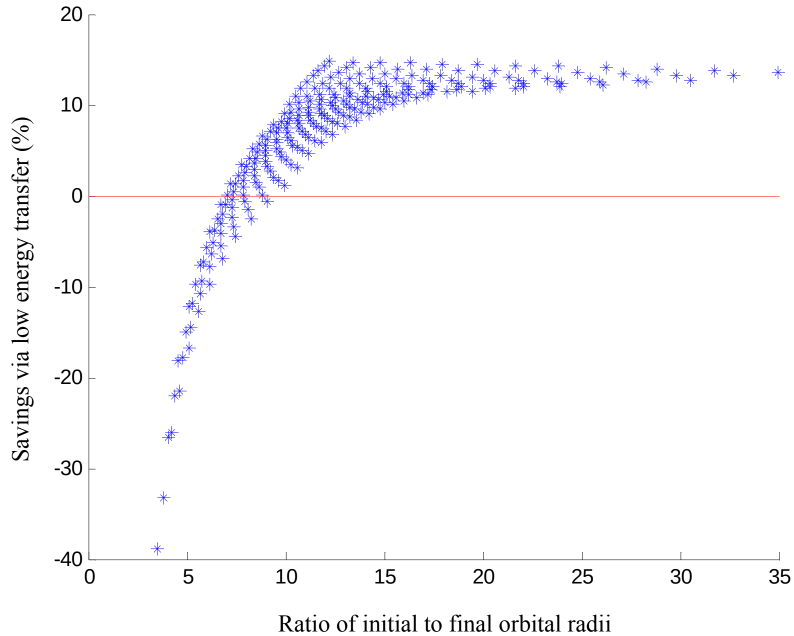

Figure 33. Savings associated with *f*16p1 transfers

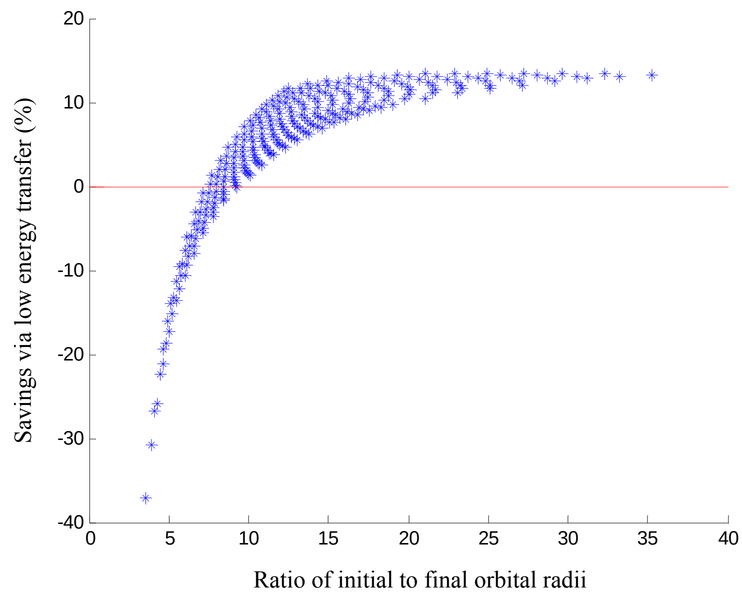

Figure 34. Savings associated with *f*17p1 transfers

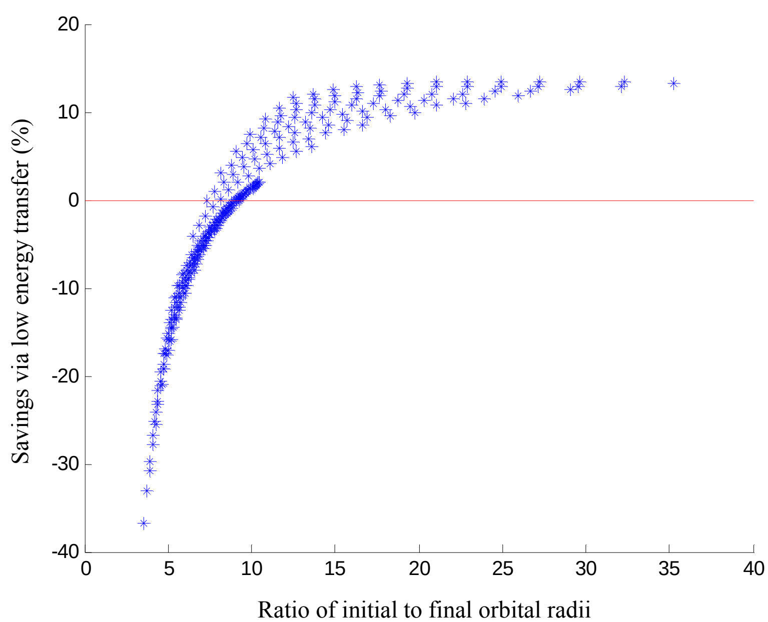

Figure 35. Savings associated with *f*18p1 transfers

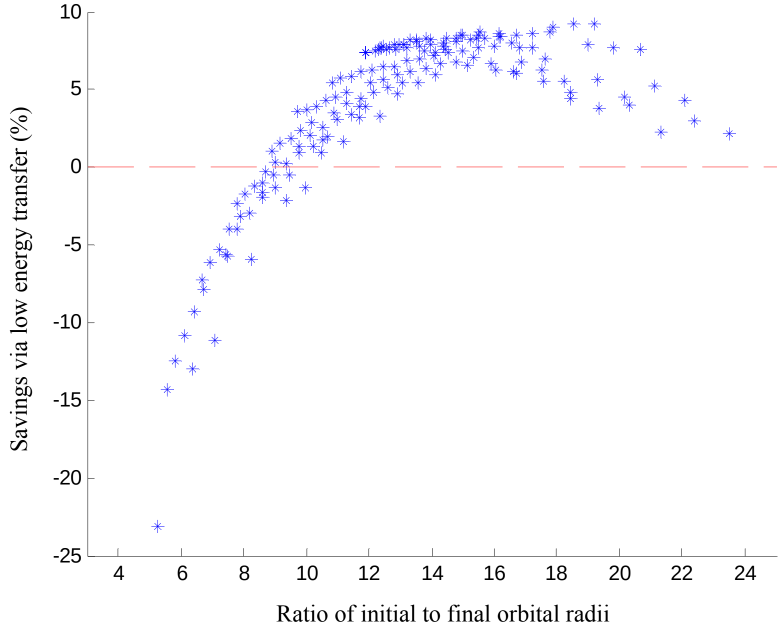

Figure 36. Savings associated with *f*16p2 transfers

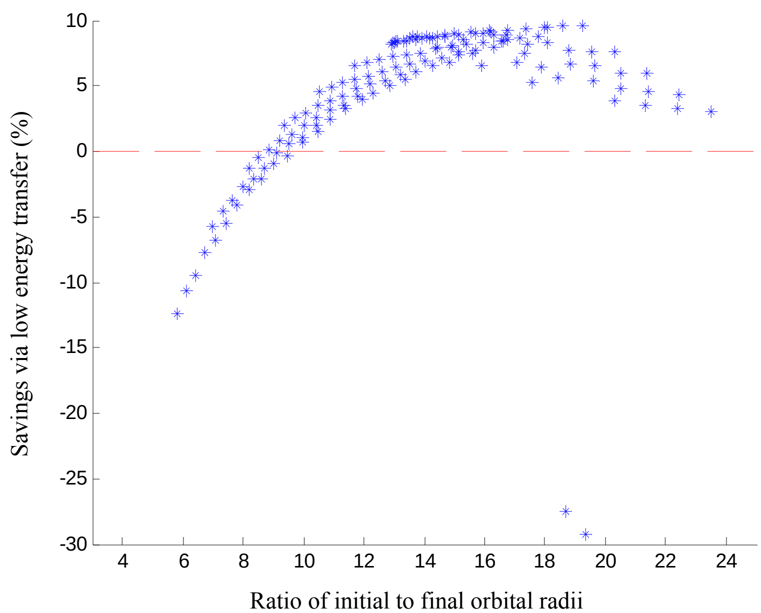

Figure 37. Savings associated with *f*17p2 transfers

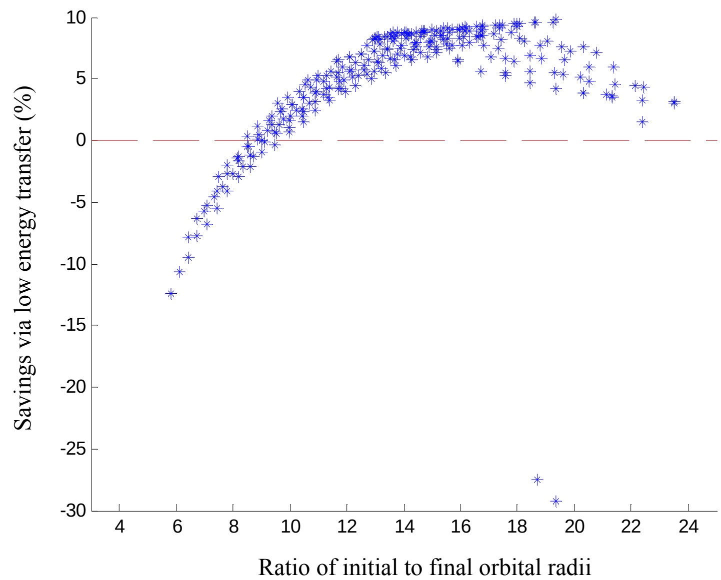

Figure 38. Savings associated with f18p2 transfers

2.8 CHAPTER CONCLUSIONS

This chapter defines a class of low energy transfers in the Earth-Sun CRTBP that is geometrically similar to BLCTs in the four body problem and exhibits similar fuel savings when compared to direct transfers. These low energy transfers are constructed by following arcs of periodic orbits that have been chosen from Markellos's catalog based on the changes in the orbit's perigees within them. A naming convention is defined, and a partial catalog of these transfers is included.

The optimality of the low energy transfers is considered in two different ways. First, primer vector theory is applied to verify their local optimality. In case of each family of transfers, the primer vector history reveals that the trajectories satisfy the necessary conditions for an optimal two-impulse transfer, although in some cases the primer vector history shows a potential for fuel savings with the addition of intermediate impulses.

Secondly, the relative optimality of the low energy transfer is examined with respect to a direct transfer between circular orbits of equal radii. Many transfers from each family are compared to their direct counterparts, and a threshold in the ratio of the initial and final orbital radii is established where savings via low energy transfer becomes possible. Additionally, a maximum in the amount of savings expected is established to be approximately fifteen percent. This maximum savings value is similar to the savings observed in the literature of a BLCT over a Hohmann transfer from low Earth orbit.

The low energy transfers defined in this chapter are not only useful in minimum fuel orbit-to-orbit transfers. The proceeding chapters in this dissertation demonstrate their utility in defining initial guesses for the problem of targeting BLCTs. Because these transfers are calculated in the Earth-Sun CRTBP, the orientation issues concerning the Sun's angle with respect to the Earth are automatically solved. With a proper consideration for phasing to orient the Moon in the rotating coordinate system, targeting BLCTs can be reduced to simply adding the Moon to the three body system.

3. Ballistic Lunar Capture Transfers in the Four-body System

3.1 Introduction

The design of a BLCT can be as simple as adding the influence of the Moon to the low energy transfers of the CRTBP. Many of the documented families of periodic orbits contain members with perigees that are raised to the distance between the Earth and the Moon. If these particular members are used as initial guesses in the BCRFBP then the solar influence will raise the perigee distance from a lower state to the desired distance to target the Moon. From there, the lunar gravity captures the spacecraft, in effect providing a free circularization of the orbit. By simply adding the Moon to the previously calculated orbit-to-orbit transfers, BLCTs can be targeted with good success.

Examples of BLCTs derived from each family of low energy transfers from the nearest perigee to the first and second subsequent perigees are presented here to demonstrate the applicability of the periodic orbits to the targeting of BLCTs. These transfers are not necessarily considered to be transfers that might be used in mission planning due to their unfavorable travel times, high initial orbits about the Earth, and the difficulty in achieving convergence to solutions, especially to the multiple-pass transfers. Alternatively, a more robust algorithm is presented in Chapter 4 for real-world BLCT design.

3.2 Definition of the Nonlinear Targeting Problem

In this body of work a targeting problem has the following definition. We wish to satisfy a constraint vector, **c**, by varying parameters in a parameter vector, **a**. Some initial guess is given, and a solution is found through iteration of the initial guess in a way that reduces the magnitude of the constraint vector. This problem is solved numerically with a nonlinear programming routine.[42]

3.3 BI-CIRCULAR RESTRICTED FOUR BODY PROBLEM

Several authors have established the bi-circular restricted four body problem (BCRFBP) as a useful system for analysis of BLCTs.[45,46] The BCRFBP, as formulated here, is a planar approximation of the Earth-Sun-Moon system with several assumptions about the interactions of the four massive bodies in the system. The first assumption is that the Earth and Moon are in two-body circular orbits around their common center of mass, unperturbed by the Sun. The second assumption is that the Sun is in a two-body circular orbit around the same Earth-Moon center of mass and in the same plane as the orbits of the Earth and Moon. Finally, the system is completed by assuming that the fourth body is infinitesimally small, such that its presence does not alter the orbits of the massive bodies.

The system described by the BCRFBP is defined as follows. A non-rotating coordinate system is established that is centered on the Earth-Moon barycenter. The Earth and Moon are in two-body circular orbits about this point. At the epoch $t = 0$, the Earth lies on the positive x-axis and the Moon on the negative x-axis. As time progresses, they rotate about each other with angular frequency ω,

$$\omega_{\mathrm{em}} = \sqrt{\frac{\mu_{\mathrm{e}} + \mu_{\mathrm{m}}}{\mathrm{r}_{\mathrm{em}}}} \tag{3.3.1}$$

where μ_{e} is the gravitational parameter of the Earth, μ_{m} is the gravitational parameter of the Moon, and r_{em} is the constant radial distance between the Earth and Moon. The position of the Earth and Moon, respectively, can be calculated using the following formulas:

$$\mathbf{r}_{\mathrm{e}} = \begin{pmatrix} \mathrm{r}_{\mathrm{em}} \cos\left(\omega_{\mathrm{em}} \mathrm{t}\right) \\ \mathrm{r}_{\mathrm{em}} \sin\left(\omega_{\mathrm{em}} \mathrm{t}\right) \end{pmatrix} \left(\frac{\mu_{\mathrm{m}}}{\mu_{\mathrm{e}} + \mu_{\mathrm{m}}} \right)$$

$$\mathbf{r}_{\mathrm{m}} = \begin{pmatrix} -\mathrm{r}_{\mathrm{em}} \cos\left(\omega_{\mathrm{em}} \mathrm{t}\right) \\ -\mathrm{r}_{\mathrm{em}} \sin\left(\omega_{\mathrm{em}} \mathrm{t}\right) \end{pmatrix} \left(\frac{\mu_{\mathrm{e}}}{\mu_{\mathrm{e}} + \mu_{\mathrm{m}}} \right). \tag{3.3.2}$$

The Sun and the Earth-Moon barycenter are in two-body circular orbits around their common center of mass. The angular frequency of their orbit is defined to be

$$\omega_{(\mathrm{em})\mathrm{s}} = \sqrt{\frac{\mu_{\mathrm{e}} + \mu_{\mathrm{s}} + \mu_{\mathrm{m}}}{\mathrm{r}_{(\mathrm{em})\mathrm{s}}}}, \tag{3.3.3}$$

where $r_{(em)s}$ is the constant orbital radius of the Earth-Moon barycenter about the Sun. The position of the Sun with respect to the center of the coordinate system is as follows:

$$\mathbf{r}_{\mathrm{s}} = \begin{pmatrix} \mathrm{r}_{(\mathrm{em})\mathrm{s}} \cos\left(\omega_{(\mathrm{em})\mathrm{s}} \mathrm{t}\right) \\ \mathrm{r}_{(\mathrm{em})\mathrm{s}} \sin\left(\omega_{(\mathrm{em})\mathrm{s}} \mathrm{t}\right) \end{pmatrix}. \tag{3.3.4}$$

It is observed that the origin of the coordinate system is accelerating. The inertial point in this system is the Earth-Sun-Moon barycenter, and the reference frame origin is in a circular orbit about this point.

The equations of motion for the small particle in this system come from the n-body equations of motion.[47] The small mass is subject to three force terms that derive directly from the gravitational attraction of each of the massive bodies, and a fourth indirect term that is the consequence of the acceleration of the coordinate system. Together, they give the BCRFBP equations of motion:

$$\ddot{\mathbf{r}} = -\frac{\mu_{\mathrm{s}}}{\mathrm{r}_{\mathrm{s/sc}}^{3}} \mathbf{r}_{\mathrm{s/sc}} - \frac{\mu_{\mathrm{e}}}{\mathrm{r}_{\mathrm{e/sc}}^{3}} \mathbf{r}_{\mathrm{e/sc}} - \frac{\mu_{\mathrm{m}}}{\mathrm{r}_{\mathrm{m/sc}}^{3}} \mathbf{r}_{\mathrm{m/sc}} - \frac{\mu_{\mathrm{s}} + \mu_{\mathrm{e}} + \mu_{\mathrm{m}}}{\mathrm{r}_{\mathrm{s}}^{3}} \mathbf{r}_{\mathrm{s}} \tag{3.3.5}$$

3.4 Iterative Nonlinear Targeting Technique

The low energy orbit-to-orbit transfers in the CRTBP can be transitioned into BLCTs in the BCRFBP by adjusting the magnitude and direction of the transfer insertion burn to account for the addition of the Moon's gravity to the dynamical system. It is comparable to the Belbruno and Carrico targeting algorithm, which will be discussed in later section.[48]

The nonlinear targeting algorithm described here is a two step process. The first step targets a state in the neighborhood of the Moon, and the second step targets a captured orbit about the Moon.

Before the nonlinear targeting algorithm is initiated, consideration must be given to the epoch of the transfer due to the time-varying equations of motion. The proper epoch synchronizes the Moon's orbit and the spacecraft's approach to the final perigee of the transfer. It can be appropriately determined by fixing the final time to be an instant when the Moon is in the proper position relative to the Earth-Sun line and then subtracting the period of the nominal three body transfer.

Once the orientation of the massive bodies is defined, the targeting of a BLCT can commence. The first step in the two-step nonlinear targeting algorithm targets a final state that is within a specified radius of the Moon. It is in this step that the dynamics of the four body problem are adjusted for by modifying the initial maneuver. The result should be a trajectory that is geometrically similar to the nominal transfer propagated in the CRTBP.

The targeting problem is defined by the parameter vector, **a**, which is adjusted through a differential correction method until the final state satisfies the constraint vector, **c**. In this first step, the parameter vector consists of three parameters,

$$\mathbf{a} = \begin{pmatrix} f & \Delta v & t_0 \end{pmatrix}^T, \tag{3.4.1}$$

where *f* is the true anomaly at which the spacecraft departs the parking orbit, Δv is the magnitude of the transfer insertion burn to be applied along the velocity vector of the spacecraft, and t_0 is the time at which the transfer insertion burn is applied. The constraint vector is of single dimension,

$$c = r_m^s - r_m , \tag{3.4.2}$$

where r_m is the distance between the spacecraft and the Moon and r_m^s is the specified target distance for the spacecraft to be within. This constraint is an inequality constraint that is satisfied when $c \leq 0$. An appropriate value for r_m^s is the distance from the Moon to the L_2 Lagrange point.

The second step in the two-step nonlinear targeting algorithm is to target a captured orbit around the Moon in the BCRFBP. This step numerically minimizes the energy of the spacecraft with respect to the Moon. The inequality constraint in the previous step is also applied to this step in order to restrict the final state of the spacecraft to be near the Moon. The vector of free parameters is modified to include the final time of the transfer. Lastly, a minimization problem is defined by introducing a performance index, $J = E_m$, where E_m is the Keplerian energy of the spacecraft with respect to the Moon and can be expressed through the vis-viva equation:

$$E_m = \frac{{v_m}^2}{2} - \frac{\mu_m}{r_m} , \tag{3.4.3}$$

where μ_m is the mass parameter of the Moon, v_m is the magnitude of the velocity of the spacecraft with respect to the Moon, and r_m is the magnitude of the position vector of the spacecraft with respect to the Moon.

Because the BLCTs are derived directly from the low energy transfers in the CRTBP, it is appropriate that they be classified based upon their nominal trajectories. It is proposed that a BLCT and the low energy orbit-to-orbit transfer in the CRTBP that was used to target it share the same naming convention. For example, a BLCT that is based on a *f*16p1 low energy orbit-to-orbit transfer be named a *f*16p1 BLCT.

3.5 NUMERICAL EXAMPLES OF BALLISTIC LUNAR CAPTURE TRANSFERS

The targeting problem can be solved by using one of a variety of numerical methods. The results presented in this section are calculated using the Matlab sequential quadratic programming function *fmincon*. In the first step, the performance index is null. In the second step, *fmincon* numerically minimizes the performance index subject to the specified constraints. The trajectories shown below have been rotated into a coordinate system that rotates with the Earth-Sun line for comparison to previous results.

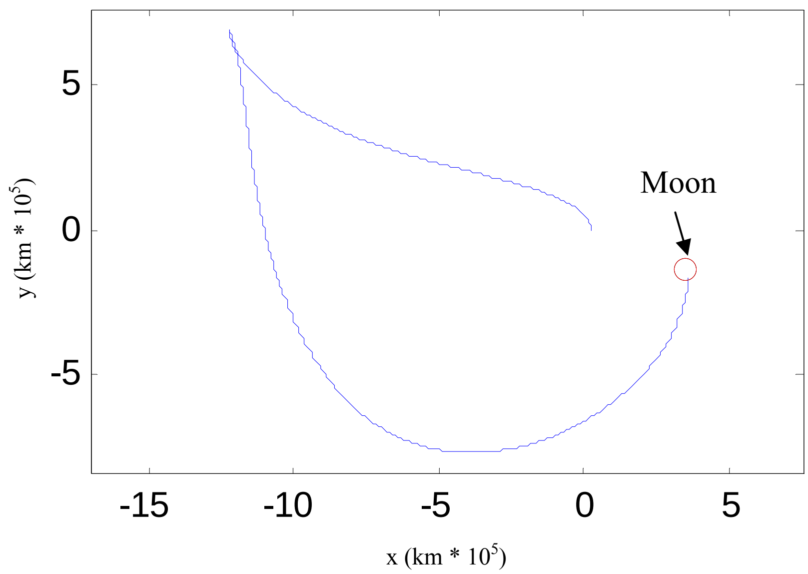

Figure 39. A *f*14p1 BLCT shown in Earth-centered rotating coordinates

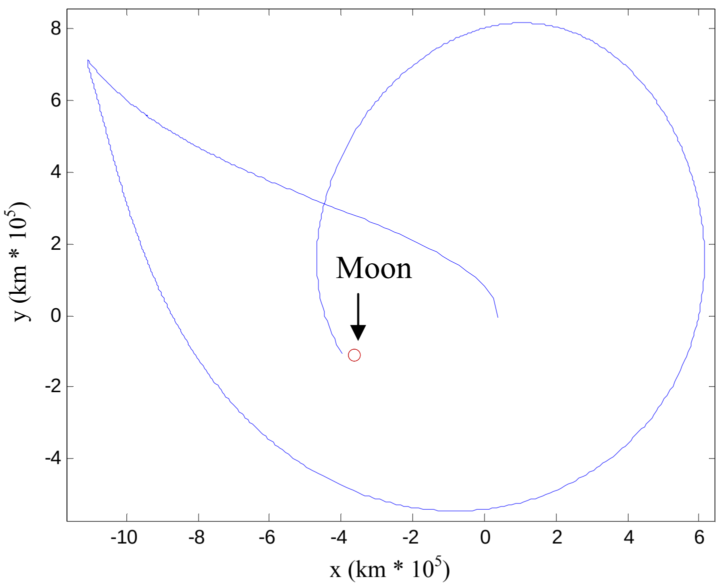

Figure 40. A *f*16p2 BLCT shown in Earth-centered rotating coordinates

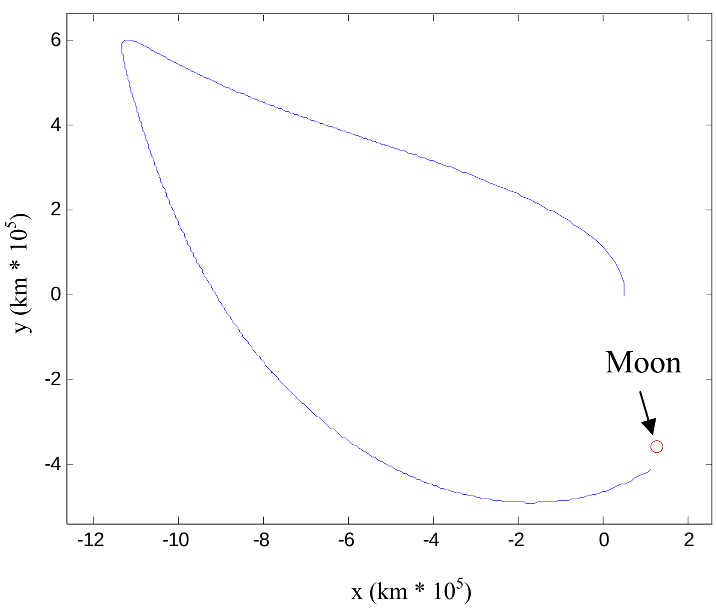

Figure 41. A *f*17p1 BLCT shown in Earth-centered rotating coordinates

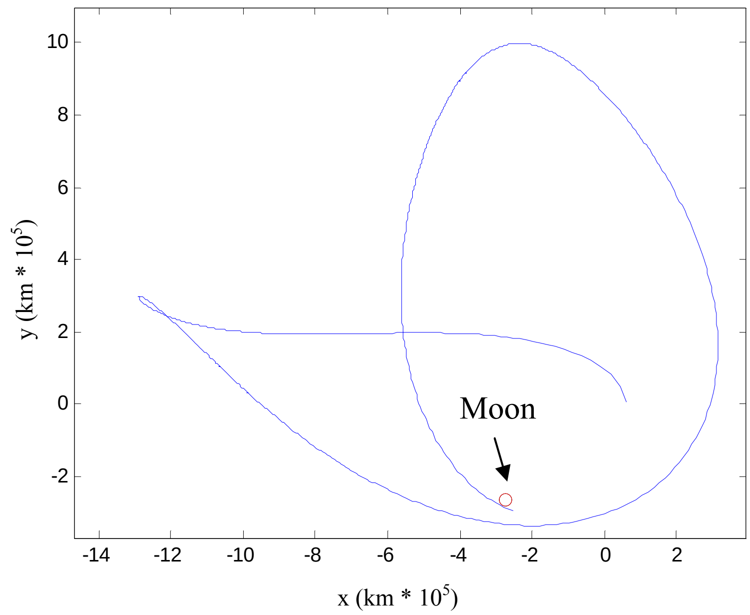

Figure 42. A *f*17p2 BLCT shown in Earth-centered rotating coordinates

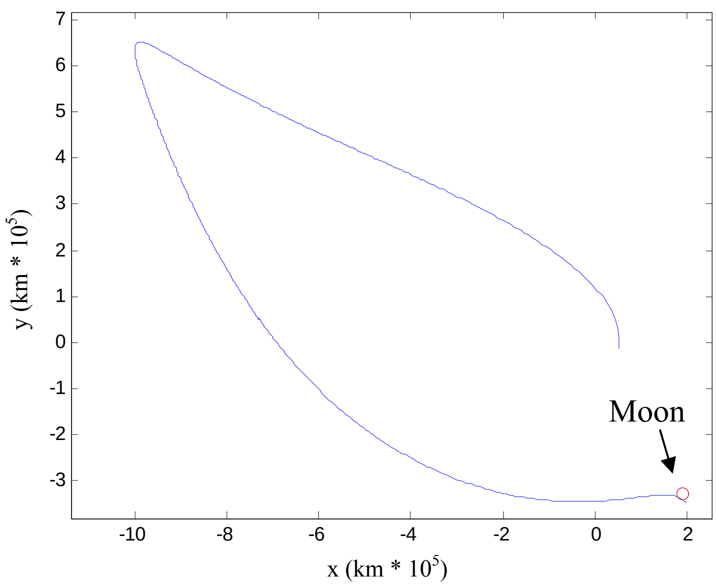

Figure 43. A *f*18p1 BLCT shown in Earth-centered rotating coordinates

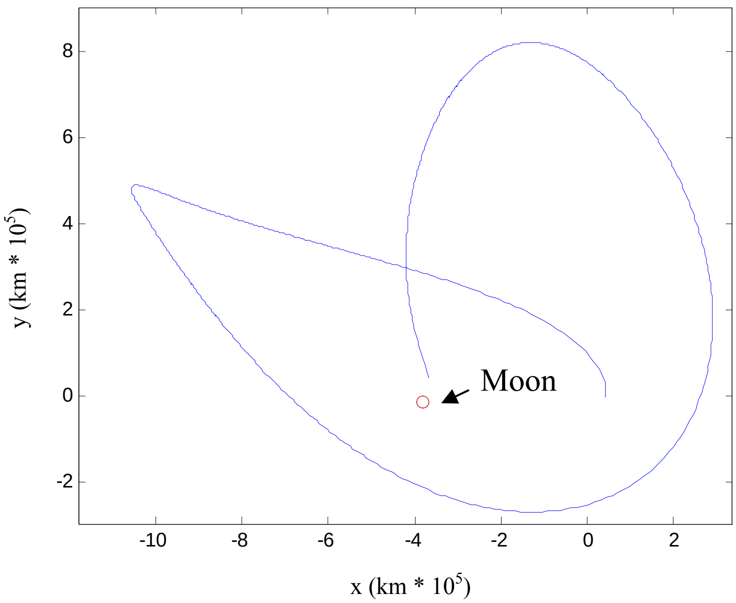

Figure 44. A *f*18p2 BLCT shown in Earth-centered rotating coordinates

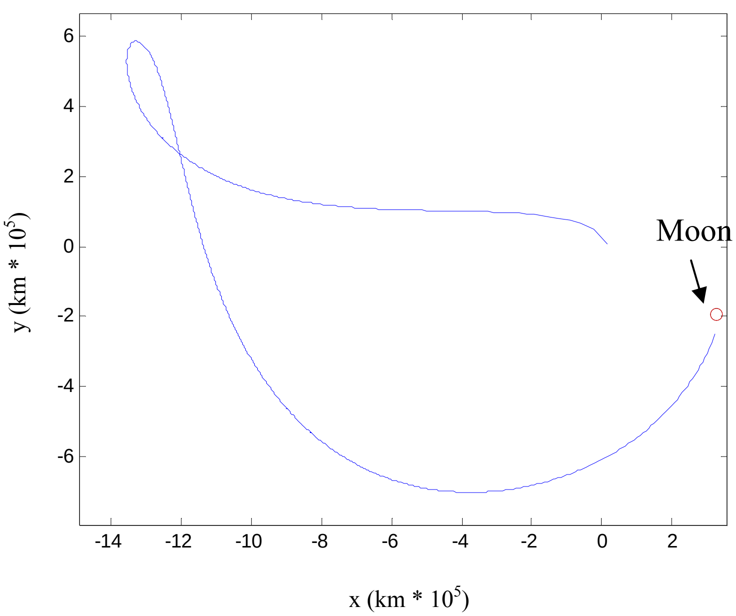

Figure 45. A *f*25p1 BLCT shown in Earth-centered rotating coordinates

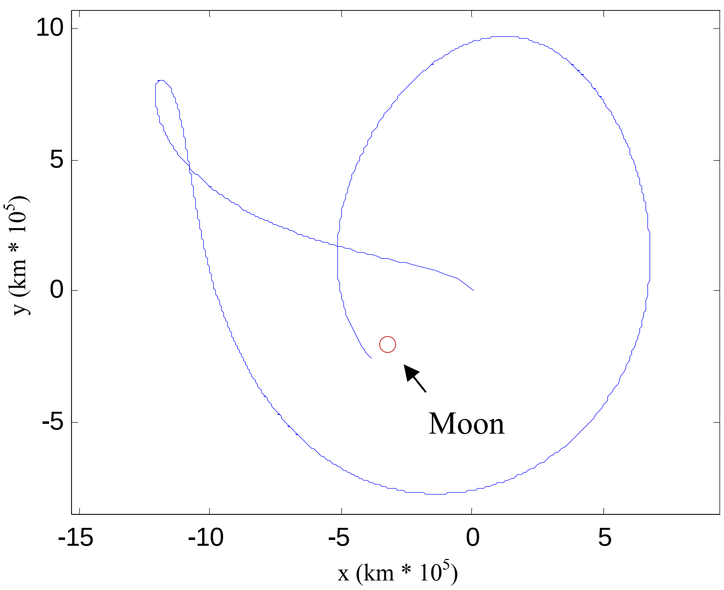

Figure 46. A *f*25p2 BLCT shown in Earth-centered rotating coordinates

3.6 CHAPTER CONCLUSIONS

The results of this chapter are conceptual and not intended to be a practical definition of a nonlinear targeting algorithm for BLCTs. The low energy transfers of the Earth-Sun CRTBP are shown to translate well into BLCTs in the BCRFBP. The BLCTs can be targeting by making small adjustments to the transfer insertion burn of the three body transfer. These small adjustments, along with proper consideration of the location of the Moon yield BLCTs based on many families of low energy transfers.

It is proposed that BLCTs be categorized based on the family from which they are constructed. The families contained in Markellos's catalog provide a naming convention sufficient to differentiate and categorize BLCTs. A more practical nonlinear targeting algorithm is explored in Chapter 4 that will use a particular member of Markellos's catalog to design real-world transfers.

4. Targeting Ballistic Lunar Capture Transfers in a Real-world System

4.1 Introduction

The goal of this chapter is to describe an algorithm that produces BLCTs in a systematic way. The algorithm differs from the targeting method in the previous chapter in that it is designed to target real world trajectories that might be employed for a lunar mission, and also has the ability to accommodate for an arbitrary date for initiation of the transfer. Its stability and potential for automation are the critical factors that determine whether the algorithm is successful.

Several methods of designing BLCTs have been documented. Belbruno and Carrico[48] have presented a two dimensional, user-in-the-loop forward targeting algorithm for transfers that approach the Moon along an orbit that has an apogee that is much greater than the distance from the Earth to the Moon. Yamakawa, et al.[49] used a shooting method to produce similar transfers in the planar restricted four body problem. Additional algorithms exist that utilize the invariant manifolds associated with libration point orbits in the Sun-Earth and Earth-Moon CRTBPs. For example, Koon, et al.[14] targeted BLCTs by finding intersections of these invariant manifolds. Similarly, Yamato and Spencer[15] approximated the invariant manifolds in a perturbed CRTBP, yielding transit orbits. Parker and Lo[17] have categorized families of BLCTs found from invariant manifolds with the intention of allowing a mission planner to choose appropriate trajectories for specific missions. Alternatively, Biesbroek et al.[50] have used genetic algorithms to successfully find WSB trajectories. Finally, Yagasaki[46] has created a non-linear boundary value problem that obtains a solution by beginning with an elliptic arc in the two body problem (mass parameters of the Sun and Moon equal to zero), and iterating the solution with increasing mass parameters of the Sun and Moon until the real-world solution is obtained.

Other low energy targeting methods exist that do not rely on solar gravitational effects. The trajectories that these transfers target approach the Moon from the direction of the interior Earth-Moon Lagrange point. Bolt and Meiss[51] developed a targeting scheme in the Earth-Moon planar CRTBP that relies on recurrence of chaotic trajectories. Macau and Grebogi[52] used a similar method to target transfers to the Moon through chaotic spaces in the restricted three body problem through elimination of recurrent orbits; however, improved transfer times over Bolt and Meiss were achieved at the expense of a second maneuver and higher fuel costs. Both of these methods rely on very large parking orbits around the Earth to achieve a low energy impulsive transfer without the need of a solar perturbation. Mengali and Quarta[53] also disregard the solar gravitational influence when they compare their planar three body bi-impulsive method to WSB transfers.

The difficulties in systematically targeting BLCTs arise from the chaotic dynamics that are present along its path. The instability inherent in the chaotic system is overcome by choosing an initial guess that automatically orients the transfer and by using precise derivatives computed from the state transition matrix. The initial guess for the algorithm is based on family *f*16p1 and *f'*16p1 low energy orbit-to-orbit transfers presented in Chapter 2. By incrementally adding the complexity of the real world restricted four body problem, the low energy transfers will be adjusted and eventually converge to BLCTs.

4.2 ALGORITHM DEFINITION

The low energy orbit to orbit transfers documented previously all share a common trait of using the Sun's gravity to change the periapse distance of the orbit. The algorithm presented here will take advantage of this periapse raising property of these low energy transfers to produce BLCTs. Even though the second periapse of the low energy transfer in the three body problem may not be equal to the lunar orbital radius, the algorithm will adjust the increase in periapse distance to coincide with the lunar orbit. The algorithm also considers the phasing of the transfer so that for an arbitrary launch date a BLCT can be computed.

The algorithm's stability depends in part on the precision of the partial derivatives of the constraints with respect to the parameter vector that are provided to the numerical targeting routine. It has been shown that derivatives based on the state transition matrix are more precise and lead to faster convergence than finite difference derivatives.[43]

One of the goals of this nonlinear targeting algorithm is to create transfers that would be useful in mission planning. To support this goal, a more precise dynamical model is used. The four body problem is defined here with the locations of the Earth, Sun and Moon determined by the JPL DE405 ephemeris.[54] The equations of motion used in the BCRFBP (Eq. (3.3.5)) still apply to this system, but the approximations of the BCRFBP regarding the masses orbiting in circular orbits are abandoned in favor of their real-world orbits.

4.2.1 Parameters of the nonlinear targeting algorithm

In order to define a ballistic lunar transfer in a given coordinate system the following parameters must be defined: two orientation angles that define the plane of the initial circular orbit, an orientation angle to designate the point on the initial orbit at which the transfer is initiated, the time at which the transfer insertion impulse is applied, the magnitude and direction of the impulse, and the transfer time. For the purposes of the nonlinear targeting algorithm, the initial time and the radius of the initial orbit are specified and are not adjusted. The other parameters are considered to be free in the nonlinear targeting algorithm. The initial step in the nonlinear targeting algorithm only uses one free variable, the magnitude of the impulse. The other free variables will be added to the algorithm an incremental way following the method that has proven to reliably converge upon BLCTs.

4.2.2 Transfer Orientation

The initial task of the nonlinear targeting algorithm is to select the proper family of generating obits from the two generating families, *f*16 and *f*16'. One of the factors of the reliability of the nonlinear targeting algorithm is the effect of the Moon's gravity on the outbound leg of the BLCT. Depending on the orientation of the Moon with respect to the trajectory, the lunar perturbation can vary greatly. To increase the stability of the algorithm, a transfer orientation with the farthest distance from the Moon is chosen in order to mitigate the effect of the Moon on this leg of the transfer. Trajectories that employ a lunar flyby on the outbound segment can have advantages in cost savings, but are not presented here due to their adverse affects on the numerical routine's convergence.

Insertion into either of the two possible transfers can be achieved with a ΔV in the direction of the velocity vector of the spacecraft in a prograde low Earth parking orbit. The selection of the appropriate class of trajectory to target on a given launch date is made based on the location of the Moon in the Sun-Earth rotating coordinate frame at the time of the transfer. If the Moon is located in the quadrants of the coordinate system farthest from the Sun, then the *f'*16 generating family should be used, maximizing the distance between the Moon and the spacecraft. If the Moon is located in a quadrant nearest to the Sun, then the generating family *f*16 is appropriate. Figure 47 and Figure 48 illustrate the selection of the reference trajectory. In the example BLCT described below, the family *f'*16 is appropriate for the given epoch.

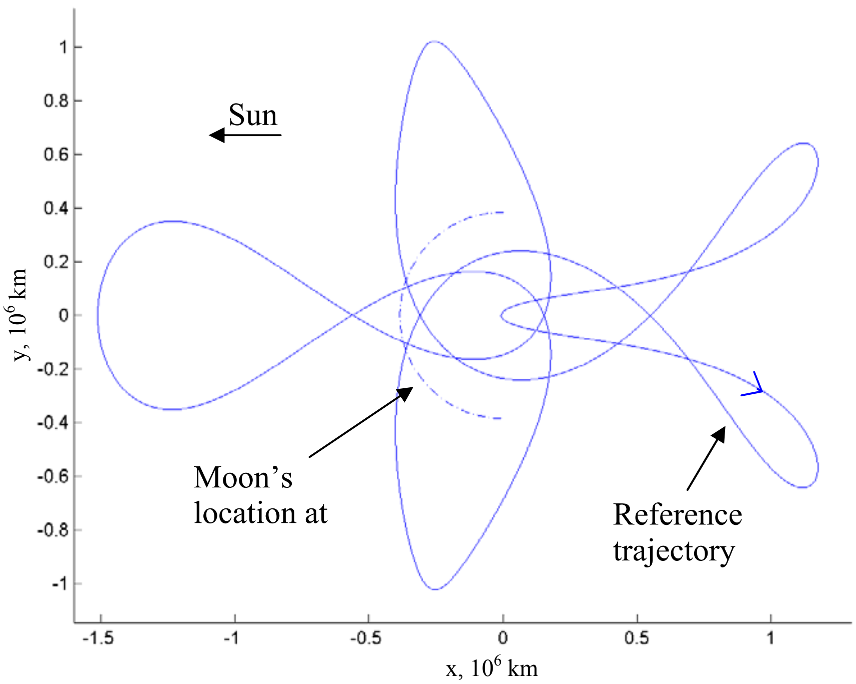

Figure 47. Lunar position for generating family *f'*16 selection in Earth centered rotating coordinates

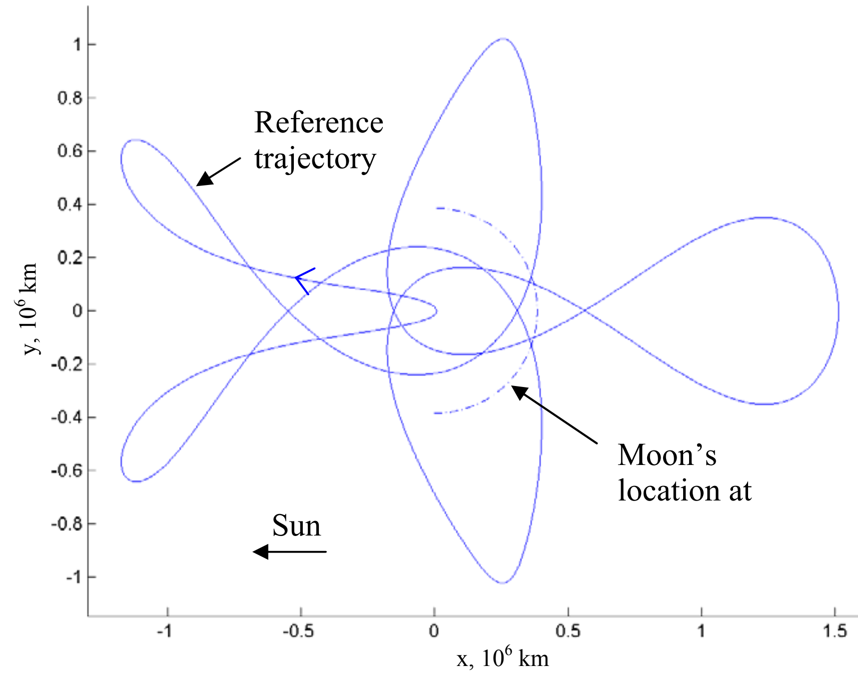

Figure 48. Lunar position for generating family *f*16 selection in Earth centered rotating coordinates

4.2.3 Estimation of Transfer Time

The first step in the nonlinear targeting algorithm is to estimate the transfer duration. The initial time is provided among the given parameters. An estimation of the final time is made so that it coincides with one of the Moon's crossings of the y-z plane in the Earth-centered Earth-Sun rotating coordinates. There are two crossings of the y-z plane each month, one in the positive-y direction and one in the negative-y direction. The appropriate crossing is selected to correspond to the orientation of the reference trajectory. For example, in the *f'*16 trajectories, the appropriate final time is the time of the Moon's passage from the third quadrant to the fourth quadrant of the rotating coordinate system. In order to allow for the appropriate phasing, the final time is selected to be that of the crossing that occurs nearest to the time of the periapse that is being used as the initial guess for the transfer. Regardless of the initial time, the final time will be chosen based on the orientation of the Moon in rotating coordinates as described. By following this procedure, the initial time is arbitrary, but a spacecraft will arrive at the Moon at one of two possible times during each month.

4.2.4 Δv Targeting in the Restricted Three Body Problem

With an estimation of the transfer time, the first targeting step can be performed. This first step adjusts $\Delta\mathbf{v}_0$ in the planar problem to account for the difference in the flight time of the nominal transfer in the CRTBP and the estimated flight time found the in the previous step, thereby phasing the transfer to arrive at the Moon. If the initial time were not a fixed parameter, then this step would be unnecessary.

Following the method developed by Pu and Edelbaum,[29] an approximate $\Delta\mathbf{v}_0$ for the four body problem is calculated in a three body problem by adding the mass of the Moon to the mass of the Earth. The combined mass is located at the Earth-Moon barycenter. The radius of the initial parking orbit is then adjusted to produce the same circular velocity around the combined mass object as the original parking orbit around the Earth as shown in Eq. (4.2.1).

$$r_2 = r_1 \frac{\mu_e + \mu_m}{\mu_e} \qquad (4.2.1)$$

This three body problem allows a good approximation of the $\Delta\mathbf{v}$ required to insert the spacecraft into the transfer. An initial guess of the Δv is determined by taking the difference between the spacecraft's circular velocity and its velocity at the closest flyby on the reference trajectory. For this step, the orientation of the parking orbit is fixed in the orbital plane of the Earth-Moon barycenter around the Sun, and only the magnitude of the initial burn is varied. The Δv is assumed to be oriented in the direction of the spacecraft's velocity vector.

In the numerical propagation of the trajectory, the Sun and the Earth-Moon combined masses are both treated as point masses. Their locations are determined from the JPL DE405 ephemeris. The trajectory of the spacecraft is propagated in a non-rotating coordinate frame centered on the Earth-Moon combined mass, and the motion in the restricted three body problem is governed by Eq. (4.2.2).

$$\ddot{\mathbf{r}}_{\mathrm{RTBP}} = -\frac{\mu_e + \mu_m}{r^3}\mathbf{r} - \frac{\mu_s}{\left|\mathbf{r}-\mathbf{r}_s\right|^3}\left(\mathbf{r}-\mathbf{r}_s\right) - \frac{\mu_s + \mu_e + \mu_m}{r_s^3}\mathbf{r}_s \tag{4.2.2}$$

A differential correction algorithm is used in this targeting step. The numerical targeting problem is defined as follows. Let **a** be the vector of parameters that are varied to satisfy the targeting constraints. The vector **c** contains the constraints that must be driven to zero in the numerical routine. The differential correction algorithm estimates the derivative of **c** with respect to **a** in order to find the correction of **a** to make **c** approach the zero vector.

For this targeting step, there is only one free parameter, $a = \Delta v$. The target is the spacecraft's crossing of the y-z plane of the Earth-Moon barycenter centered, Sun-combined mass rotating coordinate system at t_f, $c = r_x$. The transversal of the y-z plane will occur in the bottom half of the x-y plane if observed from above, resulting in a trajectory similar to the trajectory shown in Figure 49. Table 20 details the parameters of the first iteration step for the sample trajectory shown in Figure 49.

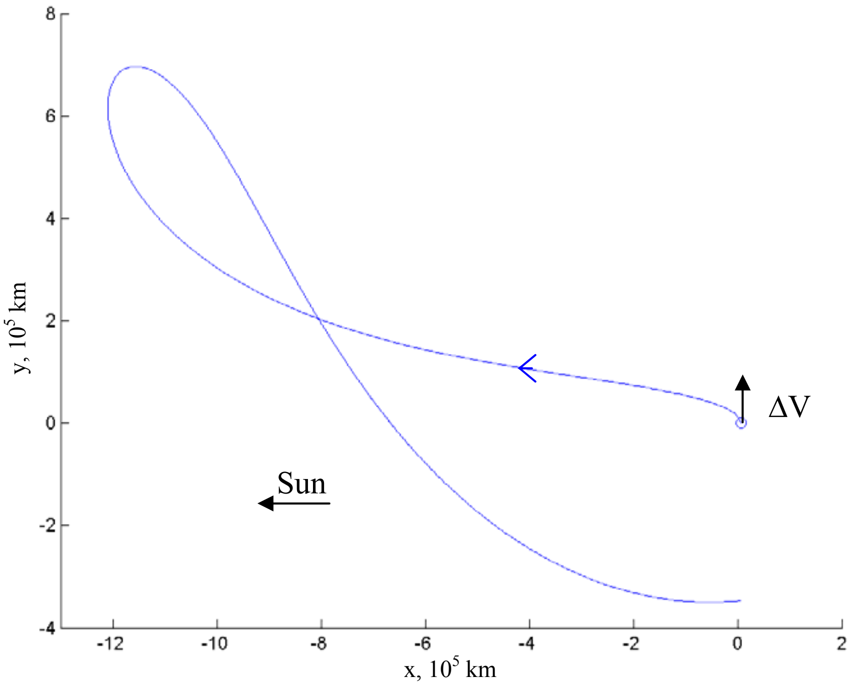

Figure 49. Crossing of the y-axis in rotating coordinates centered on the Earth-Moon combined mass

Table 20. First iteration step

	t_0 (Julian date)	parking orbit radius (km)	ΔV *(km/s)*	Δt *(days)*
initial	2453611	7200	3.08568	87.42570
solution	2453611	7200	3.04789	87.42570

4.2.5 Parking Orbit Orientation Selection in the Restricted Three Body Problem

With the result obtained above, the approximate Δv has been established for the BLCT. Continuing with Pu and Edelbaum's approximation of the four body problem, the orientation of the initial parking orbit should be obtained before attempting to propagate the trajectory in the more complicated four body dynamical system.

Let α, β and γ be the orientation angles for the parking orbit. These angles define the position of a spacecraft in a circular orbit of given radius, and are analogous to the inclination, longitude of the ascending node, and true anomaly, respectively. They differ from orbital elements only in that they are defined relative to the x-y plane of the solar system as opposed to the equatorial plane of the Earth. In the nonlinear targeting algorithm, the orientation angles represent the orientation of the spacecraft at the time of the transfer insertion burn. The transformation from a set of orientation angles to Cartesian coordinates is shown in Eq. 6.

$$\begin{aligned}\mathbf{r} &= \mathrm{r}\begin{pmatrix}\cos\beta\cos\alpha-\sin\beta\sin\alpha\cos\gamma\\ \cos\beta\sin\alpha+\sin\beta\cos\alpha\cos\gamma\\ \sin\beta\sin\gamma\end{pmatrix}\\ \mathbf{v} &= \sqrt{\frac{\mu_e}{\mathrm{r}}}\begin{pmatrix}-\cos\alpha\sin\beta-\cos\beta\sin\alpha\cos\gamma\\ -\sin\alpha\sin\beta+\cos\beta\cos\alpha\cos\gamma\\ \sin\gamma\cos\beta\end{pmatrix}\end{aligned} \tag{4.2.3}$$

Using the definition of the correction scheme described above, the four free variables in Eq. 7 are iterated from their values that were used in the last targeting step to target a point on the y-axis in the Sun-(Earth-Moon) rotating coordinate system. The target should be at a distance from the combined Earth-Moon mass equal to the distance from the Earth to the collinear Lagrange point of the Earth-Moon three body system exterior to the Moon, $\mathbf{c} = \mathbf{r} - \mathbf{r}_{L2}$. In this iteration step, four parameters are used to target a three dimensional point.

$$\mathbf{a} = \begin{pmatrix}\Delta \mathrm{V} & \alpha & \beta & \gamma\end{pmatrix}^{\mathrm{T}} \tag{4.2.4}$$

The result of the previous iterations should provide approximate solutions for the orientation of the spacecraft at the time of the transfer insertion burn as well as the magnitude of the burn, which is oriented along the direction of the velocity vector of the spacecraft. The approximation is satisfactorily accurate to converge to a solution in the complex dynamics of the Sun-Earth-Moon four body problem. Table 21 details the results of the second iteration step, continuing the targeting of the example trajectory in Figure 49.

Table 21. Second iteration step results

	t_0 (Julian date)	α (rad)	β (rad)	γ (rad)	ΔV *(km/s)*	Δt *(days)*
initial	2453611	0.00000	2.70878	0.40907	3.04789	87.42570
solution	2453611	0.62247	1.78275	0.35308	3.04889	87.42570

4.2.6 Targeting in the Restricted Four Body Problem

At this stage of the algorithm, the masses of the Moon and the Earth are decoupled, and a more realistic system is used. The numerical stability of any nonlinear targeting algorithm continues to be an issue, and the BLCT is targeted in two steps.

The four free parameters used in the previous targeting step are used again in the four body system. Equations of motion governing the four body problem are now used. The three gravitational bodies are treated as point masses located at positions determined from the JPL DE405 ephemeris. The coordinate system in which the trajectory is propagated is a non-rotating frame centered at the Earth-Moon barycenter. Equation (4.2.5) shows the restricted four body problem equations of motion with an indirect term to account for the acceleration of the origin of the coordinate system.

$$\ddot{\mathbf{r}}_{\mathrm{RFBP}} = -\frac{\mu_{\mathrm{e}}}{\left|\mathbf{r}-\mathbf{r}_{\mathrm{e}}\right|^3}\left(\mathbf{r}-\mathbf{r}_{\mathrm{e}}\right) - \frac{\mu_{\mathrm{m}}}{\left|\mathbf{r}-\mathbf{r}_{\mathrm{m}}\right|^3}\left(\mathbf{r}-\mathbf{r}_{\mathrm{m}}\right) - \frac{\mu_{\mathrm{e}}}{\left|\mathbf{r}-\mathbf{r}_{\mathrm{s}}\right|^3}\left(\mathbf{r}-\mathbf{r}_{\mathrm{s}}\right) - \frac{\mu_{\mathrm{s}}+\mu_{\mathrm{e}}+\mu_{\mathrm{m}}}{\left|\mathbf{r}_{\mathrm{s}}\right|^3}\mathbf{r}_{\mathrm{s}} \qquad (4.2.5)$$

The target in this iteration is the spacecraft's radial distance from the Moon. The iteration is considered successful if the final radial distance from the Moon is less than the distance from the Moon to L2 of the Earth-Moon three body system. In this case the constraint is an inequality constraint, $\mathrm{r}_{\mathrm{sc/m}} \le \mathrm{r}_{\mathrm{L_2/m}}$. The parameter vector **a** is the same as the previous targeting step, Eq.(4.2.4). Table 22 shows the details of the four body targeting step.

Table 22. Restricted four body targeting step

	t_0 (Julian date)	α (rad)	β (rad)	γ (rad)	Δ*V (km/s)*	Δ*t (days)*
initial	2453611	0.62247	1.78275	0.35308	3.04889	87.42570
solution	2453611	1.31989	1.22746	0.40292	3.04932	87.42570

4.2.7 Energy Minimization in the Restricted Four Body Problem

The final step of the nonlinear targeting algorithm is a constrained minimization of the spacecraft's Keplerian energy with respect to the Moon. In this minimization step, let

$J = KE_m$ be the scalar performance index to be minimized by a sequential quadratic programming algorithm[42]. The final time of the transfer is included with the initial time and the orientation angles of the parking orbit as free variables, shown in Eq. (4.2.6).

$$\mathbf{a} = \begin{pmatrix} \Delta \mathrm{V} & \alpha & \beta & \gamma & \mathrm{t_f} \end{pmatrix}^{\mathrm{T}} \tag{4.2.6}$$

The constraint in this step forces the trajectory to end at a periselene. This constraint, $c = f_m$, aids in convergence to trajectories that do not crash into the Moon and complete an orbit of the Moon without escaping. If desired, an additional inequality constraint may be added to ensure the periselene distance is sufficient to avoid collision with the lunar surface.

After a few iterations, the minimization results in a negative Keplerian energy, and thus a captured orbit around the Moon. If a particular orbit around the Moon is desired, constraints can be added to the minimization problem that will yield the desired orbit, possibly with another small burn. The results of the nonlinear targeting algorithm are shown in Table 23, and the capture trajectory is displayed in Figure 50. It is noteworthy that the trajectory already possesses negative energy with respect to the Moon before this iteration step; however, the minimization of energy is necessary to produce a trajectory that remains captured by the Moon for a significant period of time.

Table 23. Minimization of Keplerian energy

	t_0 (Julian date)	α (rad)	β (rad)	γ (rad)	ΔV *(km/s)*	Δt *(days)*	*KE* (km^2/s^2)
initial	2453611	1.31989	1.22746	0.40292	3.04932	87.42570	-0.05379
solution	2453611	1.36268	1.14152	0.40272	3.04971	99.03939	-0.10498

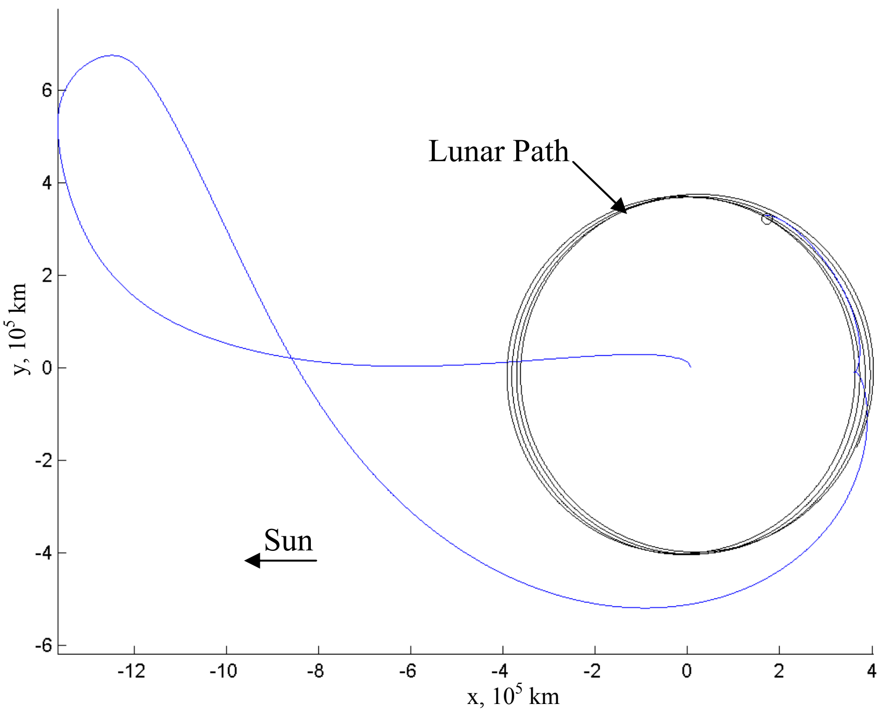

Figure 50. A *f*16p1 transfer shown in Earth centered Sun-Earth rotating coordinates

4.2.8 State Transition Matrix Derivatives for the BLCT Targeting Problem

Zimmer and Ocampo[43] have demonstrated the benefits in accuracy from using state transition matrix derivatives, as opposed to finite difference methods. First order derivatives relating the states at the initial time to the states at the final time can be calculated explicitly by solving for the state transition matrix. The state transition matrix relating to the propagation of a spacecraft through a central body force field can be solved for explicitly, and its derivatives are exact to first order. The dynamical models used in this algorithm do not yield explicit solutions to the state transition matrix, and therefore it must be numerically integrated. Error is introduced in the integration, but it is on the order of the error in the integration itself. The result is a more accurate representation of the required derivatives.

The state transition matrix of the three and four body problem is found by numerically integrating the relationship in Eq. (4.2.7), where *F* is the time dependent state propagation matrix for the desired dynamical model, shown in Eq. (4.2.8), where the state vector **X** is defined in Eq. (4.2.9).

$$\begin{aligned}\dot{\Phi}(t,t_0) &= F(t)\Phi(t,t_0)\\ \Phi(t_0,t_0) &= I\end{aligned} \tag{4.2.7}$$

$$F(t) = \frac{\partial \dot{\mathbf{X}}(t)}{\partial \mathbf{X}(t)} \tag{4.2.8}$$

$$\mathbf{X} = \begin{pmatrix}\mathbf{r}\\ \mathbf{v}\end{pmatrix} \tag{4.2.9}$$

The state propagation matrix for the three-body and four-body dynamical systems of interest to the algorithm are given in Eq. (4.2.10), where $\ddot{\mathbf{r}}$ is given in Eq. (4.2.2) for the three-body dynamical system and in Eq. (4.2.5) for the four-body dynamical system.

$$F(t) = \begin{pmatrix} 0 & I\\ \dfrac{\partial \ddot{\mathbf{r}}}{\partial \mathbf{r}} & 0\end{pmatrix} \tag{4.2.10}$$

For the non-rotating restricted four body problem, the sub-matrix in Eq. (4.2.10) that represents the partial derivative of the acceleration vector with respect to the radius vector is written as

$$\left(\frac{\partial \ddot{\mathbf{r}}}{\partial \mathbf{r}}\right)_{RFBP} = \left(\frac{3\mu_e}{r^5}\right)\mathbf{r}\mathbf{r}^T + \left(\frac{3\mu_m}{r_m^{\;5}}\right)\mathbf{r}_m\mathbf{r}_m^{\;T} + \left(\frac{3\mu_s}{r_s^{\;5}}\right)\mathbf{r}_s\mathbf{r}_s^{\;T} - \mathbf{I}\left(\frac{\mu_e}{r^3} + \frac{\mu_m}{r_m^{\;3}} + \frac{\mu_s}{r_3^{\;3}}\right) \tag{4.2.11}$$

while in the non-rotating restricted three body problem it is

$$\left(\frac{\partial \ddot{\mathbf{r}}}{\partial \mathbf{r}}\right)_{\mathrm{RTBP}} = \left(\frac{3\mu_e}{r^5}\right)\mathbf{r}\mathbf{r}^T + \left(\frac{3\mu_s}{r_s^5}\right)\mathbf{r}_s\mathbf{r}_s^T - \mathbf{I}\left(\frac{\mu_e}{r^3} + \frac{\mu_s}{r_3^3}\right). \tag{4.2.12}$$

Perturbations in the free variables are related to perturbations in the constraint values using the chain rule. The nonlinear targeting algorithm relies on partial derivatives relating both the final state and the final value of the Keplerian energy to the five control parameters listed in Eq. (4.2.6). For the orientation angle α, the partial derivatives are written in the form

$$\frac{\partial \mathrm{KE_f}}{\partial \alpha} = \frac{\partial \mathrm{KE_f}}{\partial \mathbf{X}_f}\frac{\partial \mathbf{X}_f}{\partial \mathbf{X}_0}\frac{\partial \mathbf{X}_i}{\partial \alpha} \tag{4.2.13}$$

and

$$\frac{\partial \mathbf{X}_f}{\partial \alpha} = \frac{\partial \mathbf{X}_f}{\partial \mathbf{X}_0}\frac{\partial \mathbf{X}_i}{\partial \alpha}, \tag{4.2.14}$$

where

$$\frac{\partial \mathbf{X}_f}{\partial \mathbf{X}_0} = \Phi(t_f, t_0), \tag{4.2.15}$$

$$\frac{\partial \mathrm{KE_f}}{\partial \mathbf{X}_f} = \begin{pmatrix} \frac{\mu_m}{r_m^3}(x - x_m) & \frac{\mu_m}{r_m^3}(y - y_m) & \frac{\mu_m}{r_m^3}(z - z_m) & v_x - v_{mx} & v_y - v_{my} & v_z - v_{mz} \end{pmatrix}, \tag{4.2.16}$$

and

$$\frac{\partial \mathbf{X}_i}{\partial \alpha} = \begin{pmatrix} r(-\sin\alpha\cos\beta - \cos\alpha\sin\beta\cos\gamma) \\ r(\cos\alpha\cos\beta - \sin\alpha\sin\beta\cos\gamma) \\ 0 \\ v(\sin\alpha\sin\beta - \cos\alpha\cos\beta\cos\gamma) \\ v(-\cos\alpha\sin\beta - \sin\alpha\cos\beta\cos\gamma) \\ 0 \end{pmatrix}. \tag{4.2.17}$$

Similarly, the partial derivatives relating to the orientation angle β are written as

$$\frac{\partial \mathrm{KE_f}}{\partial \beta} = \frac{\partial \mathrm{KE_f}}{\partial \mathbf{X}_f}\frac{\partial \mathbf{X}_f}{\partial \mathbf{X}_0}\frac{\partial \mathbf{X}_i}{\partial \beta} \tag{4.2.18}$$

and

$$\frac{\partial \mathbf{X}_f}{\partial \beta} = \frac{\partial \mathbf{X}_f}{\partial \mathbf{X}_0} \frac{\partial \mathbf{X}_i}{\partial \beta} \tag{4.2.19}$$

where the partial derivative of the Keplerian energy with respect to the final state is given in Eq. (4.2.16), the partial derivative of the final state with respect to the initial state is given as the state transition matrix, and

$$\frac{\partial \mathbf{X}_i}{\partial \beta} = \begin{pmatrix} r\left(-\cos\alpha\sin\beta - \sin\alpha\cos\beta\cos\gamma\right) \\ r\left(-\sin\alpha\sin\beta + \cos\alpha\cos\beta\cos\gamma\right) \\ r\cos\beta\sin\gamma \\ -v\left(\cos\alpha\cos\beta - \sin\alpha\sin\beta\cos\gamma\right) \\ -v\left(\sin\alpha\cos\beta + \cos\alpha\sin\beta\cos\gamma\right) \\ -v\sin\alpha\sin\gamma \end{pmatrix}. \tag{4.2.20}$$

The partial derivatives relating to the orientation angle γ are

$$\frac{\partial \mathrm{KE}_f}{\partial \gamma} = \frac{\partial \mathrm{KE}_f}{\partial \mathbf{X}_f} \frac{\partial \mathbf{X}_f}{\partial \mathbf{X}_0} \frac{\partial \mathbf{X}_i}{\partial \gamma} \tag{4.2.21}$$

and

$$\frac{\partial \mathbf{X}_f}{\partial \gamma} = \frac{\partial \mathbf{X}_f}{\partial \mathbf{X}_0} \frac{\partial \mathbf{X}_i}{\partial \gamma} \tag{4.2.22}$$

where the partial derivative of the Keplerian energy with respect to the final state is given in Eq. (4.2.16), the partial derivative of the final state with respect to the initial state is given as the state transition matrix, and

$$\frac{\partial \mathbf{X}_i}{\partial \gamma} = \begin{pmatrix} r\sin\alpha\sin\beta\sin\gamma \\ -r\cos\alpha\sin\beta\sin\gamma \\ r\sin\beta\cos\gamma \\ v\left(\sin\alpha\cos\beta\sin\gamma\right) \\ -v\left(\cos\alpha\cos\beta\sin\gamma\right) \\ v\cos\beta\cos\gamma \end{pmatrix}. \tag{4.2.23}$$

The initial maneuver produces the partial derivatives

$$\frac{\partial \mathrm{KE}_f}{\partial \Delta V} = \frac{\partial \mathrm{KE}_f}{\partial \mathbf{X}_f} \frac{\partial \mathbf{X}_f}{\partial \mathbf{X}_0} \frac{\partial \mathbf{X}_i}{\partial \Delta V} \tag{4.2.24}$$

and

$$\frac{\partial \mathbf{X}_f}{\partial \Delta V} = \frac{\partial \mathbf{X}_f}{\partial \mathbf{X}_0} \frac{\partial \mathbf{X}_i}{\partial \Delta V} \tag{4.2.25}$$

where the partial derivative of the Keplerian energy with respect to the final state is given in Eq. (4.2.16), the partial derivative of the final state with respect to the initial state is given as the state transition matrix, and

$$\frac{\partial \mathbf{X}_i}{\partial \Delta V} = \begin{pmatrix} \mathbf{0}_{3x1} \\ \Delta V\left(-\cos\alpha\sin\beta - \sin\alpha\cos\beta\cos\gamma\right) \\ \Delta V\left(-\sin\alpha\sin\beta + \cos\alpha\cos\beta\cos\gamma\right) \\ \Delta V\cos\beta\sin\gamma \end{pmatrix}. \tag{4.2.26}$$

Finally, the partial derivative of the final state with respect to the final time does not include the state transition matrix.

$$\frac{\partial KE_f}{\partial t_f} = \frac{\partial KE_f}{\partial \mathbf{X}_f} \frac{\partial \mathbf{X}_f}{\partial t_f} \tag{4.2.27}$$

where

$$\frac{\partial \mathbf{X}_f}{\partial t_f} = \begin{pmatrix} \mathbf{v}_f \\ \mathbf{a}_f \end{pmatrix} \tag{4.2.28}$$

and $\mathbf{a}_f$ is defined in Eq. (4.2.5) for the four body problem and Eq. (4.2.2) for the three body problem.

4.3 RESULTS

A successful nonlinear targeting algorithm will systematically produce captured trajectories. In a multibody system, however, the definition of capture is problematic. In the two body problem, a negative Keplerian energy is sufficient for ensuring the spacecraft will not escape the system. Similarly, in the restricted three body problem a Jacobi energy value that produces zero velocity surfaces that constrain the motion of the spacecraft to one of the primary bodies is an identifier of a captured spacecraft. In the four body problem used in the nonlinear targeting algorithm, there are no constant energy-like quantities that bind the motion of the spacecraft. This problem has been previously dealt with in several ways. Belbruno, in defining the WSB, defines a weakly captured trajectory as a trajectory that completes an orbit of a body, returning to the vertical plane from which it began, with negative Keplerian energy.[4] In finding WSB transfers using manifolds in the restricted three body problem, Koon et al. define a successful transfer as one that transits through the halo orbit into the system of the secondary body.[14]

The following three metrics for evaluating the capture of a spacecraft into lunar orbit are proposed to evaluate the success of the nonlinear targeting algorithm. At the conclusion of the final step of the algorithm the trajectory is at a periselene. Although termination at periselene is not required in the algorithm to have negative Keplerian energy with respect to the Moon, a successfully captured trajectory should have negative energy at this point. The first metric used to evaluate the algorithm is whether a solution is found that has negative Keplerian energy with respect to the Moon at this point. Furthermore, a second, more stringent metric is used that demands a binding of the motion of the spacecraft to the Moon. Under the second metric, the algorithm is considered to be successful if there is a second periselene with negative Keplerian energy as the trajectory is propagated forward, ensuring an orbit of the Moon. Finally, a third metric is used to ensure against collision. Under this metric, the algorithm is deemed to be successful if it meets the previous two requirements, and the periselene radii are greater than the Moon's radius.

Low energy transfers are shown to be found in a systematic way when periodic orbits in the CRTBP are used to initialize the algorithm. To validate the robustness of the technique, a computer program was designed to implement the algorithm described above with the goal of computing a low energy lunar transfer on a user supplied date. Transfers of class *f*16p1 and *f'*16p1 were computed in an automated way. The program was then supplied with a randomly generated date and time between January 1, 2010 and January 1, 2012, and a low energy lunar transfer was calculated for that date. The program was fed 1000 randomly generated dates and times. The results of the run are shown in Table 24.

Table 24. Algorithm results

Number of Runs	Criteria 1 (negative energy)	Criteria 2 (orbit)	Criteria 3 (orbit, no collision)
1000	100%	96.50%	91.10%

4.4 TARGETING ALGORITHM COMPARISON

The periodic orbit based nonlinear targeting algorithm is related to the 2x2 targeting algorithm presented by Belbruno and Carrico.[48] The main difference in the two methods is in the generation of a trajectory that serves as an initial guess. The Belbruno and Carrico method uses an initial guess based on a Hohmann transfer to an apogee of 1.5 million kilometers. The orientation of the initial guess is determined by selecting a launch window where the proper orientation of the Sun, Earth, and Moon are achieved. The Griesemer and Ocampo method presented above relies upon periodic orbits of the Earth-Moon CRTBP to estimate an initial maneuver and orient the transfer. The advantages of the latter method include greater accuracy in the initial guess due to the inclusion of the Sun's gravity, an infinite launch window, and the ability to target multiple types of BLCTs, either from different families of periodic orbits or different perigees within a family of orbits. Table 25 summarizes the differences between the two methods.

Table 25. Comparison between the Belbruno-Carrico and Griesemer-Ocampo algorithms

Targeting Algorithm Comparison		
Algorithm Property	Belbruno and Carrico	Griesemer and Ocampo
Launch widow	2x per month	Unlimited
Target	Lunar radius, inclination	Captured lunar orbit
Initial guess	Hohmann transfer ellipse	3 body low energy transfer
Transfer type	f16p1	User specified

4.5 Chapter Conclusions

A stable real-world nonlinear targeting algorithm is presented that successfully uses low energy transfers of families f16p1 and f'16p1 to find BLCTs in a four body problem with realistic ephemerides. The algorithm overcomes the numerical difficulties of targeting chaotic transfers in the four body problem by beginning with an initial guess that represents well the dynamics of the system, and by supplying precise derivatives calculated from the state transition matrix. Solutions are targeted that minimize the energy of the spacecraft with respect to the Moon at the final time.

The success of the nonlinear targeting algorithm is measured based on three different metrics. The algorithm is successful in all cases in producing transfers that terminate in a capture state with respect to the Moon. When the additional criteria are imposed that define a successful result as an trajectory that completes an orbit of the Moon without crashing into the lunar surface, the algorithm produces successful trajectories over ninety percent of the time. It is expected that in the cases where a successful trajectory is not found, numerical difficulties could be overcome with user input in the targeting process.

A comparison is made between the presented nonlinear targeting algorithm and the previously published Belbruno-Carrico targeting algorithm. While the two targeting routines share similar parameters and constraints, it is shown that the algorithm presented in this chapter is unrestricted in terms of launch window, and provides and initial guess that is expected to be nearer to the final result. Consequently, faster and more reliable convergence is expected.

5. Optimality of Ballistic Lunar Capture Transfers

5.1 INTRODUCTION

Previous chapters have shown that BLCTs can be found by inserting low energy transfers in the CRTBP into the four body problem. It has also been shown that the low energy transfers on which the BLCTs are based meet the necessary conditions of optimality for orbit-to-orbit transfers. This chapter extends the analysis to the study of the optimality of the BLCTs in the four body problem.

Primer vector theory is used to examine the local optimality of orbit-to-capture transfers. Analysis of the optimality of these types of transfers has not been previously attempted with primer vector theory. New necessary conditions are derived in this chapter to apply to the trajectory at the final time. A numerical example of a BLCT is given and shown to meet the conditions of local optimality based on the new necessary conditions. The necessary conditions that are derived agree with previous results where the theory overlaps, and are newly derived where previous theory is insufficient.

5.2 PRIMER VECTOR CONDITIONS FOR ORBIT-TO-CAPTURE TRANSFER

The derivation of the necessary conditions for the orbit-to-capture transfer involves a modification of the optimal control problem presented in Chapter 2. The cost function for the new problem is the magnitude of a single impulse at the initial time, and the constraints on the final state are modified to fit the captured final condition. Otherwise, the derivation of the necessary conditions follows from the same formulation previously presented.

5.2.1 Cost function

An optimal control will minimize the total impulse provided to the spacecraft, represented by the following performance index,

$$J = \int_{t_1^-}^{t_2} \Delta v_1 \delta(t - t_1) dt , \tag{5.2.1}$$

where Δv_1 is the magnitude of the initial impulses, and $\delta(t - t_0)$ is an impulse function in which the impulse occurs at time $t = t_0$. The definition of the impulse function is given in Eqns. (2.4.2) to (2.4.4).

5.2.2 Constraint Definitions

The controls in the orbit-to-capture problem, shown in Eq. (5.2.2), are the magnitude and direction of the transfer insertion maneuver at time t_1 and the location of the spacecraft on a specified orbit around a gravitational body, represented by the time-like parameter, τ_1, discussed below. Both t_1 and t_2 are free parameters.

$$\mathbf{u} = \begin{pmatrix} \Delta v_1 & \mathbf{l}_1^T & \tau_1 \end{pmatrix}^T \tag{5.2.2}$$

A control constraint exists on the thrust direction vector $\mathbf{l}_1$ so that its magnitude is unity.

$$C = \left(\mathbf{l}_1^T \mathbf{l}_1 - 1 \right) = \mathbf{0} \tag{5.2.3}$$

The differential constraint for the problem is the state equation,

$$\dot{\mathbf{x}} = \begin{pmatrix} \dot{\mathbf{r}} \\ \dot{\mathbf{v}} \end{pmatrix} = \begin{pmatrix} \mathbf{v} \\ \mathbf{g} + \delta(t - t_1) \Delta v_1 \mathbf{l}_1 \end{pmatrix}, \tag{5.2.4}$$

where $\mathbf{r}$ and $\mathbf{v}$ are the position vector and the velocity vector, respectively. The gravitational acceleration vector $\mathbf{g}$ represents a general force field, and although the low energy lunar transfer is typically calculated with a four body system, no assumptions about the composition of the gravity field will be made in the derivation. The acceleration vector for the spacecraft consists of the gravitational acceleration and the acceleration due to the impulsive maneuver.

The boundary conditions are such that they constrain the initial state of the spacecraft to lie on a pre-established orbit. Following the methodology of Ocampo,[28] a time-like variable, τ_1, is introduced into the problem to define a plane in phase space that contains the initial orbit. This variable is defined such that $\mathbf{r}(\tau_1)$ and $\mathbf{v}(\tau_1)$ lie on the initial orbit for any τ_1. In this way, τ_1 is similar to the true anomaly of an orbit, or any other variable that defines a spacecraft's orientation on a particular orbit. The boundary conditions that constrain the initial state of the spacecraft have the following form:

$$\boldsymbol{\theta} = \begin{pmatrix} \mathbf{r}_1 - \mathbf{r}(\tau_1) \\ \mathbf{v}_1 - \mathbf{v}(\tau_1) \end{pmatrix} = \mathbf{0} . \tag{5.2.5}$$

The boundary conditions at the final time are generalized to require the spacecraft to arrive at the second body with a specific energy state with respect to the second body less than or equal to a stated value. For a spacecraft to be captured in a two body system, the stated energy level would be less than zero. In a multi-body system, however, a stronger constraint of may be desired to increase the likelihood of the spacecraft orbiting the second body before it escapes. In the derivation, the desired energy level will be left unspecified. An inequality constraint is used to allow for the possibility of an optimal transfer arriving at the secondary body at a more strongly captured state than specified. The resulting boundary condition at the final time is

$$\psi = \mathrm{E}^* - \frac{\mathrm{v_s}^2}{2} - \frac{\mu_\mathrm{s}}{\mathrm{r_s}} \le 0 , \tag{5.2.6}$$

where r_s and v_s are the radius and velocity with respect to the secondary body, μ_s is the mass parameter of the secondary body, and E^* is the desired energy state. Following the methodology of Hull,[41] a slack variable is introduced to convert the inequality constraint in Eq. (5.2.6) into an equality constraint. A consequence of the introduction of a slack variable into the constraint is an additional parameter in the problem. The boundary condition becomes

$$\psi = \frac{v_s^{\,2}}{2} - \frac{\mu_s}{r_s} - E^* + \alpha^2 = 0 \tag{5.2.7}$$

where α is the slack variable.

The endpoint function of the problem is

$$G = \upsilon\psi + \boldsymbol{\xi}^T \boldsymbol{\theta} \tag{5.2.8}$$

where υ and $\boldsymbol{\xi}$ are a scalar and a vector, respectively, of constant Lagrange multipliers adjoined to the endpoint constraints.

An extended Hamiltonian is formed by using a Lagrange multiplier, μ, to adjoin the control constraint in Eq. (5.2.3) to the Hamiltonian,

$$\hat{H} = \boldsymbol{\lambda}_r^{\,T}\mathbf{v} + \boldsymbol{\lambda}_v^{\,T}\left(\mathbf{g} + \Delta v_1 \delta(t - t_1)\mathbf{l}_1\right) + \Delta v_1 \delta(t - t_1) + \mu C \; . \tag{5.2.9}$$

The extended Hamiltonian is constant if the gravitational acceleration vector is not a function of time. It is not necessarily an integral of the problem for the general force field.

The performance index and constraints described above form a time-free optimal control problem. The necessary conditions from the first differential are given by the Euler-Lagrange equations shown in Eqns. (2.4.14) to (2.4.16) and the necessary boundary conditions shown in Eq. (2.4.17).

The conditions of optimality pertaining to the impulse in the orbit-to-capture problem, given in Chapter 2.4, are equivalent to the orbit-to-orbit necessary conditions derived in the work of Lawden, Jezewski, and others. As previously stated, the primer vector must be aligned with the impulse at the time of the impulse, the time derivative of its magnitude must be zero, and its magnitude must be unity:

$$\mathbf{p}_1 = \frac{\mathbf{\Delta V_1}}{\Delta v_1} \tag{5.2.10}$$

$$\dot{p}_1 = 0\,, \tag{5.2.11}$$

where

$$\mathbf{p} = -\boldsymbol{\lambda}_v\,. \tag{5.2.12}$$

At time t_2, there are two complications that call for necessary conditions that differ from the conventional derivation. First, the inequality endpoint constraint does not fit neatly into the optimal control theory formulation of Hull. After extending Hull's formulation to account for an endpoint inequality constraint, the inclusion of the slack variable produces two separate cases: one when the solution is on the boundary of the constraint, and one when it is off of the boundary. Secondly, the ballistic arc that terminates in a captured orbit provides a set of conditions at the final time that have not been studied with primer vector theory. New conditions need to be derived from primer vector theory to account for the instance of reaching the targeted energy without an impulse to conclude the transfer.

Hull’s derivation allows for the endpoint function to depend on the initial and final time, the state variables, and constant Lagrange multipliers. The slack variable associated with the inequality constraint cannot be considered to belong to any class of these variables. It can be dealt with outside of Hull’s formulation by treating it as a separate class of variable and including it in a modified endpoint function. The new modified endpoint function, G', depends on the initial and final time, the state variables, constant Lagrange multipliers, and the slack variable, α.

Hull derives the first order conditions of optimality by taking the first differential of the extended cost function, which includes the endpoint function. By adding the slack variable to the parameters of the problem, a term appears in the differential that depends on $\mathrm{d}\alpha$,

$$\frac{\mathrm{d}G'}{\mathrm{d}\alpha} = 2\upsilon\alpha\,. \tag{5.2.13}$$

In order for a trajectory to be optimal, the expression in Eq. (5.2.13) must be equal to zero. This optimality condition can be satisfied in two cases. In the first case, α is equal to zero and the solution lies on the boundary of the inequality constraint. In the second case, the Lagrange multiplier is equal to zero. This condition applies to the situation where the optimality conditions are satisfied on a trajectory that terminates with an energy level less than the targeted energy level. When this condition is applied to the previous optimality condition in Eq. (5.2.14), it can be seen that the primer vector and the derivative of its magnitude must be zero. All other conditions on G' are equivalent to the conditions on G in Hull's formulation.

To determine necessary conditions appropriate for an orbit to capture trajectory at the terminal time, consider the endpoint conditions on $\boldsymbol{\lambda}$ from Eq. (2.4.17):

$$\begin{aligned} \boldsymbol{\lambda}_{r_2} &= G_{\mathbf{r}_2}{}^{T} = \upsilon\frac{\mu_s}{r_s^3}\mathbf{r}_s \\ \boldsymbol{\lambda}_{v_2} &= G_{\mathbf{v}_2}{}^{T} = \upsilon v_s \mathbf{v}_s \end{aligned}\,. \tag{5.2.14}$$

Equation (5.2.14) establishes new necessary conditions for the orbit-to-capture problem. It dictates that the primer vector at the final time is aligned with the velocity vector of the spacecraft with respect to the second gravitational body, and the rate of change of the primer vector is aligned with the radius vector of the spacecraft with respect to the gravitational body. Furthermore, because the constant Lagrange multiplier υ appears in both expressions, its value is not arbitrary. The parameter υ establishes a relationship between the magnitude of the primer vector and its time derivative. It can be solved for by post-multiplying the equation for the Lagrange multipliers associated with the velocity by the relative velocity vector,

$$\upsilon = \frac{\boldsymbol{\lambda}_{v_2}{}^{T}\mathbf{v}_s}{v_s{}^3}. \tag{5.2.15}$$

$\boldsymbol{\lambda}_{v_2}$ and $\mathbf{v}_s$ must be parallel to satisfy Eq. (5.2.14), so their inner product can be reduced to the product of their two magnitudes, resulting in a simplified expression for υ,

$$\upsilon = \pm\frac{\lambda_{v_2}}{v_s} \tag{5.2.16}$$

The sign of Eq. (5.2.16) is left to be determined by analyzing the results of a particular problem.

Together, Eqns. (5.2.14) and (5.2.13) define the necessary conditions at the final time for a ballistic capture trajectory. They apply to the magnitude and direction of both the primer vector and the time derivative of the primer vector, and represent trajectories that terminate with an energy level either equal to that targeted or less than the targeted value. Additionally, it can be shown that the magnitude of the primer vector must be less than unity to preclude an additional impulse.

5.2.3 Additional Impulses

A separate analysis can be made to determine whether an intermediate impulse has the potential to reduce the cost of the maneuver. Consider a trajectory that has an additional maneuver at time t_k. The performance index of the trajectory would be changed to

$$J = \int_{t_1^-}^{t_2^+} \Delta v_1 \delta(t - t_1) + \Delta v_k \delta(t - t_k)\,dt, \tag{5.2.17}$$

and the vector of controls would include the magnitude, time, and direction of the intermediate maneuver,

$$\mathbf{u} = \begin{pmatrix} \Delta v_1 & \mathbf{l}_1 & \tau_1 & \Delta v_k & \mathbf{l}_k & t_k \end{pmatrix}^T . \tag{5.2.18}$$

The unit thrust vectors are subject to the constraint

$$\mathbf{C} = \begin{pmatrix} \mathbf{l}_1{}^T \mathbf{l}_1 - 1 \\ \mathbf{l}_k{}^T \mathbf{l}_k - 1 \end{pmatrix} = \mathbf{0} , \tag{5.2.19}$$

and the extended Hamiltonian is

$$\begin{aligned} \hat{H} = \boldsymbol{\lambda}_r{}^T \mathbf{v} + \boldsymbol{\lambda}_v{}^T \left(\mathbf{g} + \Delta v_1 \delta(t - t_1) \mathbf{l}_1 + \Delta v_k \delta(t - t_k) \mathbf{l}_k \right) \\ + \Delta v_1 \delta(t - t_1) + \Delta v_k \delta(t - t_k) + \boldsymbol{\mu}^T \mathbf{C} \end{aligned} . \tag{5.2.20}$$

Applying the optimality condition in Eq. (2.4.16) to Eq. (5.2.20) gives the following result when considering the magnitude of the primer vector at an intermediate impulse:

$$\hat{H}_{\Delta v_k} = \delta(t - t_k) \boldsymbol{\lambda}_v{}^T \mathbf{l}_k + \delta(t - t_k) = 0 \tag{5.2.21}$$

At time t_k Eq. (5.2.21) reduces to

$$-\left| \boldsymbol{\lambda}_{v_k} \right| = -1 . \tag{5.2.22}$$

The result obtained from applying the same condition at the initial time is

$$-\left| \boldsymbol{\lambda}_{v_1} \right| = -1 . \tag{5.2.23}$$

It can be seen from comparing Eq. (5.2.22) to Eq. (5.2.23) that a necessary condition for an intermediate impulse is

$$\left| \mathbf{p}_k \right| = \left| \mathbf{p}_1 \right| . \tag{5.2.24}$$

Equation (5.2.24) requires that an optimal trajectory with an intermediate impulse have primer vectors with equal magnitudes at each impulse. Consequently, if the primer vector never again returns to unit magnitude between times t_1 and t_2, there is no potential for an improvement in the performance index via an intermediate impulse.

5.3 NUMERICAL DEMONSTRATION OF OPTIMALITY

The boundary conditions and the necessary conditions of the optimal control problem define a two point boundary value problem that can be numerically solved to yield an control that satisfies the conditions of optimality. The unknown and constraint vectors associated with the problem are shown in Eq. (5.3.1). The differential equations to be satisfied are the state and co-state equations shown in Eqns. (2.4.14) and (2.4.15).

$$\mathbf{a}_{11\mathrm{x}1} = \begin{pmatrix} \mathrm{t}_1 & \mathrm{t}_2 & \mathrm{f} & \dot{\mathbf{p}}_1 & \mathbf{\Delta v}_1 & \upsilon & \alpha \end{pmatrix}^{\mathrm{T}},$$
$$\mathbf{c}_{11\mathrm{x}1} = \begin{pmatrix} \hat{\mathrm{H}}_1 & \hat{\mathrm{H}}_2 & \mathrm{E} - \mathrm{E}^* + \alpha^2 & 2\alpha\upsilon & \boldsymbol{\lambda}_{\mathrm{r}_2} - \upsilon\dfrac{\mu_{\mathrm{s}}}{\mathrm{r}_{\mathrm{s}}^3}\mathbf{r}_{\mathrm{s}} & \boldsymbol{\lambda}_{\mathrm{v}_2} - \upsilon \mathrm{v}_{\mathrm{s}} \mathbf{v}_{\mathrm{s}} & \dot{\mathrm{p}}_1 \end{pmatrix}^{\mathrm{T}} \tag{5.3.1}$$

This system, with an appropriate initial guess, can be solved by a numerical nonlinear equation solver such as NS-12.[55]

For the constraints that are applied at the initial time, partial derivatives can be derived directly from the initial state. For the constraints at the final time, the state transition matrix can be used in conjunction with the chain rule to determine partial derivatives that relate parameters at the initial time to the final constraints. It should be noted, however, that the state transition matrix can only relate changes in the state that occur at a common time. Therefore, when relating constraint values to the variation of the initial time, an additional step must be taken. The changes in the state at $\mathrm{t} = \mathrm{t}_1$ must be related to the change in the initial time through calculating a numerical derivative that relates to change in the state at a fixed time t_1 to a change in the initial time. This derivative can be calculated with the forward difference method by simply integrating the state from a perturbed time back to t_1. The partial derivatives used in the numerical routine are shown below.

$$\frac{\partial c}{\partial a}=\begin{pmatrix} \boldsymbol{\lambda}_{v}{}^{T}\frac{\partial \mathbf{g}_1}{\partial t_1} & 0 & \frac{\partial \hat{H}_1}{\partial \mathbf{X}_1}\frac{\partial \mathbf{X}_1}{\partial f} & \mathbf{0} & \mathbf{0} & 0 & 0 \\ \frac{\partial \hat{H}_2}{\partial \mathbf{X}_2}\Phi(t_2,t_1)\frac{\partial \mathbf{X}_1}{\partial t_1} & \frac{\partial \hat{H}_2}{\partial \mathbf{X}_2}\frac{\partial \mathbf{X}_2}{\partial t_2} & \frac{\partial \hat{H}_2}{\partial \mathbf{X}_2}\Phi(t_2,t_1)\frac{\partial \mathbf{X}_1}{\partial f} & \frac{\partial \hat{H}_2}{\partial \mathbf{X}_2}\Phi(t_2,t_1)\frac{\partial \mathbf{X}_1}{\partial \dot{\mathbf{p}}_1} & \frac{\partial \hat{H}_2}{\partial \mathbf{X}_2}\Phi(t_2,t_1)\frac{\partial \mathbf{X}_1}{\partial \boldsymbol{\Delta}\mathbf{v}_1} & 0 & 0 \\ \frac{\partial c_3}{\partial \mathbf{X}_2}\Phi(t_2,t_1)\frac{\partial \mathbf{X}_1}{\partial t_1} & \frac{\partial c_3}{\partial \mathbf{X}_2}\frac{\partial \mathbf{X}_2}{\partial t_2} & \frac{\partial c_3}{\partial \mathbf{X}_2}\Phi(t_2,t_1)\frac{\partial \mathbf{X}_1}{\partial f} & \frac{\partial c_3}{\partial \mathbf{X}_2}\Phi(t_2,t_1)\frac{\partial \mathbf{X}_1}{\partial \dot{\mathbf{p}}_1} & \frac{\partial c_3}{\partial \mathbf{X}_2}\Phi(t_2,t_1)\frac{\partial \mathbf{X}_1}{\partial \boldsymbol{\Delta}\mathbf{v}_1} & 0 & -2\alpha \\ 0 & 0 & 0 & \mathbf{0} & \mathbf{0} & 0 & 2\mu \\ \frac{\partial \mathbf{c}_5}{\partial \mathbf{X}_2}\Phi(t_2,t_1)\frac{\partial \mathbf{X}_1}{\partial t_1} & \frac{\partial \mathbf{c}_5}{\partial \mathbf{X}_2}\frac{\partial \mathbf{X}_2}{\partial t_2} & \frac{\partial \mathbf{c}_5}{\partial \mathbf{X}_2}\Phi(t_2,t_1)\frac{\partial \mathbf{X}_1}{\partial f} & \frac{\partial \mathbf{c}_5}{\partial \mathbf{X}_2}\Phi(t_2,t_1)\frac{\partial \mathbf{X}_1}{\partial \dot{\mathbf{p}}_1} & \frac{\partial \mathbf{c}_5}{\partial \mathbf{X}_2}\Phi(t_2,t_1)\frac{\partial \mathbf{X}_1}{\partial \boldsymbol{\Delta}\mathbf{v}_1} & -\frac{\mu_s}{r_s^3}\mathbf{r}_s & 0 \\ \frac{\partial \mathbf{c}_6}{\partial \mathbf{X}_2}\Phi(t_2,t_1)\frac{\partial \mathbf{X}_1}{\partial t_1} & \frac{\partial \mathbf{c}_6}{\partial \mathbf{X}_2}\frac{\partial \mathbf{X}_2}{\partial t_2} & \frac{\partial \mathbf{c}_6}{\partial \mathbf{X}_2}\Phi(t_2,t_1)\frac{\partial \mathbf{X}_1}{\partial f} & \frac{\partial \mathbf{c}_6}{\partial \mathbf{X}_2}\Phi(t_2,t_1)\frac{\partial \mathbf{X}_1}{\partial \dot{\mathbf{p}}_1} & \frac{\partial \mathbf{c}_6}{\partial \mathbf{X}_2}\Phi(t_2,t_1)\frac{\partial \mathbf{X}_1}{\partial \boldsymbol{\Delta}\mathbf{v}_1} & -v_s\mathbf{v}_s & 0 \\ 0 & 0 & 0 & \frac{1}{\dot{p}_1}\dot{\mathbf{p}}_1{}^{T} & \mathbf{0} & 0 & 0 \end{pmatrix} \tag{5.3.2}$$

where

$$\frac{\partial \hat{H}_2}{\partial \mathbf{X}_2}=\begin{pmatrix} \boldsymbol{\lambda}_{v_2}{}^{T}\frac{\partial \mathbf{g}_2}{\partial \mathbf{r}_2} & \boldsymbol{\lambda}_{r_2}{}^{T} & \mathbf{g}_2{}^{T} & \mathbf{v}_2{}^{T} \end{pmatrix} \tag{5.3.3}$$

$$\frac{\partial\left(E-E^{*}+\alpha^{2}\right)}{\partial \mathbf{X}_2}=\begin{pmatrix} \frac{\mu_m}{r_m{}^3}\mathbf{r}_m{}^{T} & \mathbf{v}_m{}^{T} & \mathbf{0} & \mathbf{0} \end{pmatrix} \tag{5.3.4}$$

$$\frac{\partial \mathbf{c}_5}{\partial \mathbf{X}_2}=\begin{pmatrix} \upsilon\mu_m\left(-\frac{1}{r_m{}^3}\mathbf{I}+3\frac{1}{r_m{}^5}\mathbf{r}_m\mathbf{r}_m{}^{T}\right) & \mathbf{0} & \mathbf{0} & \mathbf{I} \end{pmatrix} \tag{5.3.5}$$

$$\frac{\partial \mathbf{c}_6}{\partial \mathbf{X}_2}=\begin{pmatrix} \mathbf{0} & -\upsilon\left(v_m\mathbf{I}+\frac{1}{v_m}\mathbf{v}_m\mathbf{v}_m{}^{T}\right) & \mathbf{I} & \mathbf{0} \end{pmatrix} \tag{5.3.6}$$

$$\frac{\partial \mathbf{X}_2}{\partial t_2}=\begin{pmatrix} \mathbf{v}^{T} & \mathbf{g}^{T} & \dot{\boldsymbol{\lambda}}_r{}^{T} & \dot{\boldsymbol{\lambda}}_v{}^{T} \end{pmatrix} \tag{5.3.7}$$

$$\frac{\partial \mathbf{X}_1}{\partial \dot{\mathbf{p}}_1}=\begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \end{pmatrix} \tag{5.3.8}$$

$$\frac{\partial \mathbf{X}_1}{\partial \boldsymbol{\Delta}\mathbf{v}_1}=\begin{pmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \end{pmatrix} \tag{5.3.9}$$

$$\frac{\partial \mathbf{X}_1}{\partial \mathrm{f}} = \begin{pmatrix} \mathrm{a}\left(-\cos\Omega\sin\mathrm{f} + \sin\Omega\cos\mathrm{f}\cos\mathrm{i}\right) \\ \mathrm{a}\left(-\sin\Omega\sin\mathrm{f} + \cos\Omega\cos\mathrm{f}\cos\mathrm{i}\right) \\ \mathrm{a}\cos\mathrm{f}\sin\mathrm{i} \\ -\mathrm{v}\left(\cos\Omega\cos\mathrm{f} - \sin\Omega\sin\mathrm{f}\cos\mathrm{i}\right) \\ -\mathrm{v}\left(\sin\Omega\cos\mathrm{f} + \cos\Omega\sin\mathrm{f}\cos\mathrm{i}\right) \\ -\mathrm{v}\sin\mathrm{f}\sin\mathrm{i} \end{pmatrix} \tag{5.3.10}$$

Note that in equations (5.3.14) to (5.3.10) the initial orbit around the Earth is circular, where *a* is the semi-major axis of the orbit, *f* is the true anomaly of the orbit, *i* is the inclination, *v* is the magnitude of the velocity, and Ω is the right ascension of the ascending node. $\Phi(t_2, t_1)$ is the state transition matrix associated with the four body equations of motion given in Eq (3.3.5). In the formulations for chapters five and six, the state vector *X* includes both the spacecraft's state and costate,

$$\mathbf{X} = \begin{pmatrix} \mathbf{r} \\ \mathbf{v} \\ \boldsymbol{\lambda}_{\mathrm{r}} \\ \boldsymbol{\lambda}_{\mathrm{v}} \end{pmatrix} \tag{5.3.11}$$

$$\dot{\mathbf{X}} = \begin{pmatrix} \mathbf{v} \\ \mathbf{a} \\ \left(\left(\frac{3\mu_{\mathrm{e}}}{\mathrm{r}^5}\right)\mathbf{r}\mathbf{r}^{\mathrm{T}} + \left(\frac{3\mu_{\mathrm{m}}}{\mathrm{r}_{\mathrm{m}}^{5}}\right)\mathbf{r}_{\mathrm{m}}\mathbf{r}_{\mathrm{m}}^{\mathrm{T}} + \left(\frac{3\mu_{\mathrm{s}}}{\mathrm{r}_{\mathrm{s}}^{5}}\right)\mathbf{r}_{\mathrm{s}}\mathbf{r}_{\mathrm{s}}^{\mathrm{T}} - \mathbf{I}\left(\frac{\mu_{\mathrm{e}}}{\mathrm{r}^3} + \frac{\mu_{\mathrm{m}}}{\mathrm{r}_{\mathrm{m}}^{3}} + \frac{\mu_{\mathrm{s}}}{\mathrm{r}_{3}^{3}}\right)\right)\boldsymbol{\lambda}_{\mathrm{v}} \\ -\boldsymbol{\lambda}_{\mathrm{r}} \end{pmatrix} \tag{5.3.12}$$

$$\mathbf{F} = \begin{pmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \frac{\partial \mathbf{a}}{\partial \mathbf{r}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \frac{\partial \dot{\boldsymbol{\lambda}}_{\mathrm{r}}}{\partial \mathbf{r}} & \mathbf{0} & \mathbf{0} & -\frac{\partial \mathbf{a}}{\partial \mathbf{r}} \\ \mathbf{0} & \mathbf{0} & -\mathbf{I} & \mathbf{0} \end{pmatrix} \tag{5.3.13}$$

These partial derivatives are employed in the targeting code to iterate from an initial guess to an optimal solution. The guesses are propagated using a numerical predictorcorrector integration method.[56] Partial derivatives based on the state transition matrix are used to increase the precision of the solver, and are provided in Appendix A.

$$\frac{\partial \mathbf{X}(t_2)}{\partial t_1} = \Phi(t_2, t_1)\frac{\partial \mathbf{X}(t_1)}{\partial t_1} \tag{5.3.14}$$

where $\partial\left[x(t_1)\right]/\partial t_1$ is calculated using the forward difference method by varying the initial time by a small amount and numerically integrating back to t_1.

An optimal transfer is sought from the low Earth orbit and detailed in Table 26 to a captured state around the Moon using a single impulsive maneuver. A previously targeted non-optimal transfer is used as an initial guess, and improved upon by targeting the necessary conditions developed above. Figure 51 shows a converged solution in non-rotating coordinates centered at the Earth-Moon barycenter.

Table 26. Parking orbit properties at t_0

Orbit property	Value
epoch (JED)	2455658.47677
a (km)	7200
e	0
i (deg)	47.188
ω (deg)	0
Ω (deg)	1.09948
f (deg)	22.6881

Following the procedure detailed in Chapter 4, a low energy transfer to a captured lunar orbit using a single impulse is calculated. The parameters of the transfer are listed in Table 27, where the subscript v refers to the component of the vector in the direction of the velocity, the subscript h refers to the component in the direction of the angular momentum, and the subscript r refers to the direction that completes the right-handed coordinate system. The final state of the transfer lies on an elliptical two body orbit about the Moon.

Table 27. Transfer properties

Transfer Property	Value
ΔV_v (km/sec)	3.05047
ΔV_r (km/sec)	0.0
ΔV_h (km/sec)	0.0
Δt (days)	117.32581

The time history of the primer vector is shown in Figure 52. This primer vector history satisfies the necessary conditions in that it begins with a magnitude of unity and the slope of zero, shown in Figure 53, and never again exceeds a magnitude of unity. Additionally, this solution lies on the boundary of the inequality constraint. The slack variable, α, has a value of zero for this transfer. It is not evident in Figure 51, but the primer vector at the final time is aligned with the velocity vector and its time derivative is aligned with the radius vector with respect to the Moon. The magnitude of the Lagrange multiplier υ satisfies the relationship in Eq. (5.2.16).

Table 28 shows the angles of the related vectors relative to their counterparts.

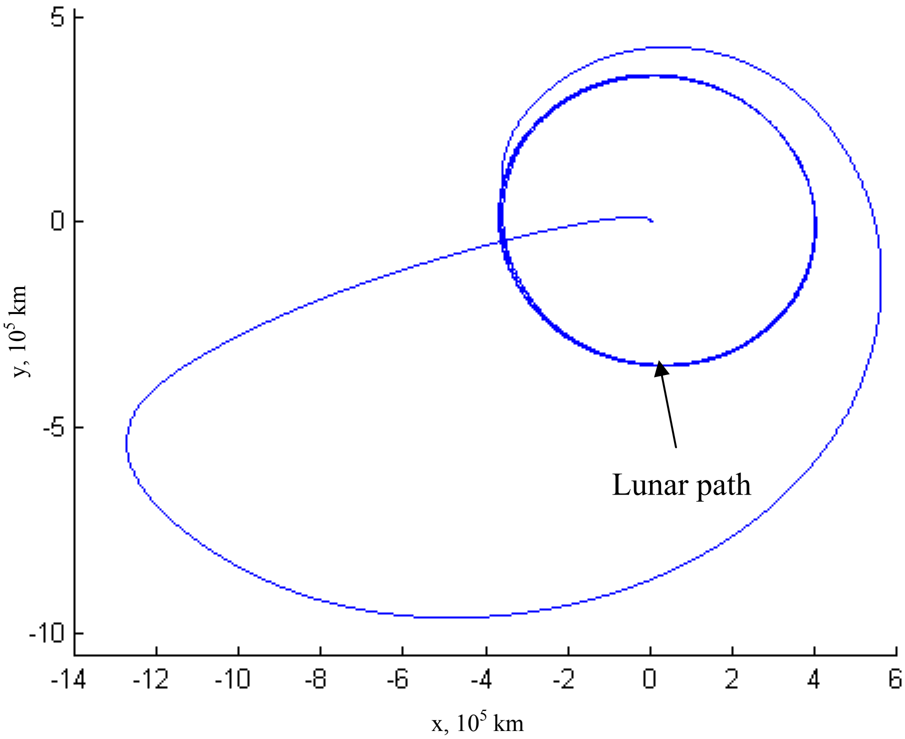

Figure 51. Single burn low energy transfer to lunar orbit

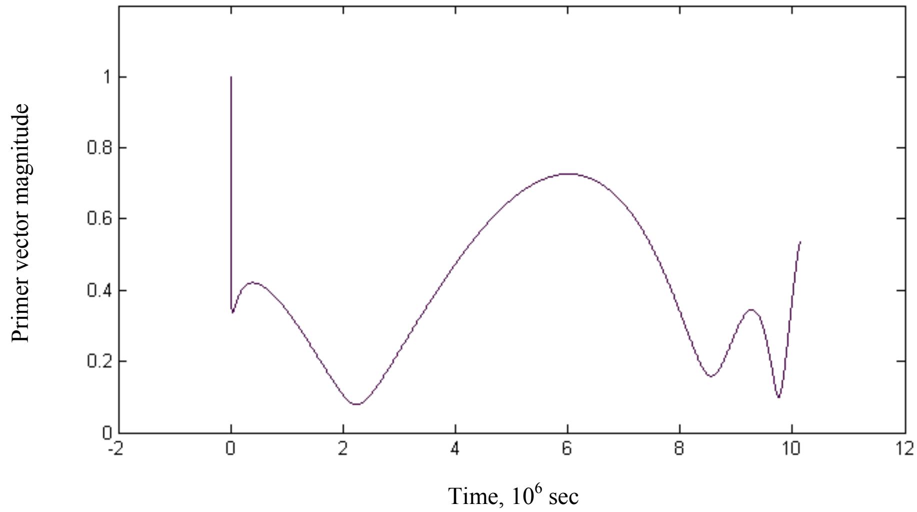

Figure 52. Primer vector magnitude of a single burn transfer

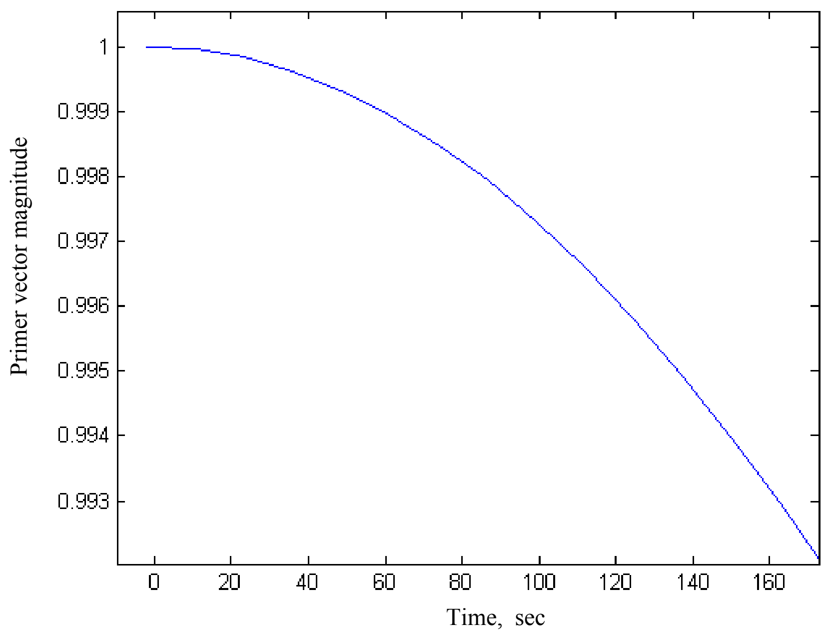

Figure 53. Zoom of the primer magnitude at the initial time

Table 28. Primer vector properties at t_f

property at t_f	value
primer vector angle with the velocity vector (deg)	0.01
primer rate vector angle with the radius vector (deg)	179.95

5.4 CHAPTER CONCLUSIONS

This chapter provides the theory for applying primer vector theory to orbit-to-capture transfers. The necessary conditions for an optimal transfer are derived using optimal control theory. Where the derivation overlaps with previous derivations of primer vector theory, the results are equivalent. The derivation is original in establishing necessary conditions for optimality for a spacecraft transfer that targets a specific energy level with an impulse followed by a ballistic segment.

The necessary conditions are used to establish a two point boundary value problem for the targeting of an optimal BLCT. The two point boundary value problem is solved for a specific case, and the ballistic lunar capture is shown to satisfy the necessary conditions. The significant conclusion that can be made from this result is that BLCTs are locally optimal, and cannot be improved with an intermediate impulse.

6. Optimal Time-Fixed Ballistic Lunar Capture Transfers

6.1 Introduction

In this section, theory and analysis are presented with the intent to provide a means of analyzing the trade space between transfer time and fuel cost in lunar transfers. The method proposed for this analysis is to remove the initial and final times from the vector of free parameters in the optimal control problem, and thus fix the transfer time. With the addition of a second impulse, optimal transfers can be constructed that are initiated by a transfer insertion burn and have a second burn that places the spacecraft on a ballistic arc that transitions to a captured state at the final time. By constructing transfers using this methodology, the transfer time can be set anywhere between an upper bound of the transfer time of a single impulse low energy transfer and lower bound of the transfer time of a direct two impulse transfer. Indeed, it is shown that the two impulse low energy transfer converges to a direct transfer at the minimum transfer time.

6.2 Definition of the Two Point Boundary Value Problem

The necessary conditions for an optimal two impulse transfer to a ballistic arc terminating at a desired energy state mirror those presented in Chapter 5 for an optimal ballistic lunar transfer. The two modifications of the problem are the removal of the initial and final times from the vector of free parameters and the addition of an intermediate impulse. The two point boundary value problem that is used to target optimal trajectories shown in Eq. (5.3.1) can be modified in the following way to target optimal time-fixed trajectories. The initial and final times are removed from the parameter vector, and the time, magnitude and direction of the intermediate impulse are added. The constraint vector is modified by eliminating the constraints on the initial and final Hamiltonians and adding the optimality conditions on the intermediate impulse (Eq. (5.2.24)). The resulting two point boundary value problem, shown in Eq. (6.2.1), is thirteen dimensional and can be solved by iterating from an initial guess using a numerical nonlinear equation solver.

$$\begin{aligned}
\mathbf{a}_{13x1} &= \begin{pmatrix} \mathrm{t_k} & \mathbf{\Delta v}_\mathrm{k} & \mathrm{f} & \dot{\mathbf{p}}_1 & \mathbf{\Delta v}_1 & \upsilon & \alpha \end{pmatrix}^\mathrm{T}, \\
\mathbf{c}_{13x1} &= \begin{pmatrix} \mathbf{p}_\mathrm{k} - \dfrac{\mathbf{\Delta v}_\mathrm{k}}{\left|\mathbf{\Delta v}_\mathrm{k}\right|} & \mathbf{p}_\mathrm{k}{}^\mathbf{T}\dot{\mathbf{p}}_\mathrm{k} & \mathrm{E} - \mathrm{E}^* + \alpha^2 & 2\alpha\mu & \boldsymbol{\lambda}_{\mathrm{r}_2} - \upsilon\dfrac{\mu_\mathrm{s}}{\mathrm{r}_\mathrm{s}^3}\mathbf{r}_\mathrm{s} & \boldsymbol{\lambda}_{\mathrm{v}_2} - \upsilon \mathrm{v}_\mathrm{s}\mathbf{v}_\mathrm{s} & \dot{\mathrm{p}}_1 \end{pmatrix}^\mathrm{T}
\end{aligned} \qquad (6.2.1)$$

6.3 GENERATION OF THE INITIAL GUESS

Convergence to optimal ballistic arc transfers is especially problematic due to the sensitivity of the problem. A very good initial guess near the solution in both the state space and the co-state space must be provided to the nonlinear targeting algorithm. A numerically optimized transfer can be used as such a guess and provide a solution within the convergence envelope.

Based on previous results of optimal orbit-to-orbit transfers, it would be expected that an optimal low energy transfer would terminate at or near a periapse with respect to the Moon. Therefore, it will be assumed that an optimal transfer to periselene is the desired target for generating an initial guess.

A transfer of this type can be found by adding a second impulse at the apogee of a low energy orbit-to-orbit transfer and then targeting a minimum $\Delta \mathrm{v}$ transfer to the Earth-Moon L_2 point as the transfer time is reduced. Figure 54 shows a transfer that was formed to meet these conditions with a transfer time of 54 days.

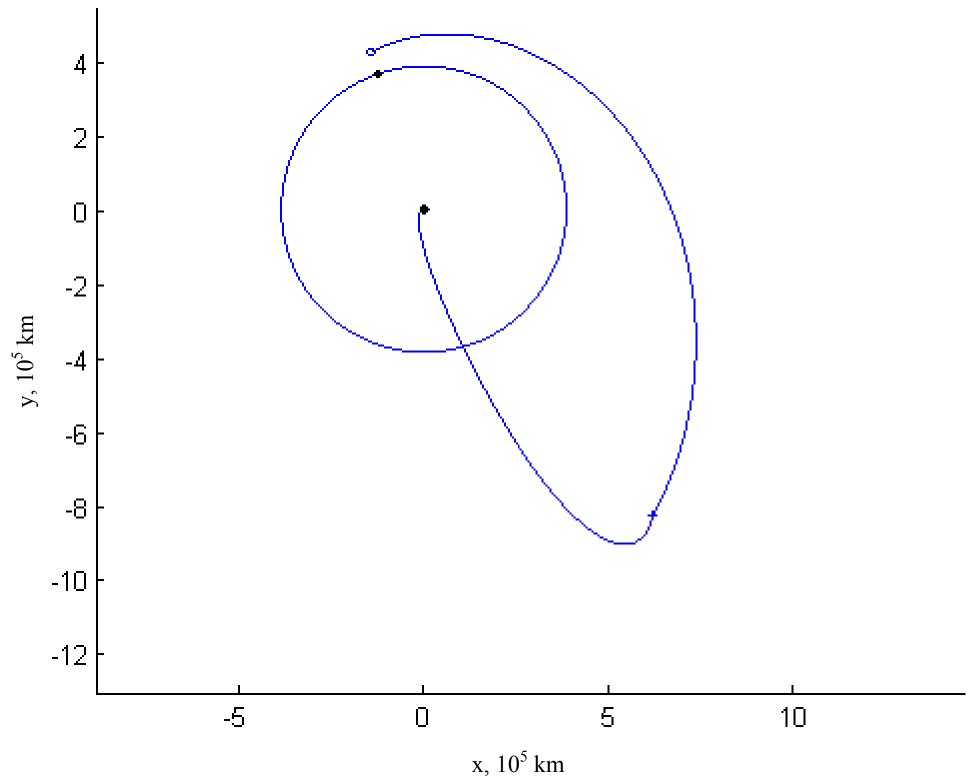

Figure 54. Time-fixed reference trajectory in non-rotating coordinates

The numerical optimization of this transfer is continued with the constraint vector altered to target a periselene. Figure 55 shows the results of the numerical optimization. This trajectory is the initial guess for the two point boundary value problem described in Eq. (6.2.1).

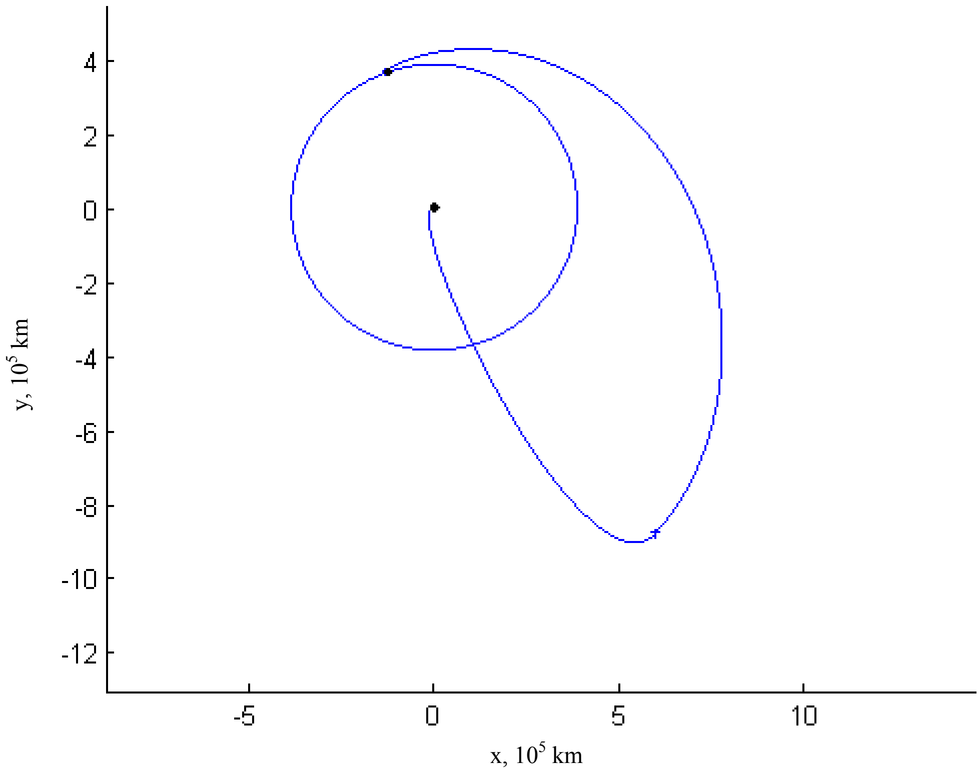

Figure 55. Fixed time initial guess in non-rotating coordinates

The initial guess for the costate parameters is formed by satisfying two of the necessary conditions. The initial primer vector is chosen to align with the velocity vector and have a unit magnitude. The primer rate vector is calculated based on the necessary condition that the primer vector be aligned with the second impulse. This condition is used along with Eq. (2.5.12), where the state transition matrix is calculated from the four body equations of motion as shown in Appendix A.

Figure 56 shows the primer vector history of the initial guess. The trajectory does not meet the conditions of optimality due to the segment near the end of the trajectory where the magnitude exceeds unity. The initial guess is iterated to an optimal solution using a numerical non-linear equation solving routine until the two point boundary value problem is solved. The primer vector history of the converged solution is shown in

Figure 57. In the process of solving the optimization problem, the magnitude of the cost was reduced from 3.30535 km/s to 3.288372 km/s.

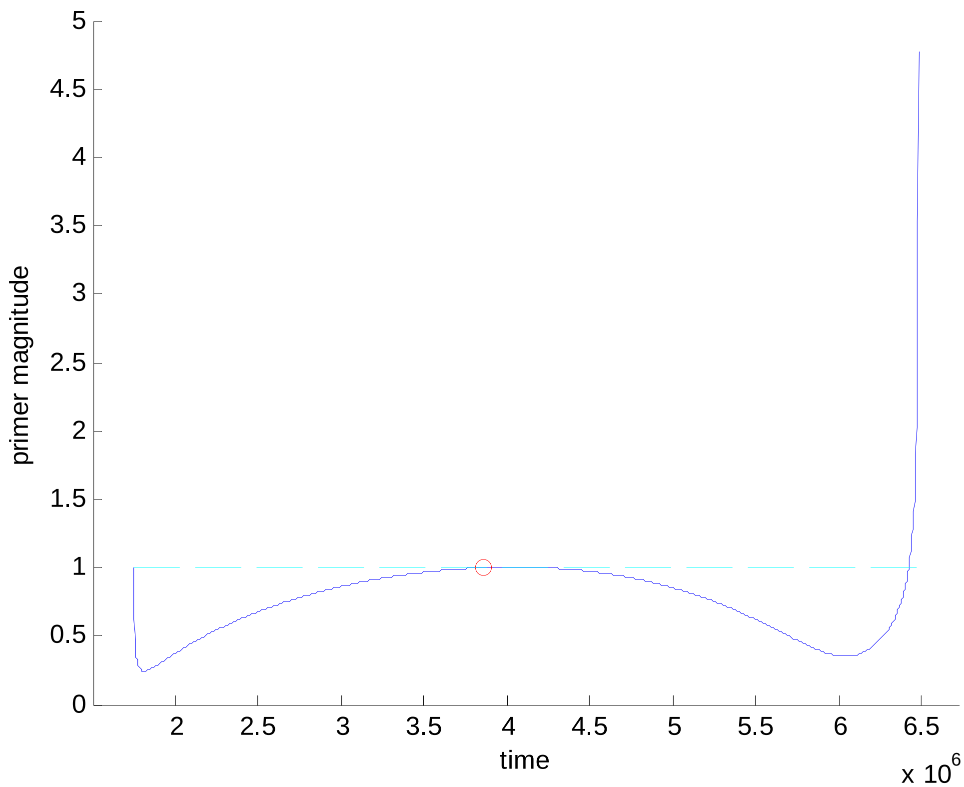

Figure 56. Primer vector history of the initial guess

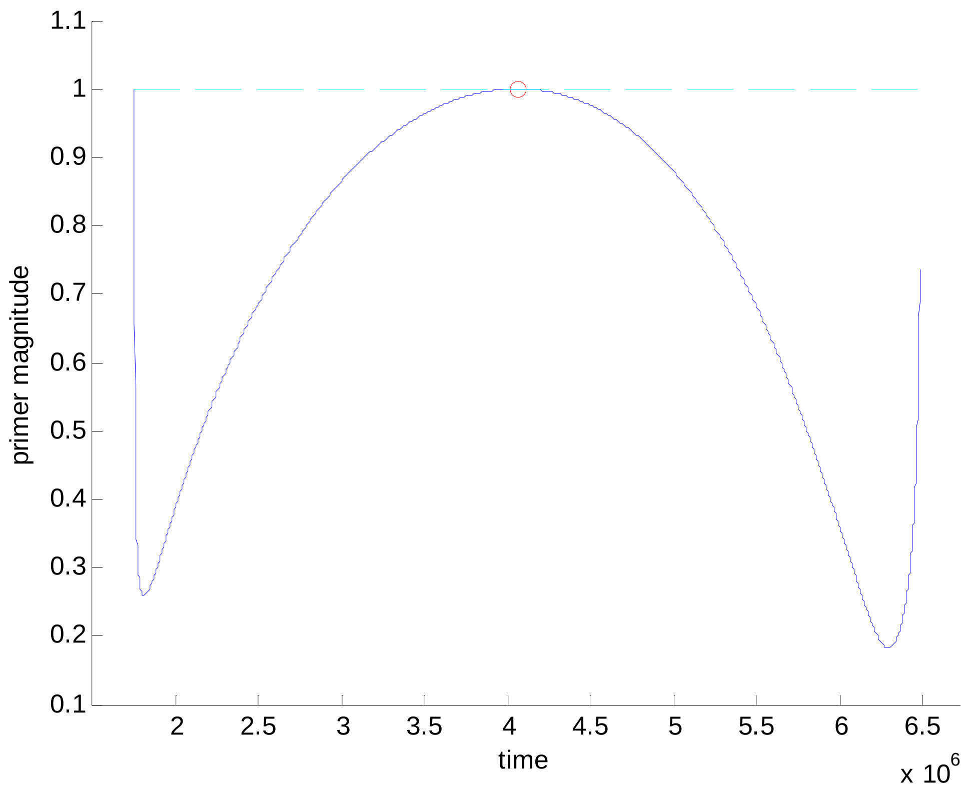

Figure 57. Primer vector history of the converged optimal solution

6.4 RESULTS

The analysis of the solution space between transfer time and fuel cost through computation of optimal time-fixed transfers in the BCRFBP. The method described above is repeated for different transfer times spanning the space between the Hohmann transfer time and the single impulse low energy transfer time. The parameters that define the converged optimal solutions are shown in Appendix B.

Figure 58 shows the results of the trade-space study. It displays a linear increase in fuel cost as transfer time is shortened. Figure 59 and Figure 60 show the change in magnitude of the first and second maneuver, respectively, for the transfers included in

Figure 58. The cost of shortening the transfer time is shown to be entirely contained in the second impulse. The initial impulse actually decreases in magnitude as the transfer is shortened. This result can be interpreted by assuming the second impulse does the work of the Sun as the transfer time is shortened and the spacecraft doesn't travel far enough from the Earth to be influenced by the Sun's dynamics. For shorter transfer times, the low energy transfer increasingly resembles a bi-elliptic transfer.

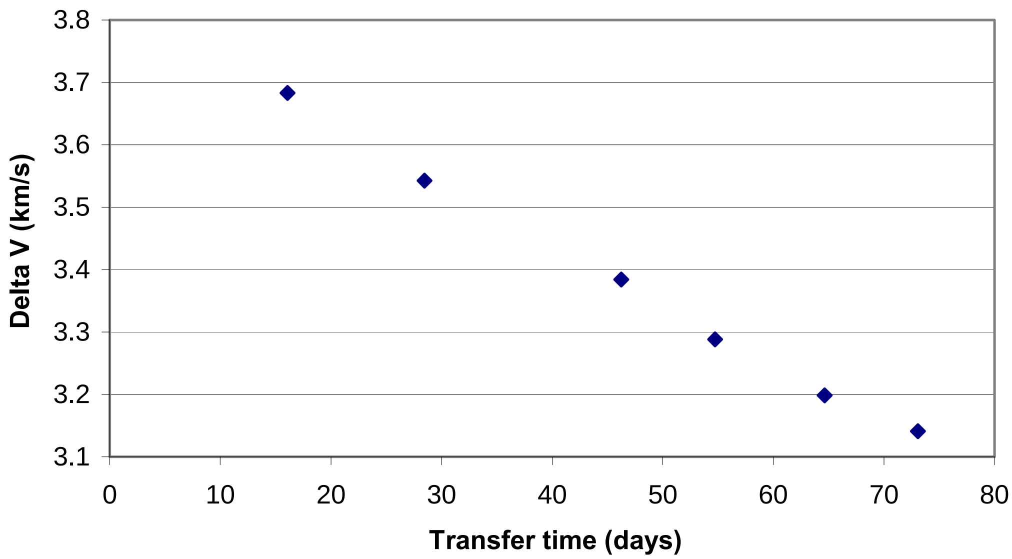

Figure 58. Optimal time-fixed transfer parameters

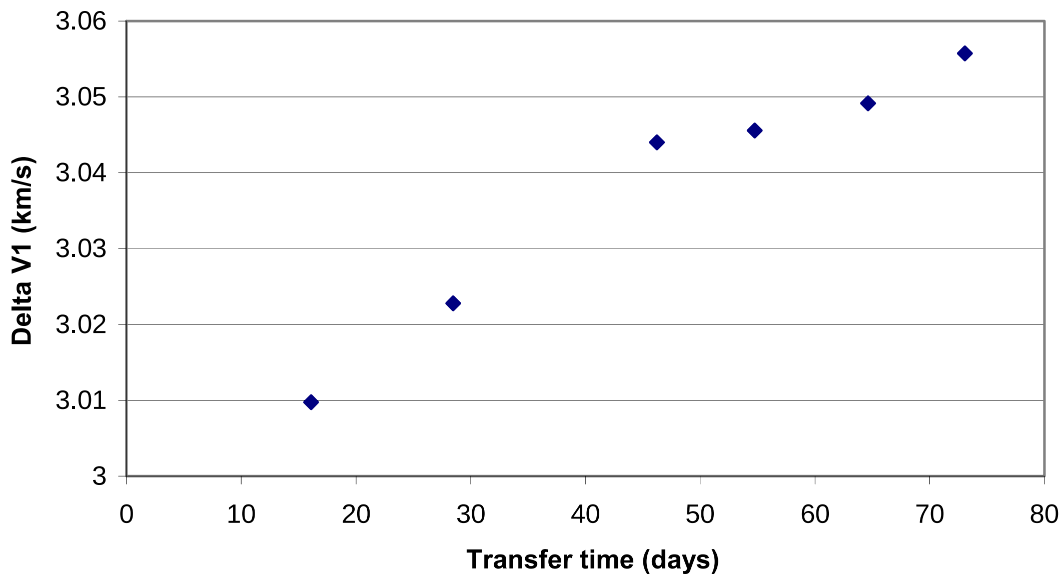

Figure 59. Time-fixed transfer initial maneuver magnitude

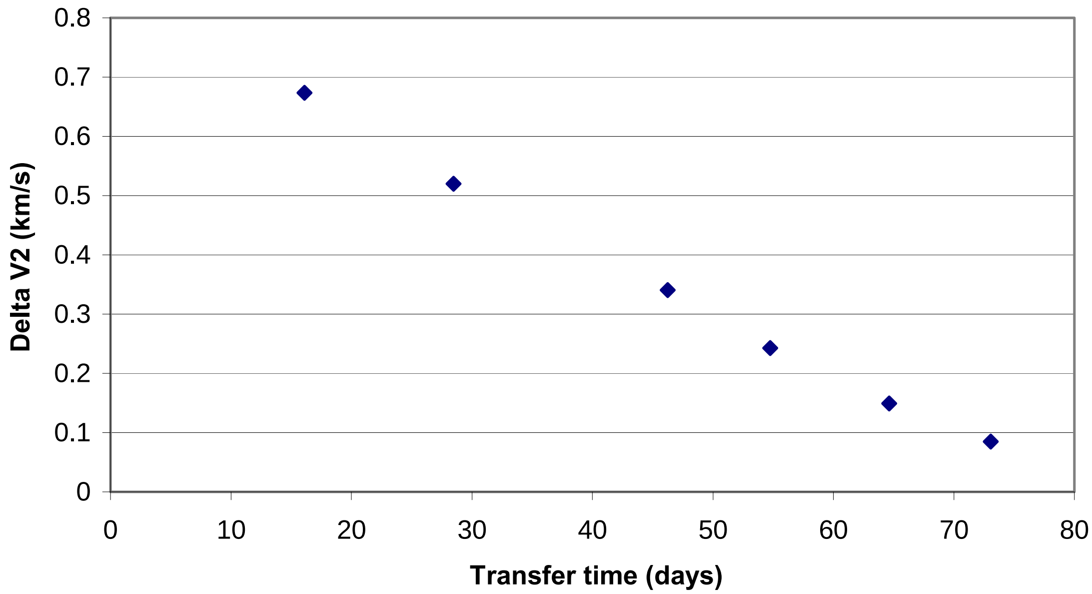

Figure 60. Time-fixed transfer intermediate maneuver magnitude

6.5 Chapter Conclusions

This chapter applies the theory developed in chapter five to analyze the cost of shortening a BLCT by applying an intermediate impulse near the apogee of the transfer. Optimal time-fixed solutions are sought by solving a two point boundary problem that is formed using the necessary conditions for optimality. As the transfer time is shortened, the cost of an optimal transfer is shown to rise linearly as the magnitude of the intermediate impulse increases.

A significant result in this chapter is found as the transfer time of the two-impulse ballistic transfer approaches the transfer time of a Hohmann transfer. Using the same targets and conditions for optimality, the direct transfer is shown to be less costly than any BLCT, no matter the transfer time. This result contradicts the expectation that a BLCT provides savings over direct transfers.

7. Conclusions

7.1 Dissertation summary

This dissertation makes possible the practical application of BLCTs by providing a simplified means of targeting different families of transfers on arbitrary launch dates and then developing theory for the analysis of the local optimality of the transfers. The methods used in the targeting and optimization algorithms overcome the difficulties of applying numerical methods to four body problem by generating reference orbits in a simplified model and providing precise derivatives to the numerical routines. Examples of optimal transfers are given, and the practicality of shortening the transfer time in a multiple impulse ballistic capture trajectory is explored.

Following an introduction, chapter two presents the theory for creating low energy transfers between circular orbits that mimic the behavior of BLCTs in a simplified three body model. These transfers are shown to meet the necessary conditions for local optimality and potentially produce savings when compared to direct orbit-to-orbit transfers that are based on Hohmann transfers. The low energy transfer are shown to be less costly than the direct transfers when the ratio of the final orbital radius to the initial orbital radius to the final orbital radius is greater than approximately six. When transferring between orbits greater than this ratio, the expected savings is anywhere from zero to fifteen percent.

Chapter three begins the analysis of BLCTs in the four body problem. The low energy transfers in the CRTBP are used to produce similar transfers that end with ballistic capture at the Moon. Chapter four continues this analysis by introducing a robust nonlinear targeting algorithm that produces transfers that would be practical to use in a lunar mission. These transfers can be found on any given launch date and have a transfer time between seventy-five and one hundred and twenty days.

Chapters five and six introduce primer vector theory to BLCTs. New necessary conditions for optimality are derived for an orbit-to-capture transfer and applied to both single and multiple impulse transfers between the Earth and the Moon. BLCTs are shown to be locally optimal, but not necessarily cost effective when compared to direct transfers.

7.2 General conclusions

This study provides a new method for analysis of BLCTs. Periodic orbits around the secondary mass in the CRTBP display similar traits to BLCTs in both geometry and the underlying dynamics. These periodic orbits are used to create low energy transfers from one orbit about the secondary mass to another that stratify the necessary conditions for optimality and also are potentially less expensive than direct transfers. The introduction of the low energy transfers to the study of BLCTs enables a mission planner to approximate the full transfer without targeting the chaotic trajectories much like Hohmann transfers are used to approximate direct transfers.

Additionally, the theory contained in chapters five and six enable the application of primer vector theory to a ballistic capture transfer. The necessary conditions developed in this work apply to any arc of an orbit that terminates at a desired energy. For the first time, ballistic lunar capture transfers are confirmed to be optimal transfers through analytic, or indirect, methods.

These two contributions are tools for the practical implementation of BLCTs in future missions. The speed and accuracy of targeting low energy lunar transfers has been increased, an a method provided for verifying that the best solution has been found.

7.3 FUTURE WORK

The results of this dissertation provide several opportunities for future research in low energy transfers. For example, the low energy transfers of family f25p1 and f25p2 display primer vector histories that meet the necessary conditions for an optimal two impulse transfer, but exceed unity in several instances. It should be possible to reduce the fuel cost of these transfers by introducing additional impulses. If multiple-impulse low energy transfer in the three body problem can be found, it would suggest the existence of optimal time-free multiple-impulse BLCTs in the four body problem. Transfers of this type have not yet been observed in the literature.

A large area of work that deserves further research is the quantification of the parameters that lead to savings via BLCT versus direct transfer. It has been observed both in this document and in other works that BLCTs produce unpredictable savings. The analysis within this dissertation provides some tools for making a conclusion about when and where to expect savings, but no attempt has been made to investigate the critical factors that make one form of transfer advantageous to the other.

Appendix A: Low energy transfer parameters

The following tables provide the parameters that define the low energy orbit-to-orbit transfers used in the analysis in chapter 2.

Table 29. *f*16p1 low energy transfers

Inner Radius	Outer Radius	Delta V1x	Delta V1y	Delta V2x	Delta V2y	Initial *f*	Time of flight
5.101992E-05	1.478144E-03	-1.363173E-03	9.990466E-02	-1.203183E-02	-5.205869E-03	1.421912E-03	1.328142E+00
5.101992E-05	1.392252E-03	-2.701954E-03	9.988456E-02	-1.256376E-02	-5.728024E-03	5.581760E-03	1.312804E+00
5.101992E-05	1.306360E-03	-2.730396E-03	9.987914E-02	-1.314264E-02	-6.278092E-03	6.058117E-03	1.289462E+00
5.101992E-05	1.220468E-03	-2.909894E-03	9.987169E-02	-1.376746E-02	-6.883399E-03	6.321638E-03	1.265798E+00
5.101992E-05	1.134576E-03	-3.427120E-03	9.985363E-02	-1.443412E-02	-7.579089E-03	9.416850E-03	1.243408E+00
5.101992E-05	1.048685E-03	-3.755282E-03	9.983925E-02	-1.517130E-02	-8.339644E-03	1.092081E-02	1.218389E+00
5.101992E-05	9.627927E-04	-3.970600E-03	9.982215E-02	-1.599407E-02	-9.183490E-03	1.426738E-02	1.191936E+00
5.101992E-05	8.769008E-04	-5.586999E-03	9.973777E-02	-1.675373E-02	-1.041202E-02	3.091972E-02	1.173893E+00
5.101992E-05	7.910089E-04	-6.559902E-03	9.968527E-02	-1.768573E-02	-1.167734E-02	3.346854E-02	1.146486E+00
5.101992E-05	7.051171E-04	-7.989233E-03	9.960839E-02	-1.871696E-02	-1.319976E-02	3.462386E-02	1.118017E+00
5.101992E-05	6.192252E-04	-5.911009E-03	9.971100E-02	-1.991950E-02	-1.497391E-02	2.328374E-02	1.066521E+00
5.101992E-05	5.333333E-04	-5.431542E-03	9.972660E-02	-2.141913E-02	-1.705802E-02	1.659868E-02	1.019460E+00
5.101992E-05	1.518531E-03	5.526740E-04	9.990978E-02	-1.180682E-02	-4.943886E-03	-6.521889E-04	1.329251E+00
5.101992E-05	1.473027E-03	-4.505595E-05	9.990847E-02	-1.208674E-02	-5.178816E-03	-9.298398E-04	1.320177E+00
5.101992E-05	1.427522E-03	-8.567593E-04	9.990590E-02	-1.236755E-02	-5.446377E-03	-2.389385E-03	1.311417E+00
5.101992E-05	1.382018E-03	-2.287407E-04	9.990320E-02	-1.267532E-02	-5.688242E-03	-1.839455E-03	1.296659E+00
5.101992E-05	1.336513E-03	3.968518E-04	9.989976E-02	-1.299339E-02	-5.946535E-03	-1.687117E-03	1.281584E+00
5.101992E-05	1.291009E-03	-8.125754E-04	9.989917E-02	-1.329057E-02	-6.293475E-03	-2.267914E-03	1.274043E+00
5.101992E-05	1.245504E-03	2.954392E-04	9.989561E-02	-1.364616E-02	-6.557011E-03	-1.702836E-03	1.256503E+00
5.101992E-05	1.200000E-03	-2.155595E-05	9.989355E-02	-1.398111E-02	-6.913707E-03	-2.482010E-03	1.244414E+00
5.357092E-05	1.493911E-03	-4.173308E-04	9.744124E-02	-1.190639E-02	-5.162961E-03	3.792874E-03	1.327349E+00
5.357092E-05	1.406586E-03	-1.760120E-03	9.742878E-02	-1.244162E-02	-5.681257E-03	4.189245E-03	1.310656E+00
5.357092E-05	1.319261E-03	-1.870936E-03	9.742196E-02	-1.304782E-02	-6.179923E-03	5.554687E-03	1.287760E+00
5.357092E-05	1.231935E-03	-3.024040E-03	9.740124E-02	-1.363533E-02	-6.870606E-03	7.317360E-03	1.268594E+00
5.357092E-05	1.144610E-03	-3.651305E-03	9.737911E-02	-1.431196E-02	-7.554967E-03	1.107359E-02	1.246612E+00
5.357092E-05	1.057285E-03	-4.119704E-03	9.735743E-02	-1.502644E-02	-8.366630E-03	1.432567E-02	1.222332E+00
5.357092E-05	9.699596E-04	-4.903866E-03	9.732925E-02	-1.579469E-02	-9.313460E-03	1.559812E-02	1.197292E+00
5.357092E-05	8.826343E-04	-1.102877E-02	9.718583E-02	-1.656763E-02	-1.054216E-02	5.146273E-03	1.191558E+00
5.357092E-05	7.953091E-04	-1.268176E-02	9.709062E-02	-1.749500E-02	-1.183274E-02	7.899489E-03	1.167104E+00
5.357092E-05	7.079838E-04	-1.170578E-02	9.729139E-02	-1.859024E-02	-1.326214E-02	-1.241079E-02	1.119823E+00
5.357092E-05	6.206586E-04	-1.162275E-02	9.749714E-02	-1.999921E-02	-1.483806E-02	-3.448235E-02	1.073131E+00
5.357092E-05	5.333333E-04	-1.232155E-02	9.766190E-02	-2.105477E-02	-1.743764E-02	-5.064252E-02	1.027335E+00
5.357092E-05	1.533582E-03	-4.270615E-04	9.744335E-02	-1.166869E-02	-4.956032E-03	1.076612E-03	1.336728E+00
5.357092E-05	1.485927E-03	-9.363467E-04	9.743794E-02	-1.194738E-02	-5.223613E-03	5.183218E-03	1.328131E+00
5.357092E-05	1.438273E-03	-1.530125E-03	9.743183E-02	-1.223319E-02	-5.510540E-03	4.557664E-03	1.318134E+00
5.357092E-05	1.390618E-03	-2.351150E-03	9.741779E-02	-1.253120E-02	-5.808823E-03	8.127381E-03	1.310337E+00
5.357092E-05	1.342964E-03	-2.363852E-03	9.741516E-02	-1.285110E-02	-6.099050E-03	8.146739E-03	1.297352E+00
5.357092E-05	1.295309E-03	-2.575739E-03	9.741092E-02	-1.318999E-02	-6.393910E-03	7.286980E-03	1.284763E+00
5.357092E-05	1.247655E-03	-2.844071E-03	9.740452E-02	-1.352773E-02	-6.738472E-03	7.397516E-03	1.272385E+00
5.357092E-05	1.200000E-03	-3.012563E-03	9.739909E-02	-1.388094E-02	-7.101809E-03	7.536992E-03	1.259226E+00
5.624946E-05	1.510206E-03	-6.411672E-04	9.503296E-02	-1.176987E-02	-5.139448E-03	5.218321E-05	1.330049E+00
5.624946E-05	1.421399E-03	-4.643880E-04	9.502885E-02	-1.233138E-02	-5.601779E-03	2.495927E-04	1.305912E+00
5.624946E-05	1.332593E-03	-4.352727E-04	9.502397E-02	-1.292365E-02	-6.134577E-03	6.720478E-04	1.281667E+00
5.624946E-05	1.243786E-03	-1.244997E-03	9.501559E-02	-1.354260E-02	-6.767220E-03	1.607970E-03	1.260491E+00
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5.624946E-05	6.221399E-04	-7.832752E-03	9.484693E-02	-1.968932E-02	-1.512998E-02	3.630643E-03	1.066240E+00
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5.624946E-05	1.549136E-03	1.189301E-03	9.503043E-02	-1.156772E-02	-4.874224E-03	-4.077457E-03	1.330005E+00
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5.624946E-05	1.449383E-03	3.384088E-04	9.503010E-02	-1.216763E-02	-5.411665E-03	-2.310934E-03	1.308758E+00
5.624946E-05	1.399506E-03	-4.316323E-04	9.502842E-02	-1.247893E-02	-5.716152E-03	-1.635494E-03	1.299280E+00
5.624946E-05	1.349630E-03	-9.346751E-04	9.502223E-02	-1.279857E-02	-6.046485E-03	3.553629E-03	1.289664E+00
5.624946E-05	1.299753E-03	-7.243281E-04	9.502096E-02	-1.317131E-02	-6.311696E-03	1.870250E-03	1.274284E+00
5.624946E-05	1.249877E-03	-1.901512E-03	9.501082E-02	-1.348572E-02	-6.749484E-03	2.765976E-03	1.265701E+00
5.624946E-05	1.200000E-03	-2.341434E-03	9.500119E-02	-1.385113E-02	-7.136881E-03	4.819800E-03	1.253755E+00
5.906194E-05	1.527053E-03	-1.266568E-03	9.267923E-02	-1.162375E-02	-5.131347E-03	-4.712552E-05	1.336129E+00
5.906194E-05	1.436715E-03	-2.007148E-03	9.266642E-02	-1.217115E-02	-5.629593E-03	3.523793E-03	1.317164E+00
5.906194E-05	1.346377E-03	-3.423612E-03	9.264241E-02	-1.272460E-02	-6.249076E-03	5.316625E-03	1.299931E+00
5.906194E-05	1.256038E-03	-4.356859E-03	9.261304E-02	-1.336230E-02	-6.848192E-03	8.938172E-03	1.280001E+00
5.906194E-05	1.165700E-03	-4.844766E-03	9.259417E-02	-1.401766E-02	-7.584380E-03	1.003961E-02	1.255686E+00
5.906194E-05	1.075362E-03	-5.784633E-03	9.255201E-02	-1.471082E-02	-8.450569E-03	1.422001E-02	1.233205E+00
5.906194E-05	9.850240E-04	-7.386639E-03	9.243662E-02	-1.548734E-02	-9.410018E-03	3.015729E-02	1.216347E+00
5.906194E-05	8.946859E-04	-8.701975E-03	9.231127E-02	-1.632076E-02	-1.055196E-02	4.574680E-02	1.196036E+00
5.906194E-05	8.043478E-04	-8.671254E-03	9.229074E-02	-1.725087E-02	-1.186683E-02	5.015469E-02	1.162791E+00
5.906194E-05	7.140096E-04	-9.284947E-03	9.222319E-02	-1.831822E-02	-1.340756E-02	5.605440E-02	1.130428E+00
5.906194E-05	6.236715E-04	-9.956675E-03	9.214992E-02	-1.949191E-02	-1.534487E-02	6.038009E-02	1.094075E+00
5.906194E-05	5.333333E-04	-6.819683E-03	9.237087E-02	-2.099291E-02	-1.756310E-02	4.229385E-02	1.026266E+00
5.906194E-05	1.565217E-03	-7.791122E-04	9.268260E-02	-1.140639E-02	-4.926906E-03	-1.580200E-04	1.343493E+00
5.906194E-05	1.513043E-03	-1.315356E-03	9.267756E-02	-1.170679E-02	-5.204335E-03	8.539465E-04	1.333029E+00
5.906194E-05	1.460869E-03	-1.180543E-03	9.267532E-02	-1.202213E-02	-5.488526E-03	1.230442E-03	1.318724E+00
5.906194E-05	1.408696E-03	-2.011926E-03	9.266749E-02	-1.235611E-02	-5.777694E-03	1.479720E-03	1.308969E+00
5.906194E-05	1.356522E-03	-2.560842E-03	9.265663E-02	-1.268047E-02	-6.133562E-03	3.678076E-03	1.298003E+00
5.906194E-05	1.304348E-03	-2.920173E-03	9.264831E-02	-1.302340E-02	-6.499670E-03	4.450607E-03	1.285304E+00
5.906194E-05	1.252174E-03	-3.201157E-03	9.264002E-02	-1.339909E-02	-6.853923E-03	5.284258E-03	1.271936E+00
5.906194E-05	1.200000E-03	-3.391845E-03	9.263258E-02	-1.381510E-02	-7.191074E-03	6.003986E-03	1.257747E+00
6.201503E-05	1.544476E-03	-1.019958E-03	9.038282E-02	-1.147984E-02	-5.110661E-03	6.690651E-04	1.338314E+00
6.201503E-05	1.452554E-03	-1.311145E-03	9.037626E-02	-1.202898E-02	-5.607282E-03	1.547088E-03	1.315861E+00
6.201503E-05	1.360632E-03	-2.132883E-03	9.036507E-02	-1.263501E-02	-6.121561E-03	2.126914E-03	1.295213E+00
6.201503E-05	1.268710E-03	-2.272268E-03	9.035552E-02	-1.324478E-02	-6.780279E-03	4.131489E-03	1.270288E+00
6.201503E-05	1.176788E-03	-2.886572E-03	9.033810E-02	-1.391778E-02	-7.492138E-03	6.818795E-03	1.246849E+00
6.201503E-05	1.084866E-03	-3.678624E-03	9.030840E-02	-1.463038E-02	-8.334432E-03	1.234999E-02	1.223744E+00
6.201503E-05	9.929438E-04	-4.454676E-03	9.026916E-02	-1.541068E-02	-9.294923E-03	1.941057E-02	1.199368E+00
6.201503E-05	9.010217E-04	-5.501243E-03	9.022349E-02	-1.625292E-02	-1.043421E-02	2.189782E-02	1.172834E+00
6.201503E-05	8.090996E-04	-8.483861E-03	9.009779E-02	-1.706595E-02	-1.196412E-02	2.293265E-02	1.153173E+00
6.201503E-05	7.171775E-04	-9.983903E-03	9.001860E-02	-1.818550E-02	-1.345301E-02	2.294250E-02	1.122847E+00
6.201503E-05	6.252554E-04	-1.092462E-02	8.990168E-02	-1.965584E-02	-1.501193E-02	3.260449E-02	1.089078E+00
6.201503E-05	5.333333E-04	-9.498174E-03	9.009486E-02	-2.092903E-02	-1.763001E-02	7.740642E-03	1.026099E+00
6.201503E-05	1.581849E-03	-9.432014E-04	9.038486E-02	-1.126327E-02	-4.929495E-03	6.541396E-04	1.347525E+00
6.201503E-05	1.527299E-03	-1.486171E-03	9.037910E-02	-1.157064E-02	-5.220427E-03	1.494034E-03	1.336497E+00
6.201503E-05	1.472749E-03	-1.482310E-03	9.037667E-02	-1.189974E-02	-5.506040E-03	1.225804E-03	1.322013E+00
6.201503E-05	1.418199E-03	-1.717068E-03	9.037040E-02	-1.224183E-02	-5.810949E-03	3.106929E-03	1.309176E+00
6.201503E-05	1.363649E-03	-2.382961E-03	9.035532E-02	-1.257878E-02	-6.177603E-03	7.844155E-03	1.299071E+00
6.201503E-05	1.309100E-03	-2.465252E-03	9.035092E-02	-1.295478E-02	-6.516269E-03	7.939116E-03	1.284152E+00
6.201503E-05	1.254550E-03	-2.989595E-03	9.034110E-02	-1.333276E-02	-6.910619E-03	7.323868E-03	1.270807E+00
6.201503E-05	1.200000E-03	-3.615722E-03	9.032573E-02	-1.372052E-02	-7.349300E-03	8.324687E-03	1.258043E+00
6.511579E-05	1.562504E-03	-1.930989E-03	8.813538E-02	-1.131776E-02	-5.124058E-03	4.185060E-04	1.346421E+00
6.511579E-05	1.468943E-03	-1.051514E-03	8.814225E-02	-1.191161E-02	-5.525480E-03	-7.031745E-03	1.314749E+00
6.511579E-05	1.375382E-03	-8.483594E-04	8.813455E-02	-1.250845E-02	-6.062061E-03	-4.982456E-03	1.288849E+00
6.511579E-05	1.281821E-03	-2.094329E-03	8.812067E-02	-1.311858E-02	-6.729674E-03	-9.858502E-04	1.269991E+00
6.511579E-05	1.188260E-03	-2.097914E-03	8.810885E-02	-1.379931E-02	-7.432793E-03	2.760573E-03	1.243368E+00
6.511579E-05	1.094699E-03	-3.071926E-03	8.808878E-02	-1.451394E-02	-8.280785E-03	4.296022E-03	1.219536E+00
6.511579E-05	1.001138E-03	-3.996874E-03	8.805213E-02	-1.533597E-02	-9.185978E-03	1.087991E-02	1.195426E+00
6.511579E-05	9.075772E-04	-5.500689E-03	8.798037E-02	-1.617237E-02	-1.035895E-02	2.075052E-02	1.173205E+00
6.511579E-05	8.140162E-04	-7.347973E-03	8.783220E-02	-1.700824E-02	-1.187578E-02	4.394364E-02	1.154699E+00
6.511579E-05	7.204553E-04	-9.729926E-03	8.762049E-02	-1.809306E-02	-1.346412E-02	6.095520E-02	1.134364E+00
6.511579E-05	6.268943E-04	-8.998555E-03	8.766149E-02	-1.931310E-02	-1.540622E-02	6.007561E-02	1.086981E+00
6.511579E-05	5.333333E-04	-9.638127E-03	8.759632E-02	-2.071068E-02	-1.790015E-02	6.029576E-02	1.042073E+00
6.511579E-05	1.599057E-03	-1.218656E-03	8.813999E-02	-1.112117E-02	-4.922500E-03	1.666866E-03	1.352472E+00
6.511579E-05	1.542049E-03	-1.249319E-03	8.813684E-02	-1.145024E-02	-5.194424E-03	2.016483E-03	1.338031E+00
6.511579E-05	1.485041E-03	-1.619898E-03	8.813008E-02	-1.178028E-02	-5.510710E-03	3.682825E-03	1.325490E+00
6.511579E-05	1.428032E-03	-2.005032E-03	8.812285E-02	-1.212942E-02	-5.835419E-03	4.478443E-03	1.312424E+00
6.511579E-05	1.371024E-03	-3.006532E-03	8.809897E-02	-1.248574E-02	-6.200757E-03	1.012483E-02	1.303869E+00

6.511579E-05	1.314016E-03	-3.308199E-03	8.809058E-02	-1.283566E-02	-6.631854E-03	9.986968E-03	1.289122E+00
6.511579E-05	1.257008E-03	-3.258106E-03	8.808893E-02	-1.327414E-02	-6.951708E-03	9.337395E-03	1.272183E+00
6.511579E-05	1.200000E-03	-3.617093E-03	8.807550E-02	-1.371920E-02	-7.334822E-03	1.102674E-02	1.257632E+00
6.837158E-05	1.581165E-03	-4.698254E-05	8.595164E-02	-1.120198E-02	-5.025868E-03	-2.119978E-05	1.339642E+00
6.837158E-05	1.485907E-03	3.118890E-04	8.594636E-02	-1.177197E-02	-5.484510E-03	-5.567316E-04	1.312546E+00
6.837158E-05	1.390650E-03	-6.693379E-04	8.594036E-02	-1.235391E-02	-6.057824E-03	2.365907E-04	1.292143E+00
6.837158E-05	1.295393E-03	-3.746561E-04	8.593473E-02	-1.299965E-02	-6.655349E-03	3.171100E-04	1.263595E+00
6.837158E-05	1.200135E-03	2.496058E-04	8.592778E-02	-1.370620E-02	-7.315130E-03	-2.856185E-04	1.231933E+00
6.837158E-05	1.104878E-03	1.725774E-04	8.591988E-02	-1.445464E-02	-8.110344E-03	-1.221568E-03	1.202373E+00
6.837158E-05	1.009620E-03	-8.902983E-04	8.590949E-02	-1.523916E-02	-9.092095E-03	1.369308E-03	1.177180E+00
6.837158E-05	9.143629E-04	-2.531538E-03	8.588473E-02	-1.609300E-02	-1.025206E-02	4.239035E-03	1.152911E+00
6.837158E-05	8.191055E-04	-4.394465E-03	8.582742E-02	-1.707073E-02	-1.158086E-02	1.192612E-02	1.128794E+00
6.837158E-05	7.238481E-04	-5.838207E-03	8.571982E-02	-1.796800E-02	-1.346374E-02	3.359622E-02	1.103624E+00
6.837158E-05	6.285907E-04	-6.902651E-03	8.567082E-02	-1.923215E-02	-1.539521E-02	2.871678E-02	1.063891E+00
6.837158E-05	5.333333E-04	-5.738331E-03	8.572170E-02	-2.078862E-02	-1.774519E-02	1.873172E-02	1.004995E+00
6.837158E-05	1.616869E-03	3.788235E-04	8.595315E-02	-1.100576E-02	-4.847822E-03	-7.764084E-05	1.346450E+00
6.837158E-05	1.557317E-03	-1.133335E-03	8.594785E-02	-1.131493E-02	-5.194490E-03	5.679589E-04	1.339543E+00
6.837158E-05	1.497764E-03	-1.059639E-03	8.594473E-02	-1.167273E-02	-5.482321E-03	1.043761E-03	1.323627E+00
6.837158E-05	1.438211E-03	-1.495113E-03	8.593875E-02	-1.203412E-02	-5.816547E-03	1.599098E-03	1.310130E+00
6.837158E-05	1.378658E-03	-1.394814E-03	8.593517E-02	-1.241204E-02	-6.172631E-03	2.381842E-03	1.293370E+00
6.837158E-05	1.319106E-03	-1.432828E-03	8.593103E-02	-1.282143E-02	-6.528199E-03	2.562298E-03	1.276809E+00
6.837158E-05	1.259553E-03	-3.492675E-03	8.589977E-02	-1.318511E-02	-7.045288E-03	5.750066E-03	1.271508E+00
6.837158E-05	1.200000E-03	-3.709019E-03	8.588577E-02	-1.363695E-02	-7.463248E-03	8.686787E-03	1.255781E+00
7.179015E-05	1.600490E-03	-1.289334E-03	8.380553E-02	-1.101940E-02	-5.069831E-03	5.327388E-03	1.351970E+00
7.179015E-05	1.503476E-03	-2.096535E-03	8.379049E-02	-1.156828E-02	-5.576661E-03	7.431729E-03	1.331703E+00
7.179015E-05	1.406462E-03	-3.342785E-03	8.374184E-02	-1.212403E-02	-6.206607E-03	2.570651E-02	1.318386E+00
7.179015E-05	1.309447E-03	-4.932547E-03	8.369258E-02	-1.277420E-02	-6.804631E-03	2.189048E-02	1.298531E+00
7.179015E-05	1.212433E-03	-5.547038E-03	8.366347E-02	-1.344122E-02	-7.544541E-03	2.272345E-02	1.273181E+00
7.179015E-05	1.115419E-03	-6.409998E-03	8.361145E-02	-1.415744E-02	-8.402466E-03	2.712976E-02	1.248950E+00
7.179015E-05	1.018405E-03	-7.411318E-03	8.352847E-02	-1.496148E-02	-9.358200E-03	3.680569E-02	1.225562E+00
7.179015E-05	9.213904E-04	-7.545808E-03	8.350161E-02	-1.577861E-02	-1.056854E-02	4.027367E-02	1.192753E+00
7.179015E-05	8.243761E-04	-8.044309E-03	8.344646E-02	-1.673908E-02	-1.191874E-02	4.568374E-02	1.160129E+00
7.179015E-05	7.273619E-04	-1.010179E-02	8.331964E-02	-1.779430E-02	-1.358919E-02	4.229670E-02	1.130139E+00
7.179015E-05	6.303476E-04	-1.124827E-02	8.318372E-02	-1.910854E-02	-1.551256E-02	5.101452E-02	1.094842E+00
7.179015E-05	5.333333E-04	-1.009503E-02	8.331977E-02	-2.047014E-02	-1.814158E-02	3.466629E-02	1.031458E+00
7.179015E-05	1.635316E-03	-1.025696E-03	8.381071E-02	-1.083691E-02	-4.886475E-03	2.442642E-03	1.358485E+00
7.179015E-05	1.573128E-03	-9.902963E-04	8.380764E-02	-1.118635E-02	-5.172955E-03	2.765433E-03	1.342310E+00
7.179015E-05	1.510940E-03	-5.850456E-04	8.380629E-02	-1.155905E-02	-5.461205E-03	2.506533E-03	1.323541E+00
7.179015E-05	1.448752E-03	-1.735102E-05	8.380416E-02	-1.195267E-02	-5.761943E-03	-9.690857E-04	1.302475E+00
7.179015E-05	1.386564E-03	-3.235291E-04	8.380039E-02	-1.234755E-02	-6.123783E-03	-8.797580E-04	1.287051E+00
7.179015E-05	1.324376E-03	4.905942E-04	8.379537E-02	-1.277270E-02	-6.489407E-03	-9.225212E-04	1.264966E+00
7.179015E-05	1.262188E-03	-1.048864E-03	8.378827E-02	-1.316835E-02	-6.991373E-03	2.979795E-03	1.256450E+00
7.179015E-05	1.200000E-03	-1.013077E-03	8.378427E-02	-1.362152E-02	-7.457223E-03	2.033060E-03	1.237207E+00
7.537966E-05	1.620515E-03	2.645384E-05	8.172330E-02	-1.089019E-02	-4.987755E-03	2.072311E-03	1.347084E+00
7.537966E-05	1.521680E-03	-1.318866E-04	8.171773E-02	-1.145957E-02	-5.454605E-03	1.163706E-03	1.321797E+00
7.537966E-05	1.422845E-03	-5.031489E-04	8.171093E-02	-1.205343E-02	-6.009980E-03	2.081972E-03	1.297334E+00
7.537966E-05	1.324011E-03	-5.735157E-04	8.170390E-02	-1.269650E-02	-6.623962E-03	3.104094E-03	1.270063E+00
7.537966E-05	1.225176E-03	-4.638852E-04	8.169681E-02	-1.339128E-02	-7.320405E-03	4.175713E-03	1.240493E+00
7.537966E-05	1.126341E-03	-4.191806E-05	8.168933E-02	-1.416242E-02	-8.088489E-03	3.278763E-04	1.206130E+00
7.537966E-05	1.027507E-03	-7.213245E-04	8.167830E-02	-1.496936E-02	-9.052104E-03	1.769637E-03	1.177672E+00
7.537966E-05	9.286720E-04	-2.016023E-03	8.165189E-02	-1.581660E-02	-1.025171E-02	9.617821E-03	1.152305E+00
7.537966E-05	8.298373E-04	-2.896018E-03	8.162327E-02	-1.678161E-02	-1.162854E-02	1.164944E-02	1.120028E+00
7.537966E-05	7.310027E-04	-3.606089E-03	8.158828E-02	-1.787023E-02	-1.328803E-02	1.531749E-02	1.083821E+00
7.537966E-05	6.321680E-04	-5.206678E-03	8.149886E-02	-1.908182E-02	-1.539517E-02	2.830510E-02	1.051013E+00
7.537966E-05	5.333333E-04	-5.829613E-03	8.141860E-02	-2.048258E-02	-1.808309E-02	4.578072E-02	1.008085E+00
7.537966E-05	1.654431E-03	-1.377346E-04	8.172486E-02	-1.070241E-02	-4.840429E-03	3.928221E-03	1.357414E+00
7.537966E-05	1.589512E-03	-3.407389E-04	8.172083E-02	-1.105880E-02	-5.141856E-03	3.782628E-03	1.341783E+00
7.537966E-05	1.524593E-03	-4.785525E-04	8.171682E-02	-1.143552E-02	-5.454768E-03	3.229020E-03	1.325283E+00
7.537966E-05	1.459675E-03	-7.651154E-04	8.171202E-02	-1.181822E-02	-5.814926E-03	2.637093E-03	1.309171E+00
7.537966E-05	1.394756E-03	-1.328405E-03	8.170445E-02	-1.221209E-02	-6.217056E-03	3.322375E-03	1.294614E+00
7.537966E-05	1.329837E-03	-1.959425E-03	8.169087E-02	-1.262131E-02	-6.659170E-03	7.350583E-03	1.281040E+00
7.537966E-05	1.264919E-03	-3.035875E-03	8.166140E-02	-1.304220E-02	-7.156889E-03	1.405210E-02	1.270356E+00
7.537966E-05	1.200000E-03	-3.041474E-03	8.165744E-02	-1.352681E-02	-7.617300E-03	1.327295E-02	1.250393E+00
7.914864E-05	1.641275E-03	1.336985E-04	7.968247E-02	-1.073716E-02	-4.950879E-03	-4.400551E-04	1.349451E+00
7.914864E-05	1.540553E-03	1.691653E-04	7.967686E-02	-1.131236E-02	-5.408820E-03	-1.400704E-03	1.322543E+00
7.914864E-05	1.439831E-03	-1.438683E-03	7.966569E-02	-1.187116E-02	-6.040933E-03	1.789490E-03	1.305714E+00
7.914864E-05	1.339109E-03	-1.484927E-03	7.965759E-02	-1.252245E-02	-6.642566E-03	3.021284E-03	1.277833E+00

7.914864E-05	1.238387E-03	-2.705225E-03	7.963345E-02	-1.318935E-02	-7.400090E-03	6.208262E-03	1.256136E+00
7.914864E-05	1.137665E-03	-3.058871E-03	7.961372E-02	-1.392819E-02	-8.234882E-03	9.916532E-03	1.227861E+00
7.914864E-05	1.036943E-03	-4.069164E-03	7.957779E-02	-1.471313E-02	-9.241737E-03	1.304088E-02	1.201309E+00
7.914864E-05	9.362213E-04	-4.854627E-03	7.953682E-02	-1.561572E-02	-1.035990E-02	1.722739E-02	1.171512E+00
7.914864E-05	8.354993E-04	-6.015018E-03	7.945939E-02	-1.658545E-02	-1.175226E-02	2.692906E-02	1.142872E+00
7.914864E-05	7.347773E-04	-6.829126E-03	7.940308E-02	-1.758989E-02	-1.354605E-02	2.930073E-02	1.106017E+00
7.914864E-05	6.340553E-04	-8.656597E-03	7.928262E-02	-1.883788E-02	-1.563502E-02	3.286482E-02	1.071058E+00
7.914864E-05	5.333333E-04	-1.011899E-02	7.914580E-02	-2.022846E-02	-1.838829E-02	3.924428E-02	1.028501E+00
7.914864E-05	1.674247E-03	7.036284E-04	7.968332E-02	-1.057734E-02	-4.766553E-03	-1.029017E-04	1.354650E+00
7.914864E-05	1.606498E-03	9.346218E-04	7.967809E-02	-1.095136E-02	-5.060208E-03	-1.690303E-03	1.335253E+00
7.914864E-05	1.538748E-03	1.099863E-04	7.967685E-02	-1.132100E-02	-5.421770E-03	-1.819987E-03	1.322265E+00
7.914864E-05	1.470999E-03	-1.112303E-03	7.967312E-02	-1.169612E-02	-5.834205E-03	-2.715788E-03	1.310882E+00
7.914864E-05	1.403249E-03	-1.119813E-03	7.966577E-02	-1.211643E-02	-6.221790E-03	9.187266E-04	1.293324E+00
7.914864E-05	1.335499E-03	-1.176927E-03	7.966031E-02	-1.255505E-02	-6.648645E-03	1.672928E-03	1.274525E+00
7.914864E-05	1.267750E-03	-1.573915E-03	7.965167E-02	-1.300435E-02	-7.138648E-03	3.038169E-03	1.257297E+00
7.914864E-05	1.200000E-03	-1.901076E-03	7.964115E-02	-1.348564E-02	-7.660976E-03	5.297709E-03	1.239283E+00
8.310608E-05	1.662813E-03	-1.265068E-03	7.768478E-02	-1.054752E-02	-4.985143E-03	1.937787E-03	1.363142E+00
8.310608E-05	1.560133E-03	-1.133210E-03	7.767916E-02	-1.111702E-02	-5.457239E-03	3.168638E-03	1.335674E+00
8.310608E-05	1.457453E-03	-1.636948E-03	7.766695E-02	-1.171745E-02	-6.006532E-03	5.808930E-03	1.311715E+00
8.310608E-05	1.354773E-03	-1.841201E-03	7.765764E-02	-1.234993E-02	-6.649324E-03	6.149537E-03	1.283835E+00
8.310608E-05	1.252093E-03	-2.816115E-03	7.763376E-02	-1.302802E-02	-7.390692E-03	8.616236E-03	1.260084E+00
8.310608E-05	1.149413E-03	-3.763897E-03	7.759216E-02	-1.374770E-02	-8.269524E-03	1.692572E-02	1.236442E+00
8.310608E-05	1.046733E-03	-4.340001E-03	7.756547E-02	-1.456263E-02	-9.234365E-03	1.774257E-02	1.206025E+00
8.310608E-05	9.440530E-04	-5.039880E-03	7.752413E-02	-1.543209E-02	-1.041556E-02	2.166449E-02	1.174979E+00
8.310608E-05	8.413731E-04	-5.469159E-03	7.747744E-02	-1.640399E-02	-1.181431E-02	2.944687E-02	1.140627E+00
8.310608E-05	7.386932E-04	-6.704782E-03	7.738938E-02	-1.753210E-02	-1.347885E-02	3.638104E-02	1.107232E+00
8.310608E-05	6.360133E-04	-8.496986E-03	7.724752E-02	-1.862599E-02	-1.580204E-02	4.398739E-02	1.072582E+00
8.310608E-05	5.333333E-04	-6.546324E-03	7.740686E-02	-2.024238E-02	-1.831473E-02	1.798578E-02	9.966673E-01
8.310608E-05	1.694806E-03	-1.219176E-03	7.768689E-02	-1.036435E-02	-4.876693E-03	1.657800E-03	1.370899E+00
8.310608E-05	1.624119E-03	-2.606070E-03	7.767235E-02	-1.074603E-02	-5.183353E-03	1.364657E-03	1.361434E+00
8.310608E-05	1.553433E-03	-2.320214E-03	7.766497E-02	-1.113638E-02	-5.531159E-03	5.793318E-03	1.342281E+00
8.310608E-05	1.482746E-03	-2.283407E-03	7.765872E-02	-1.154521E-02	-5.907459E-03	7.880979E-03	1.323455E+00
8.310608E-05	1.412060E-03	-3.979422E-03	7.760379E-02	-1.193580E-02	-6.395921E-03	1.829035E-02	1.317897E+00
8.310608E-05	1.341373E-03	-3.959073E-03	7.759547E-02	-1.238884E-02	-6.833822E-03	2.074634E-02	1.298178E+00
8.310608E-05	1.270687E-03	-4.292226E-03	7.757997E-02	-1.286216E-02	-7.329011E-03	2.104547E-02	1.279186E+00
8.310608E-05	1.200000E-03	-4.230828E-03	7.757735E-02	-1.335930E-02	-7.873071E-03	2.046731E-02	1.256694E+00
8.726138E-05	1.685172E-03	-8.021926E-04	7.574130E-02	-1.039581E-02	-4.929225E-03	5.328472E-05	1.364075E+00
8.726138E-05	1.580459E-03	-1.585692E-03	7.573017E-02	-1.096375E-02	-5.410098E-03	1.749117E-03	1.342253E+00
8.726138E-05	1.475746E-03	-3.166696E-03	7.570179E-02	-1.151734E-02	-6.057065E-03	4.708594E-03	1.324765E+00
8.726138E-05	1.371034E-03	-3.146456E-03	7.568971E-02	-1.216940E-02	-6.666199E-03	7.588105E-03	1.295756E+00
8.726138E-05	1.266321E-03	-3.712029E-03	7.566498E-02	-1.284346E-02	-7.418126E-03	1.075953E-02	1.269160E+00
8.726138E-05	1.161609E-03	-4.583368E-03	7.562814E-02	-1.358573E-02	-8.263984E-03	1.393211E-02	1.242884E+00
8.726138E-05	1.056896E-03	-5.336574E-03	7.557179E-02	-1.436651E-02	-9.296390E-03	2.308683E-02	1.215819E+00
8.726138E-05	9.521836E-04	-6.005940E-03	7.552638E-02	-1.524099E-02	-1.048082E-02	2.621427E-02	1.183693E+00
8.726138E-05	8.474710E-04	-6.449708E-03	7.546853E-02	-1.621957E-02	-1.188766E-02	3.524408E-02	1.149118E+00
8.726138E-05	7.427585E-04	-8.885403E-03	7.526277E-02	-1.724511E-02	-1.372218E-02	5.076708E-02	1.125910E+00
8.726138E-05	6.380459E-04	-7.647505E-03	7.536464E-02	-1.854560E-02	-1.579058E-02	3.714335E-02	1.063422E+00
8.726138E-05	5.333333E-04	-9.259477E-03	7.517535E-02	-2.006488E-02	-1.852713E-02	5.516786E-02	1.023575E+00
8.726138E-05	1.716149E-03	1.424861E-04	7.574416E-02	-1.025039E-02	-4.763615E-03	-1.618973E-04	1.365798E+00
8.726138E-05	1.642413E-03	-1.592872E-03	7.573332E-02	-1.061591E-02	-5.134795E-03	1.995647E-03	1.358828E+00
8.726138E-05	1.568678E-03	-2.611217E-03	7.571925E-02	-1.099238E-02	-5.544531E-03	2.270795E-03	1.345845E+00
8.726138E-05	1.494942E-03	-2.474038E-03	7.571743E-02	-1.143394E-02	-5.891539E-03	1.630730E-03	1.324609E+00
8.726138E-05	1.421207E-03	-2.298606E-03	7.571410E-02	-1.186955E-02	-6.325921E-03	1.829014E-03	1.302750E+00
8.726138E-05	1.347471E-03	-1.695140E-03	7.571358E-02	-1.235908E-02	-6.742108E-03	2.253517E-03	1.277773E+00
8.726138E-05	1.273736E-03	-2.061649E-03	7.570255E-02	-1.285881E-02	-7.238955E-03	4.009799E-03	1.258715E+00
8.726138E-05	1.200000E-03	-2.063614E-03	7.569490E-02	-1.336420E-02	-7.827715E-03	5.140997E-03	1.236231E+00
9.162445E-05	1.708402E-03	-3.152581E-04	7.384030E-02	-1.023211E-02	-4.890649E-03	2.351489E-03	1.366325E+00
9.162445E-05	1.601577E-03	-1.476614E-03	7.382645E-02	-1.077719E-02	-5.420308E-03	4.583713E-03	1.346764E+00
9.162445E-05	1.494753E-03	-1.894950E-03	7.381288E-02	-1.137988E-02	-5.972004E-03	7.508019E-03	1.321384E+00
9.162445E-05	1.387929E-03	-2.643434E-03	7.379277E-02	-1.201120E-02	-6.627478E-03	9.214450E-03	1.296488E+00
9.162445E-05	1.281104E-03	-3.836824E-03	7.375736E-02	-1.267554E-02	-7.404836E-03	1.151190E-02	1.273261E+00
9.162445E-05	1.174280E-03	-5.226459E-03	7.368788E-02	-1.339830E-02	-8.294720E-03	2.043710E-02	1.251967E+00
9.162445E-05	1.067455E-03	-6.423595E-03	7.358029E-02	-1.421322E-02	-9.291396E-03	3.851335E-02	1.230629E+00
9.162445E-05	9.606310E-04	-8.023409E-03	7.341623E-02	-1.500729E-02	-1.061981E-02	5.974363E-02	1.210604E+00
9.162445E-05	8.538066E-04	-1.058998E-02	7.318816E-02	-1.596646E-02	-1.208457E-02	6.346051E-02	1.188188E+00
9.162445E-05	7.469822E-04	-9.241405E-03	7.334107E-02	-1.707761E-02	-1.378352E-02	4.735653E-02	1.127200E+00
9.162445E-05	6.401577E-04	-1.149328E-02	7.307131E-02	-1.846757E-02	-1.585027E-02	6.414470E-02	1.098108E+00

9.162445E-05	5.333333E-04	-1.127924E-02	7.309452E-02	-2.003822E-02	-1.859869E-02	5.767586E-02	1.035731E+00
9.162445E-05	1.738323E-03	-3.138854E-05	7.384256E-02	-1.008368E-02	-4.755988E-03	2.400842E-03	1.372143E+00
9.162445E-05	1.661420E-03	1.882848E-04	7.383847E-02	-1.049055E-02	-5.068360E-03	1.386639E-03	1.350342E+00
9.162445E-05	1.584516E-03	3.544393E-04	7.383378E-02	-1.091636E-02	-5.409509E-03	9.042026E-04	1.328684E+00
9.162445E-05	1.507613E-03	-6.356654E-04	7.382743E-02	-1.133351E-02	-5.843425E-03	2.047676E-03	1.314668E+00
9.162445E-05	1.430710E-03	-2.015972E-03	7.381294E-02	-1.176953E-02	-6.321826E-03	2.419543E-03	1.302327E+00
9.162445E-05	1.353807E-03	-2.590310E-03	7.379794E-02	-1.225416E-02	-6.797327E-03	4.547681E-03	1.284568E+00
9.162445E-05	1.276903E-03	-2.636748E-03	7.378930E-02	-1.274799E-02	-7.352061E-03	5.792056E-03	1.262109E+00
9.162445E-05	1.200000E-03	-3.330938E-03	7.376428E-02	-1.326243E-02	-7.984863E-03	9.864418E-03	1.244010E+00
9.620567E-05	1.732558E-03	-1.881997E-03	7.196890E-02	-1.003786E-02	-4.908623E-03	6.093499E-03	1.383810E+00
9.620567E-05	1.623538E-03	-2.151492E-03	7.195793E-02	-1.058476E-02	-5.434665E-03	7.482473E-03	1.357121E+00
9.620567E-05	1.514518E-03	-2.602700E-03	7.193790E-02	-1.119522E-02	-5.975084E-03	1.258304E-02	1.332235E+00
9.620567E-05	1.405497E-03	-3.623693E-03	7.190880E-02	-1.182802E-02	-6.632761E-03	1.270063E-02	1.308183E+00
9.620567E-05	1.296477E-03	-4.026966E-03	7.188912E-02	-1.250301E-02	-7.392340E-03	1.349765E-02	1.278512E+00
9.620567E-05	1.187456E-03	-4.377028E-03	7.186226E-02	-1.325304E-02	-8.237327E-03	1.715697E-02	1.247816E+00
9.620567E-05	1.078436E-03	-4.992106E-03	7.182086E-02	-1.406189E-02	-9.243309E-03	2.085104E-02	1.216811E+00
9.620567E-05	9.694152E-04	-5.487232E-03	7.178431E-02	-1.494320E-02	-1.044526E-02	2.366372E-02	1.182040E+00
9.620567E-05	8.603947E-04	-6.024885E-03	7.174259E-02	-1.592855E-02	-1.188148E-02	2.610377E-02	1.144178E+00
9.620567E-05	7.513743E-04	-6.419605E-03	7.169344E-02	-1.700172E-02	-1.368994E-02	3.040148E-02	1.101758E+00
9.620567E-05	6.423538E-04	-7.949479E-03	7.158207E-02	-1.825787E-02	-1.593487E-02	3.391525E-02	1.061285E+00
9.620567E-05	5.333333E-04	-9.618554E-03	7.133218E-02	-1.952779E-02	-1.908568E-02	5.716911E-02	1.021132E+00
9.620567E-05	1.761382E-03	-2.588541E-03	7.196653E-02	-9.874606E-03	-4.828681E-03	2.686784E-03	1.395093E+00
9.620567E-05	1.681184E-03	-2.730743E-03	7.195156E-02	-1.026662E-02	-5.199564E-03	8.096175E-03	1.376911E+00
9.620567E-05	1.600987E-03	-2.421984E-03	7.195074E-02	-1.070692E-02	-5.542776E-03	9.029806E-03	1.353518E+00
9.620567E-05	1.520789E-03	-2.773166E-03	7.193718E-02	-1.114301E-02	-5.972147E-03	1.097427E-02	1.334531E+00
9.620567E-05	1.440592E-03	-3.364841E-03	7.191690E-02	-1.160060E-02	-6.445559E-03	1.282728E-02	1.316545E+00
9.620567E-05	1.360395E-03	-3.972818E-03	7.188835E-02	-1.208759E-02	-6.959722E-03	1.704313E-02	1.298774E+00
9.620567E-05	1.280197E-03	-4.468555E-03	7.185597E-02	-1.259284E-02	-7.545771E-03	2.268657E-02	1.279814E+00
9.620567E-05	1.200000E-03	-4.924141E-03	7.182596E-02	-1.316423E-02	-8.139464E-03	2.609696E-02	1.258884E+00
1.010160E-04	1.757703E-03	-5.265498E-04	7.017705E-02	-9.896832E-03	-4.804382E-03	-1.304238E-02	1.372396E+00
1.010160E-04	1.646397E-03	-1.149041E-03	7.017474E-02	-1.044280E-02	-5.336704E-03	-1.338043E-02	1.347282E+00
1.010160E-04	1.535091E-03	-7.803369E-04	7.016110E-02	-1.107232E-02	-5.841868E-03	-9.511334E-03	1.315747E+00
1.010160E-04	1.423784E-03	-4.649667E-04	7.014860E-02	-1.174018E-02	-6.437126E-03	-5.222784E-03	1.283538E+00
1.010160E-04	1.312478E-03	-1.983533E-04	7.013803E-02	-1.245466E-02	-7.135118E-03	-3.641766E-03	1.249295E+00
1.010160E-04	1.201172E-03	-4.751220E-04	7.012712E-02	-1.320969E-02	-7.984447E-03	-2.056877E-03	1.216992E+00
1.010160E-04	1.089865E-03	-6.381998E-04	7.011453E-02	-1.404312E-02	-8.964948E-03	-1.699258E-03	1.181356E+00
1.010160E-04	9.785588E-04	-3.652896E-04	7.009917E-02	-1.497344E-02	-1.011227E-02	9.765534E-04	1.140965E+00
1.010160E-04	8.672524E-04	-2.164729E-03	7.006030E-02	-1.591264E-02	-1.165876E-02	1.270508E-02	1.113744E+00
1.010160E-04	7.559461E-04	-3.593763E-03	7.000479E-02	-1.699786E-02	-1.350037E-02	1.693952E-02	1.077274E+00
1.010160E-04	6.446397E-04	-4.346821E-03	6.995152E-02	-1.826732E-02	-1.577886E-02	2.053302E-02	1.030180E+00
1.010160E-04	5.333333E-04	-6.390741E-03	6.978066E-02	-1.958959E-02	-1.896301E-02	4.484483E-02	9.906653E-01
1.010160E-04	1.785384E-03	-9.690915E-04	7.016399E-02	-9.721901E-03	-4.772275E-03	8.199250E-03	1.390569E+00
1.010160E-04	1.701757E-03	-9.764534E-04	7.015972E-02	-1.014633E-02	-5.106804E-03	7.062821E-03	1.368281E+00
1.010160E-04	1.618131E-03	-1.066977E-03	7.015383E-02	-1.059360E-02	-5.469922E-03	7.339150E-03	1.346654E+00
1.010160E-04	1.534505E-03	-1.451398E-03	7.014424E-02	-1.104105E-02	-5.913974E-03	7.815585E-03	1.326510E+00
1.010160E-04	1.450879E-03	-1.769575E-03	7.013434E-02	-1.152300E-02	-6.382313E-03	8.154452E-03	1.305183E+00
1.010160E-04	1.367252E-03	-2.017593E-03	7.012402E-02	-1.203435E-02	-6.897402E-03	8.831309E-03	1.282673E+00
1.010160E-04	1.283626E-03	-3.307202E-03	7.008897E-02	-1.254874E-02	-7.524370E-03	1.254495E-02	1.267645E+00
1.010160E-04	1.200000E-03	-3.754752E-03	7.006454E-02	-1.307990E-02	-8.242515E-03	1.601656E-02	1.245376E+00
1.060668E-04	1.783907E-03	-1.295600E-03	6.838970E-02	-9.678135E-03	-4.851660E-03	2.663608E-03	1.389560E+00
1.060668E-04	1.670218E-03	-2.854041E-03	6.836417E-02	-1.024004E-02	-5.356564E-03	4.117662E-03	1.371848E+00
1.060668E-04	1.556530E-03	-4.211276E-03	6.832291E-02	-1.082269E-02	-5.959254E-03	8.027906E-03	1.352179E+00
1.060668E-04	1.442841E-03	-4.246677E-03	6.830553E-02	-1.146244E-02	-6.608035E-03	1.150836E-02	1.320905E+00
1.060668E-04	1.329153E-03	-4.323144E-03	6.828796E-02	-1.216006E-02	-7.338482E-03	1.431207E-02	1.288275E+00
1.060668E-04	1.215464E-03	-4.661954E-03	6.826245E-02	-1.290295E-02	-8.209370E-03	1.660957E-02	1.255597E+00
1.060668E-04	1.101776E-03	-5.918366E-03	6.816365E-02	-1.368470E-02	-9.284182E-03	3.193570E-02	1.232218E+00
1.060668E-04	9.880873E-04	-7.047973E-03	6.808391E-02	-1.458037E-02	-1.049724E-02	3.537328E-02	1.200870E+00
1.060668E-04	8.743988E-04	-8.033681E-03	6.799063E-02	-1.551846E-02	-1.203857E-02	4.146622E-02	1.165866E+00
1.060668E-04	7.607103E-04	-1.027719E-02	6.766379E-02	-1.653866E-02	-1.399530E-02	7.844627E-02	1.147768E+00
1.060668E-04	6.470218E-04	-1.250232E-02	6.736893E-02	-1.775724E-02	-1.637260E-02	8.814176E-02	1.113933E+00
1.060668E-04	5.333333E-04	-1.565150E-02	6.682548E-02	-1.914840E-02	-1.953977E-02	1.102937E-01	1.085174E+00
1.060668E-04	1.810396E-03	-8.684957E-04	6.839422E-02	-9.562615E-03	-4.721676E-03	2.030828E-03	1.393001E+00
1.060668E-04	1.723196E-03	-5.516689E-04	6.839098E-02	-1.000317E-02	-5.053067E-03	1.309857E-03	1.367424E+00
1.060668E-04	1.635997E-03	-1.251457E-03	6.838071E-02	-1.044392E-02	-5.462510E-03	3.593867E-03	1.350117E+00
1.060668E-04	1.548798E-03	-9.846327E-04	6.837650E-02	-1.092095E-02	-5.885160E-03	4.300527E-03	1.324279E+00
1.060668E-04	1.461598E-03	-1.183221E-03	6.836897E-02	-1.142057E-02	-6.361789E-03	3.935485E-03	1.300835E+00
1.060668E-04	1.374399E-03	-1.584066E-03	6.835882E-02	-1.194161E-02	-6.906856E-03	3.873905E-03	1.278101E+00

1.060668E-04	1.287199E-03	-1.853579E-03	6.834890E-02	-1.249871E-02	-7.508534E-03	3.632723E-03	1.253352E+00
1.060668E-04	1.200000E-03	-1.921006E-03	6.833988E-02	-1.310135E-02	-8.170206E-03	3.465941E-03	1.226048E+00
1.113701E-04	1.811248E-03	-6.674412E-04	6.666164E-02	-9.511320E-03	-4.783181E-03	1.927301E-03	1.390519E+00
1.113701E-04	1.695074E-03	-7.783782E-04	6.665419E-02	-1.008562E-02	-5.263336E-03	2.257440E-03	1.360604E+00
1.113701E-04	1.578900E-03	-1.390508E-03	6.664156E-02	-1.068096E-02	-5.842334E-03	3.824020E-03	1.333975E+00
1.113701E-04	1.462726E-03	-2.967476E-03	6.660378E-02	-1.130145E-02	-6.536609E-03	9.999906E-03	1.315172E+00
1.113701E-04	1.346552E-03	-3.930637E-03	6.657363E-02	-1.197338E-02	-7.319475E-03	1.011109E-02	1.287688E+00
1.113701E-04	1.230378E-03	-4.915069E-03	6.650767E-02	-1.269741E-02	-8.232871E-03	2.131911E-02	1.262542E+00
1.113701E-04	1.114204E-03	-5.217149E-03	6.648284E-02	-1.354725E-02	-9.202185E-03	2.203385E-02	1.226065E+00
1.113701E-04	9.980296E-04	-5.782527E-03	6.643927E-02	-1.443143E-02	-1.044151E-02	2.467638E-02	1.189366E+00
1.113701E-04	8.818556E-04	-6.947041E-03	6.631671E-02	-1.538803E-02	-1.198582E-02	4.137322E-02	1.158765E+00
1.113701E-04	7.656815E-04	-7.339431E-03	6.626572E-02	-1.653477E-02	-1.378669E-02	4.377203E-02	1.112346E+00
1.113701E-04	6.495074E-04	-8.945715E-03	6.605660E-02	-1.769957E-02	-1.627966E-02	6.580670E-02	1.076226E+00
1.113701E-04	5.333333E-04	-1.027444E-02	6.590508E-02	-1.912976E-02	-1.945047E-02	6.683876E-02	1.021207E+00
1.113701E-04	1.836494E-03	-6.528032E-04	6.666340E-02	-9.390761E-03	-4.687095E-03	1.137736E-03	1.396703E+00
1.113701E-04	1.745567E-03	-1.317653E-03	6.665346E-02	-9.816583E-03	-5.077231E-03	2.874542E-03	1.378623E+00
1.113701E-04	1.654639E-03	-1.369666E-03	6.664727E-02	-1.027970E-02	-5.473876E-03	3.169365E-03	1.354555E+00
1.113701E-04	1.563711E-03	-1.220891E-03	6.664251E-02	-1.078762E-02	-5.875663E-03	2.921165E-03	1.328185E+00
1.113701E-04	1.472783E-03	-2.086021E-03	6.662774E-02	-1.128138E-02	-6.406450E-03	3.267630E-03	1.308923E+00
1.113701E-04	1.381856E-03	-2.210737E-03	6.661395E-02	-1.182820E-02	-6.953345E-03	7.188464E-03	1.284378E+00
1.113701E-04	1.290928E-03	-3.182134E-03	6.658620E-02	-1.238988E-02	-7.606315E-03	9.427153E-03	1.264531E+00
1.113701E-04	1.200000E-03	-2.844918E-03	6.657887E-02	-1.298239E-02	-8.343968E-03	1.263781E-02	1.233758E+00
1.169386E-04	1.839820E-03	-1.309336E-05	6.497116E-02	-9.332895E-03	-4.727983E-03	7.419673E-04	1.391035E+00
1.169386E-04	1.721049E-03	-4.580911E-04	6.496298E-02	-9.904429E-03	-5.216434E-03	2.204080E-03	1.363632E+00
1.169386E-04	1.602277E-03	-1.050594E-03	6.495137E-02	-1.051552E-02	-5.769636E-03	3.740658E-03	1.336359E+00
1.169386E-04	1.483506E-03	-1.508696E-03	6.493827E-02	-1.116014E-02	-6.421021E-03	4.447238E-03	1.306276E+00
1.169386E-04	1.364734E-03	-1.766730E-03	6.492626E-02	-1.186801E-02	-7.147211E-03	3.887819E-03	1.272623E+00
1.169386E-04	1.245963E-03	-2.908680E-03	6.489351E-02	-1.258948E-02	-8.074528E-03	8.005937E-03	1.245530E+00
1.169386E-04	1.127191E-03	-2.950576E-03	6.487580E-02	-1.342573E-02	-9.075582E-03	1.027410E-02	1.206905E+00
1.169386E-04	1.008420E-03	-3.281377E-03	6.485258E-02	-1.435489E-02	-1.026541E-02	1.077401E-02	1.166982E+00
1.169386E-04	8.896480E-04	-3.972982E-03	6.481233E-02	-1.538347E-02	-1.173170E-02	1.349798E-02	1.126813E+00
1.169386E-04	7.708764E-04	-6.273459E-03	6.466877E-02	-1.634349E-02	-1.382452E-02	2.861667E-02	1.098306E+00
1.169386E-04	6.521049E-04	-1.061509E-02	6.428162E-02	-1.751089E-02	-1.638772E-02	5.011074E-02	1.082918E+00
1.169386E-04	5.333333E-04	-1.161112E-02	6.411362E-02	-1.929368E-02	-1.920690E-02	5.930496E-02	1.026960E+00
1.169386E-04	1.863768E-03	2.996645E-04	6.497196E-02	-9.229075E-03	-4.622312E-03	-2.712893E-03	1.393391E+00
1.169386E-04	1.768944E-03	-1.504594E-04	6.496741E-02	-9.680429E-03	-4.990950E-03	-2.734238E-03	1.372151E+00
1.169386E-04	1.674120E-03	-4.334685E-04	6.496090E-02	-1.015460E-02	-5.399977E-03	-9.256566E-04	1.349560E+00
1.169386E-04	1.579296E-03	-2.880404E-04	6.495411E-02	-1.066385E-02	-5.836587E-03	5.897668E-04	1.322708E+00
1.169386E-04	1.484472E-03	-7.075664E-04	6.494565E-02	-1.118727E-02	-6.354595E-03	1.090838E-03	1.299093E+00
1.169386E-04	1.389648E-03	-2.123981E-03	6.491593E-02	-1.169431E-02	-7.024081E-03	1.164217E-02	1.285812E+00
1.169386E-04	1.294824E-03	-2.471281E-03	6.490066E-02	-1.228827E-02	-7.673373E-03	1.200906E-02	1.259282E+00
1.169386E-04	1.200000E-03	-2.857063E-03	6.488103E-02	-1.292729E-02	-8.407070E-03	1.372714E-02	1.232169E+00
1.227855E-04	1.869731E-03	-2.853542E-04	6.331749E-02	-9.134143E-03	-4.702240E-03	-3.009764E-04	1.399597E+00
1.227855E-04	1.748241E-03	-1.269810E-03	6.330552E-02	-9.702438E-03	-5.200224E-03	1.365844E-03	1.376192E+00
1.227855E-04	1.626750E-03	-1.390549E-03	6.329218E-02	-1.030781E-02	-5.765805E-03	6.994515E-03	1.345694E+00
1.227855E-04	1.505259E-03	-2.438413E-03	6.326549E-02	-1.093974E-02	-6.444901E-03	9.853664E-03	1.320566E+00
1.227855E-04	1.383769E-03	-3.573630E-03	6.323427E-02	-1.160270E-02	-7.253298E-03	9.119826E-03	1.293038E+00
1.227855E-04	1.262278E-03	-4.435329E-03	6.316227E-02	-1.232974E-02	-8.175944E-03	2.646564E-02	1.268064E+00
1.227855E-04	1.140787E-03	-4.814085E-03	6.313347E-02	-1.321880E-02	-9.109962E-03	2.695119E-02	1.230491E+00
1.227855E-04	1.019296E-03	-5.451761E-03	6.308839E-02	-1.413672E-02	-1.033403E-02	2.828208E-02	1.192179E+00
1.227855E-04	8.978055E-04	-6.594808E-03	6.298797E-02	-1.507378E-02	-1.194307E-02	3.779478E-02	1.157077E+00
1.227855E-04	7.763148E-04	-7.868586E-03	6.285101E-02	-1.616784E-02	-1.388324E-02	4.943057E-02	1.118885E+00
1.227855E-04	6.548241E-04	-1.088667E-02	6.238036E-02	-1.732990E-02	-1.650153E-02	1.000954E-01	1.104435E+00
1.227855E-04	5.333333E-04	-1.077942E-02	6.233062E-02	-1.931680E-02	-1.927289E-02	1.093440E-01	1.036439E+00
1.227855E-04	1.892319E-03	5.402777E-05	6.331891E-02	-9.036319E-03	-4.608474E-03	-8.189230E-04	1.402402E+00
1.227855E-04	1.793417E-03	-5.580728E-04	6.331228E-02	-9.489771E-03	-5.001701E-03	2.978484E-04	1.381866E+00
1.227855E-04	1.694514E-03	-7.278820E-04	6.330521E-02	-9.976397E-03	-5.418889E-03	8.179662E-04	1.356647E+00
1.227855E-04	1.595611E-03	-6.620147E-04	6.329827E-02	-1.051593E-02	-5.837529E-03	1.352541E-03	1.328862E+00
1.227855E-04	1.496708E-03	-7.566858E-04	6.328973E-02	-1.105004E-02	-6.377204E-03	2.429020E-03	1.301550E+00
1.227855E-04	1.397806E-03	-8.363852E-04	6.328025E-02	-1.162363E-02	-6.973733E-03	3.539359E-03	1.273127E+00
1.227855E-04	1.298903E-03	-1.842764E-03	6.326167E-02	-1.219987E-02	-7.712722E-03	4.551212E-03	1.250766E+00
1.227855E-04	1.200000E-03	-2.202423E-03	6.324558E-02	-1.289377E-02	-8.428295E-03	5.499284E-03	1.221849E+00
1.289248E-04	1.901109E-03	-1.603540E-03	6.168890E-02	-8.891661E-03	-4.748457E-03	7.020463E-03	1.421597E+00
1.289248E-04	1.776766E-03	-2.232318E-03	6.166912E-02	-9.463607E-03	-5.240682E-03	1.018076E-02	1.394838E+00
1.289248E-04	1.652423E-03	-2.476806E-03	6.165659E-02	-1.005659E-02	-5.829061E-03	1.022133E-02	1.362130E+00
1.289248E-04	1.528080E-03	-3.253554E-03	6.162514E-02	-1.071917E-02	-6.456517E-03	1.404719E-02	1.334545E+00
1.289248E-04	1.403736E-03	-4.329747E-03	6.156998E-02	-1.141026E-02	-7.225487E-03	2.107763E-02	1.309000E+00

1.289248E-04	1.279393E-03	-4.894495E-03	6.151712E-02	-1.217146E-02	-8.100528E-03	3.026498E-02	1.277742E+00
1.289248E-04	1.155050E-03	-6.596428E-03	6.140127E-02	-1.292801E-02	-9.249778E-03	3.747195E-02	1.252990E+00
1.289248E-04	1.030706E-03	-7.966625E-03	6.122163E-02	-1.376224E-02	-1.060931E-02	6.190410E-02	1.229034E+00
1.289248E-04	9.063632E-04	-8.395829E-03	6.113976E-02	-1.489832E-02	-1.197710E-02	7.183715E-02	1.187390E+00
1.289248E-04	7.820199E-04	-9.057451E-03	6.105138E-02	-1.593357E-02	-1.400124E-02	7.401617E-02	1.139227E+00
1.289248E-04	6.576766E-04	-1.337109E-02	6.063377E-02	-1.710582E-02	-1.663709E-02	6.614763E-02	1.113427E+00
1.289248E-04	5.333333E-04	-1.558319E-02	6.014865E-02	-1.849712E-02	-2.010750E-02	9.327440E-02	1.073347E+00
1.289248E-04	1.922271E-03	-5.624913E-05	6.170319E-02	-8.837920E-03	-4.593508E-03	-8.849031E-04	1.410051E+00
1.289248E-04	1.819090E-03	-2.913311E-04	6.169716E-02	-9.318074E-03	-4.964007E-03	-1.458067E-03	1.384509E+00
1.289248E-04	1.715908E-03	-3.807548E-04	6.168966E-02	-9.820343E-03	-5.384749E-03	1.236866E-03	1.358261E+00
1.289248E-04	1.612726E-03	-3.985125E-04	6.168218E-02	-1.035020E-02	-5.857619E-03	1.297462E-03	1.329581E+00
1.289248E-04	1.509545E-03	-3.204805E-04	6.167402E-02	-1.091880E-02	-6.378425E-03	1.618276E-03	1.299324E+00
1.289248E-04	1.406363E-03	-6.680752E-04	6.166359E-02	-1.151138E-02	-6.993440E-03	2.036293E-03	1.271548E+00
1.289248E-04	1.303182E-03	-5.600613E-04	6.165335E-02	-1.215549E-02	-7.675051E-03	2.515132E-03	1.238679E+00
1.289248E-04	1.200000E-03	-8.817386E-04	6.164010E-02	-1.283755E-02	-8.478709E-03	2.387896E-03	1.207498E+00
1.353710E-04	1.934109E-03	5.990745E-05	6.012315E-02	-8.728609E-03	-4.629211E-03	-2.530253E-03	1.410343E+00
1.353710E-04	1.806766E-03	-8.685450E-04	6.011644E-02	-9.293634E-03	-5.135890E-03	-4.179145E-03	1.383967E+00
1.353710E-04	1.679422E-03	-6.531043E-04	6.010512E-02	-9.933939E-03	-5.643261E-03	8.659219E-04	1.348775E+00
1.353710E-04	1.552079E-03	-6.719697E-04	6.009448E-02	-1.060712E-02	-6.255165E-03	2.428390E-03	1.313050E+00
1.353710E-04	1.424736E-03	-9.634836E-04	6.008089E-02	-1.134475E-02	-6.950470E-03	3.786032E-03	1.278123E+00
1.353710E-04	1.297393E-03	-1.264777E-03	6.006568E-02	-1.209536E-02	-7.848229E-03	3.587124E-03	1.240467E+00
1.353710E-04	1.170049E-03	-2.179646E-03	6.003947E-02	-1.291203E-02	-8.917697E-03	5.147256E-03	1.206158E+00
1.353710E-04	1.042706E-03	-2.194264E-03	6.001754E-02	-1.384656E-02	-1.014298E-02	7.875218E-03	1.161435E+00
1.353710E-04	9.153630E-04	-3.225700E-03	5.996826E-02	-1.484529E-02	-1.171567E-02	1.350479E-02	1.122010E+00
1.353710E-04	7.880198E-04	-4.112573E-03	5.990867E-02	-1.597037E-02	-1.368120E-02	1.814836E-02	1.075376E+00
1.353710E-04	6.606766E-04	-5.156009E-03	5.982050E-02	-1.719077E-02	-1.629711E-02	2.537420E-02	1.023091E+00
1.353710E-04	5.333333E-04	-7.242894E-03	5.959296E-02	-1.854203E-02	-1.990526E-02	5.179125E-02	9.752694E-01
1.353710E-04	1.953771E-03	-2.043988E-04	6.012427E-02	-8.636545E-03	-4.571196E-03	-9.411846E-04	1.418723E+00
1.353710E-04	1.846089E-03	-1.685126E-04	6.011774E-02	-9.129857E-03	-4.945557E-03	-3.806863E-04	1.389789E+00
1.353710E-04	1.738408E-03	-2.754798E-04	6.011039E-02	-9.651209E-03	-5.363745E-03	3.971884E-04	1.361680E+00
1.353710E-04	1.630726E-03	-2.240960E-03	6.007614E-02	-1.012573E-02	-5.977221E-03	1.152706E-02	1.353348E+00
1.353710E-04	1.523045E-03	-2.794838E-03	6.005108E-02	-1.068907E-02	-6.551601E-03	1.574972E-02	1.328434E+00
1.353710E-04	1.415363E-03	-3.829460E-03	5.999568E-02	-1.123594E-02	-7.292273E-03	2.640178E-02	1.308887E+00
1.353710E-04	1.307682E-03	-4.911154E-03	5.992373E-02	-1.187453E-02	-8.046685E-03	3.587060E-02	1.287978E+00
1.353710E-04	1.200000E-03	-5.807568E-03	5.984067E-02	-1.258064E-02	-8.890610E-03	4.769332E-02	1.264502E+00
1.421396E-04	1.968919E-03	-1.956940E-03	5.855651E-02	-8.438212E-03	-4.734108E-03	1.248697E-02	1.444642E+00
1.421396E-04	1.838411E-03	-3.240184E-03	5.850875E-02	-8.985680E-03	-5.273958E-03	2.030648E-02	1.424573E+00
1.421396E-04	1.707904E-03	-4.060716E-03	5.846078E-02	-9.589056E-03	-5.850861E-03	2.748032E-02	1.398463E+00
1.421396E-04	1.577396E-03	-5.810897E-03	5.832305E-02	-1.017092E-02	-6.624480E-03	4.854900E-02	1.385538E+00
1.421396E-04	1.446888E-03	-7.113292E-03	5.817076E-02	-1.090294E-02	-7.344346E-03	7.113277E-02	1.367547E+00
1.421396E-04	1.316380E-03	-7.752376E-03	5.809487E-02	-1.163819E-02	-8.265990E-03	7.519792E-02	1.332893E+00
1.421396E-04	1.185872E-03	-8.511199E-03	5.800999E-02	-1.244737E-02	-9.343365E-03	7.596384E-02	1.295573E+00
1.421396E-04	1.055365E-03	-9.157656E-03	5.791393E-02	-1.336202E-02	-1.060615E-02	8.192675E-02	1.256118E+00
1.421396E-04	9.248568E-04	-1.026307E-02	5.770383E-02	-1.434193E-02	-1.221712E-02	1.048618E-01	1.224020E+00
1.421396E-04	7.943490E-04	-1.269261E-02	5.737717E-02	-1.543342E-02	-1.425093E-02	1.031572E-01	1.189627E+00
1.421396E-04	6.638411E-04	-1.592613E-02	5.681030E-02	-1.668784E-02	-1.689400E-02	1.155713E-01	1.162852E+00
1.421396E-04	5.333333E-04	-1.843111E-02	5.616419E-02	-1.844171E-02	-2.022145E-02	1.413326E-01	1.124925E+00
1.421396E-04	1.986999E-03	-1.251792E-03	5.857102E-02	-8.377315E-03	-4.641572E-03	8.773719E-03	1.440915E+00
1.421396E-04	1.874570E-03	-1.879822E-03	5.855306E-02	-8.871785E-03	-5.042604E-03	1.174468E-02	1.417766E+00
1.421396E-04	1.762142E-03	-2.138596E-03	5.853754E-02	-9.391981E-03	-5.495168E-03	1.568125E-02	1.390517E+00
1.421396E-04	1.649714E-03	-2.434048E-03	5.851940E-02	-9.957737E-03	-5.980921E-03	1.949930E-02	1.362781E+00
1.421396E-04	1.537285E-03	-3.137698E-03	5.848817E-02	-1.051508E-02	-6.607591E-03	2.214967E-02	1.337134E+00
1.421396E-04	1.424857E-03	-3.927652E-03	5.845294E-02	-1.113443E-02	-7.279376E-03	2.218316E-02	1.310011E+00
1.421396E-04	1.312428E-03	-5.180227E-03	5.837150E-02	-1.179034E-02	-8.071971E-03	3.184864E-02	1.289386E+00
1.421396E-04	1.200000E-03	-7.225059E-03	5.820104E-02	-1.245568E-02	-9.058326E-03	4.726875E-02	1.276571E+00
1.492466E-04	2.005779E-03	-7.348908E-04	5.706950E-02	-8.264661E-03	-4.607455E-03	3.103034E-03	1.438051E+00
1.492466E-04	1.871921E-03	-1.686175E-03	5.705085E-02	-8.845697E-03	-5.089878E-03	5.697189E-03	1.412183E+00
1.492466E-04	1.738062E-03	-2.712786E-03	5.702098E-02	-9.433432E-03	-5.696903E-03	9.231486E-03	1.385986E+00
1.492466E-04	1.604203E-03	-2.542023E-03	5.700892E-02	-1.009837E-02	-6.327757E-03	1.313974E-02	1.346653E+00
1.492466E-04	1.470344E-03	-2.869961E-03	5.698752E-02	-1.084055E-02	-7.024147E-03	1.488697E-02	1.309985E+00
1.492466E-04	1.336486E-03	-4.167675E-03	5.692522E-02	-1.156443E-02	-7.979450E-03	2.119066E-02	1.282081E+00
1.492466E-04	1.202627E-03	-5.255666E-03	5.685121E-02	-1.236927E-02	-9.081734E-03	2.798219E-02	1.249610E+00
1.492466E-04	1.068768E-03	-6.294111E-03	5.674527E-02	-1.331648E-02	-1.032929E-02	4.102025E-02	1.215777E+00
1.492466E-04	9.349095E-04	-7.211192E-03	5.664728E-02	-1.427270E-02	-1.198605E-02	4.738923E-02	1.173460E+00
1.492466E-04	8.010508E-04	-8.207921E-03	5.649525E-02	-1.534944E-02	-1.406454E-02	6.338903E-02	1.129795E+00
1.492466E-04	6.671921E-04	-1.012215E-02	5.621616E-02	-1.657555E-02	-1.677710E-02	8.205942E-02	1.088177E+00
1.492466E-04	5.333333E-04	-1.305121E-02	5.563812E-02	-1.797251E-02	-2.052634E-02	1.227136E-01	1.055229E+00

1.492466E-04	2.022183E-03	-2.827252E-04	5.707340E-02	-8.216153E-03	-4.515941E-03	-1.220398E-03	1.436062E+00
1.492466E-04	1.904728E-03	-1.100003E-03	5.706087E-02	-8.712804E-03	-4.942034E-03	3.161754E-03	1.414261E+00
1.492466E-04	1.787274E-03	-8.216735E-04	5.705469E-02	-9.260694E-03	-5.377608E-03	3.824310E-03	1.378989E+00
1.492466E-04	1.669819E-03	-6.948518E-04	5.704673E-02	-9.850759E-03	-5.857787E-03	3.713426E-03	1.344312E+00
1.492466E-04	1.552364E-03	-2.426092E-03	5.701224E-02	-1.037668E-02	-6.585301E-03	8.985039E-03	1.328321E+00
1.492466E-04	1.434909E-03	-2.718806E-03	5.699087E-02	-1.103272E-02	-7.248490E-03	1.252761E-02	1.296452E+00
1.492466E-04	1.317455E-03	-3.370775E-03	5.696762E-02	-1.172041E-02	-8.046268E-03	1.039458E-02	1.263984E+00
1.492466E-04	1.200000E-03	-3.897834E-03	5.693847E-02	-1.246064E-02	-8.982519E-03	1.139093E-02	1.229365E+00

Table 30. *f*16p2 low energy transfers

Inner Radius	Outer Radius	Delta V1x	Delta V1y	Delta V2x	Delta V2y	Initial *f*	Time of flight
5.101992E-05	8.442077E-04	4.470293E-03	9.986303E-02	-1.463526E-02	1.431394E-02	-8.866108E-03	2.263209E+00
5.101992E-05	7.820329E-04	5.162599E-03	9.982921E-02	-1.443535E-02	1.609748E-02	-1.949808E-02	2.347568E+00
5.101992E-05	7.198580E-04	9.065713E-03	9.972440E-02	-1.470921E-02	1.753082E-02	-1.464921E-02	2.378091E+00
5.101992E-05	6.576831E-04	9.374008E-03	9.967661E-02	-1.443331E-02	1.956308E-02	-2.273696E-02	2.465604E+00
5.101992E-05	5.955082E-04	1.129009E-02	9.960120E-02	-1.450381E-02	2.149641E-02	-2.149857E-02	2.528976E+00
5.101992E-05	5.333333E-04	1.545805E-02	9.949827E-02	-1.498317E-02	2.340824E-02	-9.402302E-03	2.546601E+00
5.101992E-05	9.797870E-04	-8.560297E-03	9.993702E-02	-1.445431E-02	1.127245E-02	-2.040372E-02	2.200731E+00
5.101992E-05	1.053191E-03	-1.745532E-02	1.005181E-01	-1.763623E-02	5.612633E-03	-7.217109E-02	2.052522E+00
5.101992E-05	1.126596E-03	-1.885541E-02	1.005322E-01	-2.184035E-02	4.329641E-03	-7.299027E-02	2.057331E+00
5.101992E-05	1.200000E-03	-1.901294E-02	1.005056E-01	-2.438947E-02	4.969610E-03	-7.155670E-02	2.057444E+00
5.357092E-05	8.575124E-04	3.367307E-03	9.743393E-02	-1.418170E-02	1.446302E-02	-1.340406E-03	2.304904E+00
5.357092E-05	7.926765E-04	7.081058E-03	9.742221E-02	-1.426025E-02	1.598638E-02	6.915837E-03	2.357642E+00
5.357092E-05	7.278407E-04	1.089851E-02	9.741085E-02	-1.424268E-02	1.765519E-02	1.480725E-02	2.395675E+00
5.357092E-05	6.630049E-04	1.475288E-02	9.741735E-02	-1.525639E-02	1.939362E-02	2.400403E-02	2.428072E+00
5.357092E-05	5.981691E-04	1.802803E-02	9.737510E-02	-1.488430E-02	2.105975E-02	2.795125E-02	2.457788E+00
5.357092E-05	5.333333E-04	2.164550E-02	9.737305E-02	-1.554261E-02	2.337789E-02	3.569456E-02	2.482858E+00
5.357092E-05	9.917611E-04	-6.089973E-03	9.746464E-02	-1.484931E-02	1.048535E-02	-1.719313E-02	2.172959E+00
5.357092E-05	1.061174E-03	-1.151570E-02	9.748310E-02	-1.823882E-02	6.526184E-03	-2.785811E-02	2.115379E+00
5.357092E-05	1.130587E-03	-1.305368E-02	9.754210E-02	-2.168127E-02	4.401366E-03	-3.618213E-02	2.046582E+00
5.357092E-05	1.200000E-03	-1.259131E-02	9.757399E-02	-2.426837E-02	5.155161E-03	-3.838630E-02	2.045668E+00
5.624946E-05	8.713443E-04	2.428937E-03	9.501402E-02	-1.412156E-02	1.419254E-02	-2.667642E-02	2.295524E+00
5.624946E-05	8.037421E-04	4.224880E-03	9.495870E-02	-1.416032E-02	1.579493E-02	-4.895568E-02	2.347437E+00
5.624946E-05	7.361399E-04	9.234794E-03	9.471438E-02	-1.456280E-02	1.719732E-02	-4.076392E-02	2.357676E+00
5.624946E-05	6.685377E-04	1.012524E-02	9.465451E-02	-1.437714E-02	1.927467E-02	-4.433879E-02	2.445338E+00
5.624946E-05	6.009355E-04	1.356502E-02	9.445063E-02	-1.488853E-02	2.103517E-02	-4.067678E-02	2.471832E+00
5.624946E-05	5.333333E-04	1.568998E-02	9.423041E-02	-1.515806E-02	2.328785E-02	-4.904429E-02	2.507636E+00
5.624946E-05	1.004210E-03	-6.717631E-03	9.501677E-02	-1.408889E-02	1.118990E-02	-1.070576E-02	2.213879E+00
5.624946E-05	1.069473E-03	-1.810909E-02	9.586356E-02	-1.537595E-02	7.326809E-03	-8.635127E-02	2.044233E+00
5.624946E-05	1.134737E-03	-2.016652E-02	9.597066E-02	-2.015636E-02	3.991113E-03	-9.121440E-02	2.055080E+00
5.624946E-05	1.200000E-03	-1.980151E-02	9.600892E-02	-2.293742E-02	4.133697E-03	-9.317045E-02	2.049858E+00
5.906194E-05	8.857323E-04	4.469698E-03	9.266983E-02	-1.405142E-02	1.393911E-02	2.269857E-03	2.301168E+00
5.906194E-05	8.152525E-04	6.538651E-03	9.256337E-02	-1.396858E-02	1.568658E-02	-1.738975E-02	2.355040E+00
5.906194E-05	7.447727E-04	8.011046E-03	9.242708E-02	-1.408986E-02	1.736988E-02	-4.163817E-02	2.403682E+00
5.906194E-05	6.742929E-04	1.088453E-02	9.233640E-02	-1.413569E-02	1.929102E-02	-2.830543E-02	2.474779E+00
5.906194E-05	6.038131E-04	1.249534E-02	9.219399E-02	-1.429090E-02	2.135958E-02	-3.673286E-02	2.535996E+00
5.906194E-05	5.333333E-04	1.448699E-02	9.200378E-02	-1.438927E-02	2.378254E-02	-4.442161E-02	2.580563E+00
5.906194E-05	1.017159E-03	-2.777155E-03	9.265646E-02	-1.226941E-02	1.521117E-02	6.991656E-03	2.226884E+00
5.906194E-05	1.078106E-03	-2.150466E-03	9.266080E-02	-1.321732E-02	1.774132E-02	3.939633E-03	2.171184E+00
5.906194E-05	1.139053E-03	-7.499695E-03	9.259810E-02	-1.457000E-02	1.592849E-02	-4.924005E-03	2.093380E+00
5.906194E-05	1.200000E-03	-9.698347E-03	9.257345E-02	-1.623525E-02	1.664436E-02	-9.291066E-03	2.071016E+00
6.201503E-05	9.007078E-04	3.228429E-03	9.036450E-02	-1.360261E-02	1.404710E-02	-8.813036E-03	2.324783E+00
6.201503E-05	8.272329E-04	4.031607E-03	9.032004E-02	-1.362194E-02	1.573271E-02	-3.706546E-02	2.407233E+00
6.201503E-05	7.537580E-04	5.840348E-03	9.022535E-02	-1.364157E-02	1.749989E-02	-6.982389E-02	2.433729E+00
6.201503E-05	6.802831E-04	8.655832E-03	9.000426E-02	-1.425296E-02	1.902463E-02	-8.802671E-02	2.444408E+00
6.201503E-05	6.068082E-04	1.030393E-02	8.983330E-02	-1.502759E-02	2.075906E-02	-1.146395E-01	2.466626E+00
6.201503E-05	5.333333E-04	1.156225E-02	8.968601E-02	-1.501548E-02	2.337949E-02	-1.308949E-01	2.516500E+00
6.201503E-05	1.030637E-03	-1.030970E-03	9.038217E-02	-1.870341E-02	9.405601E-03	6.877940E-03	2.241663E+00
6.201503E-05	1.087091E-03	-2.249970E-03	9.035564E-02	-1.947314E-02	7.202930E-03	6.904022E-03	2.137725E+00
6.201503E-05	1.143546E-03	-4.431052E-03	9.032082E-02	-2.128710E-02	6.977033E-03	6.917985E-03	2.138362E+00
6.201503E-05	1.200000E-03	-6.083727E-03	9.027172E-02	-2.256822E-02	6.372739E-03	8.955815E-03	2.107204E+00
6.511579E-05	9.163043E-04	2.877992E-03	8.810811E-02	-1.370855E-02	1.358329E-02	-4.003447E-02	2.288032E+00
6.511579E-05	8.397101E-04	5.013653E-03	8.802634E-02	-1.334291E-02	1.566298E-02	-4.449509E-02	2.378768E+00
6.511579E-05	7.631159E-04	7.046557E-03	8.789030E-02	-1.375622E-02	1.716604E-02	-7.170363E-02	2.410787E+00
6.511579E-05	6.865217E-04	8.880321E-03	8.773142E-02	-1.397322E-02	1.905259E-02	-9.060298E-02	2.453684E+00
6.511579E-05	6.099275E-04	9.795655E-03	8.763757E-02	-1.423545E-02	2.118988E-02	-1.157402E-01	2.505974E+00
6.511579E-05	5.333333E-04	1.274380E-02	8.726999E-02	-1.495235E-02	2.341722E-02	-1.281433E-01	2.507940E+00
6.511579E-05	1.044674E-03	-1.663813E-03	8.814114E-02	-1.811239E-02	9.447110E-03	-8.553740E-05	2.256013E+00
6.511579E-05	1.096449E-03	-6.310200E-03	8.810558E-02	-1.699989E-02	7.472278E-03	-8.346713E-03	2.164904E+00
6.511579E-05	1.148225E-03	-1.523051E-02	8.827070E-02	-1.733492E-02	5.106360E-03	-4.390037E-02	2.091631E+00
6.511579E-05	1.200000E-03	-1.367074E-02	8.842476E-02	-2.074247E-02	4.015653E-03	-5.436812E-02	2.052649E+00
6.837158E-05	9.325589E-04	2.798814E-03	8.594894E-02	-1.286655E-02	1.408169E-02	-4.159648E-03	2.375092E+00
6.837158E-05	8.527138E-04	5.111305E-03	8.582718E-02	-1.329150E-02	1.541572E-02	-4.767102E-02	2.386397E+00
6.837158E-05	7.728687E-04	6.581811E-03	8.573000E-02	-1.348928E-02	1.713928E-02	-7.999850E-02	2.430435E+00
6.837158E-05	6.930235E-04	9.565946E-03	8.546471E-02	-1.378082E-02	1.900886E-02	-9.221283E-02	2.448289E+00
6.837158E-05	6.131784E-04	1.151490E-02	8.522880E-02	-1.437253E-02	2.098436E-02	-1.154479E-01	2.469407E+00
6.837158E-05	5.333333E-04	1.255055E-02	8.507775E-02	-1.522124E-02	2.325698E-02	-1.431739E-01	2.507131E+00
6.837158E-05	1.059303E-03	-5.106639E-03	8.595367E-02	-1.317031E-02	1.117506E-02	-1.129672E-02	2.258835E+00
6.837158E-05	1.106202E-03	-1.082266E-02	8.596546E-02	-1.320178E-02	1.019444E-02	-2.553929E-02	2.229265E+00
6.837158E-05	1.153101E-03	-1.213858E-02	8.602887E-02	-1.232443E-02	1.382812E-02	-3.479689E-02	2.190270E+00
6.837158E-05	1.200000E-03	-1.745815E-02	8.600024E-02	-1.340890E-02	1.342895E-02	-4.263095E-02	2.136681E+00
7.179015E-05	9.495115E-04	4.056736E-03	8.374883E-02	-1.314028E-02	1.344423E-02	-2.153247E-02	2.305672E+00
7.179015E-05	8.662759E-04	6.224745E-03	8.362179E-02	-1.324436E-02	1.514231E-02	-4.930994E-02	2.359193E+00
7.179015E-05	7.830402E-04	1.153484E-02	8.333225E-02	-1.384975E-02	1.657294E-02	-3.936626E-02	2.357092E+00

7.179015E-05	6.998046E-04	1.303182E-02	8.305204E-02	-1.412464E-02	1.854943E-02	-6.933641E-02	2.392228E+00
7.179015E-05	6.165690E-04	1.470283E-02	8.287661E-02	-1.418747E-02	2.100096E-02	-7.270251E-02	2.467758E+00
7.179015E-05	5.333333E-04	1.685554E-02	8.264303E-02	-1.466582E-02	2.359797E-02	-7.412673E-02	2.524385E+00
7.179015E-05	1.074560E-03	-3.673993E-03	8.378108E-02	-1.296426E-02	1.113126E-02	4.072904E-03	2.269586E+00
7.179015E-05	1.116374E-03	-7.808851E-03	8.377582E-02	-1.314191E-02	1.005423E-02	-1.278637E-02	2.211821E+00
7.179015E-05	1.158187E-03	-1.417921E-02	8.379008E-02	-1.362973E-02	8.395610E-03	-3.132896E-02	2.142799E+00
7.179015E-05	1.200000E-03	-1.335900E-02	8.382298E-02	-1.792430E-02	5.514436E-03	-3.221084E-02	2.145714E+00
7.537966E-05	9.672062E-04	6.705423E-03	8.167857E-02	-1.312592E-02	1.307881E-02	5.034094E-03	2.273685E+00
7.537966E-05	8.804316E-04	1.220620E-02	8.173619E-02	-1.398009E-02	1.480471E-02	2.720480E-02	2.315360E+00
7.537966E-05	7.936570E-04	1.676982E-02	8.177046E-02	-1.366450E-02	1.645866E-02	3.967525E-02	2.343361E+00
7.537966E-05	7.068825E-04	2.096007E-02	8.179175E-02	-1.479173E-02	1.859465E-02	4.976015E-02	2.366912E+00
7.537966E-05	6.201079E-04	2.480163E-02	8.180304E-02	-1.465338E-02	2.090841E-02	5.843877E-02	2.390927E+00
7.537966E-05	5.333333E-04	2.844186E-02	8.180341E-02	-1.544705E-02	2.307606E-02	6.625339E-02	2.414364E+00
7.537966E-05	1.090486E-03	-4.263205E-03	8.172407E-02	-1.248800E-02	1.135982E-02	-8.787979E-03	2.291814E+00
7.537966E-05	1.126990E-03	-7.450662E-03	8.171565E-02	-1.267812E-02	1.046989E-02	-1.620395E-02	2.249486E+00
7.537966E-05	1.163495E-03	-1.307112E-02	8.171293E-02	-1.282678E-02	9.579301E-03	-2.941979E-02	2.240965E+00
7.537966E-05	1.200000E-03	-1.416222E-02	8.189434E-02	-1.649223E-02	5.300097E-03	-4.802807E-02	2.117818E+00
7.914864E-05	9.856912E-04	4.749590E-03	7.957513E-02	-1.308445E-02	1.275072E-02	-3.075125E-02	2.276809E+00
7.914864E-05	8.952196E-04	9.212336E-03	7.922721E-02	-1.375699E-02	1.396564E-02	-6.738412E-02	2.233951E+00
7.914864E-05	8.047480E-04	1.185160E-02	7.887028E-02	-1.429902E-02	1.558885E-02	-1.051703E-01	2.236843E+00
7.914864E-05	7.142765E-04	1.411107E-02	7.850173E-02	-1.496679E-02	1.741739E-02	-1.352511E-01	2.254044E+00
7.914864E-05	6.238049E-04	1.689095E-02	7.806966E-02	-1.539090E-02	1.985233E-02	-1.383406E-01	2.294165E+00
7.914864E-05	5.333333E-04	1.744405E-02	7.788116E-02	-1.570638E-02	2.288978E-02	-1.661080E-01	2.355011E+00
7.914864E-05	1.107122E-03	-2.621074E-03	7.967533E-02	-1.251123E-02	1.104036E-02	-1.497653E-03	2.274319E+00
7.914864E-05	1.138081E-03	-4.336947E-03	7.967039E-02	-1.462175E-02	8.767746E-03	-7.452361E-03	2.239963E+00
7.914864E-05	1.169041E-03	-8.115312E-03	7.963767E-02	-1.402185E-02	8.258380E-03	-1.361924E-02	2.208986E+00
7.914864E-05	1.200000E-03	-8.389928E-03	7.966397E-02	-1.669218E-02	6.946824E-03	-1.820219E-02	2.207463E+00
8.310608E-05	1.005020E-03	3.376442E-03	7.764693E-02	-1.212958E-02	1.334423E-02	-2.338536E-02	2.374976E+00
8.310608E-05	9.106825E-04	8.378202E-03	7.734167E-02	-1.308307E-02	1.430278E-02	-5.235569E-02	2.305987E+00
8.310608E-05	8.163452E-04	9.197201E-03	7.723466E-02	-1.280801E-02	1.663371E-02	-7.335179E-02	2.424955E+00
8.310608E-05	7.220079E-04	1.014824E-02	7.709014E-02	-1.299749E-02	1.879396E-02	-1.030665E-01	2.498097E+00
8.310608E-05	6.276706E-04	1.174688E-02	7.684936E-02	-1.366681E-02	2.100033E-02	-1.409382E-01	2.496748E+00
8.310608E-05	5.333333E-04	1.457552E-02	7.640805E-02	-1.463811E-02	2.361073E-02	-1.498673E-01	2.508941E+00
8.310608E-05	1.124518E-03	-1.897491E-03	7.768948E-02	-1.245934E-02	1.077596E-02	-3.758019E-03	2.265507E+00
8.310608E-05	1.149679E-03	-3.643129E-03	7.767772E-02	-1.258893E-02	1.013138E-02	-6.506250E-03	2.228214E+00
8.310608E-05	1.174839E-03	-4.752545E-03	7.766850E-02	-1.507235E-02	8.081885E-03	-7.107028E-03	2.234349E+00
8.310608E-05	1.200000E-03	-8.150870E-03	7.767439E-02	-1.481916E-02	7.446506E-03	-1.871294E-02	2.210611E+00
8.726138E-05	1.025251E-03	3.617865E-03	7.568070E-02	-1.206310E-02	1.302428E-02	-3.221993E-02	2.352362E+00
8.726138E-05	9.268677E-04	6.428138E-03	7.550164E-02	-1.248155E-02	1.452952E-02	-6.930358E-02	2.369992E+00
8.726138E-05	8.284841E-04	1.004428E-02	7.522025E-02	-1.272967E-02	1.641722E-02	-6.879594E-02	2.415369E+00
8.726138E-05	7.301005E-04	1.301044E-02	7.494164E-02	-1.308089E-02	1.851813E-02	-7.078202E-02	2.462977E+00
8.726138E-05	6.317169E-04	1.534461E-02	7.458091E-02	-1.369271E-02	2.085489E-02	-8.911686E-02	2.481790E+00
8.726138E-05	5.333333E-04	1.565247E-02	7.424354E-02	-1.469459E-02	2.356817E-02	-1.542500E-01	2.474082E+00
8.726138E-05	1.142726E-03	-1.485593E-03	7.574310E-02	-1.208579E-02	1.089011E-02	-1.104953E-03	2.286416E+00
8.726138E-05	1.161817E-03	-2.191186E-03	7.572807E-02	-1.234956E-02	1.021145E-02	9.761808E-04	2.236961E+00
8.726138E-05	1.180909E-03	-3.172029E-03	7.570625E-02	-1.338859E-02	8.724884E-03	3.823446E-03	2.207144E+00
8.726138E-05	1.200000E-03	-3.771901E-03	7.570424E-02	-1.499135E-02	7.452440E-03	1.607487E-04	2.194283E+00
9.162445E-05	1.046451E-03	4.995874E-03	7.377014E-02	-1.189888E-02	1.279478E-02	-1.236274E-02	2.349255E+00
9.162445E-05	9.438278E-04	6.964643E-03	7.358941E-02	-1.195021E-02	1.466036E-02	-5.212223E-02	2.417955E+00
9.162445E-05	8.412042E-04	9.071365E-03	7.335734E-02	-1.241062E-02	1.639098E-02	-8.568067E-02	2.445089E+00
9.162445E-05	7.385806E-04	1.165516E-02	7.301815E-02	-1.311876E-02	1.826892E-02	-1.115265E-01	2.441886E+00
9.162445E-05	6.359570E-04	1.317079E-02	7.278050E-02	-1.340651E-02	2.091659E-02	-1.284009E-01	2.510845E+00
9.162445E-05	5.333333E-04	1.388765E-02	7.260316E-02	-1.413115E-02	2.392575E-02	-1.642986E-01	2.557551E+00
9.162445E-05	1.161806E-03	-6.594619E-04	7.384252E-02	-1.189731E-02	1.077468E-02	4.343844E-03	2.291809E+00
9.162445E-05	1.174538E-03	-1.410350E-03	7.383346E-02	-1.218734E-02	1.025944E-02	8.305293E-03	2.289894E+00
9.162445E-05	1.187269E-03	-2.051778E-03	7.382123E-02	-1.276610E-02	9.510466E-03	1.002378E-02	2.275403E+00
9.162445E-05	1.200000E-03	-2.764909E-03	7.380637E-02	-1.322338E-02	8.915874E-03	1.046288E-02	2.261380E+00
9.620567E-05	1.068694E-03	4.232430E-03	7.189989E-02	-1.164657E-02	1.264590E-02	-2.952743E-02	2.359690E+00
9.620567E-05	9.616220E-04	6.543723E-03	7.173735E-02	-1.168956E-02	1.454364E-02	-6.374980E-02	2.436787E+00
9.620567E-05	8.545499E-04	8.130560E-03	7.156643E-02	-1.207423E-02	1.636597E-02	-1.054139E-01	2.472895E+00
9.620567E-05	7.474777E-04	9.579003E-03	7.139537E-02	-1.234934E-02	1.859914E-02	-1.407861E-01	2.507595E+00
9.620567E-05	6.404055E-04	1.268740E-02	7.097515E-02	-1.314925E-02	2.096402E-02	-1.298353E-01	2.570134E+00
9.620567E-05	5.333333E-04	1.405137E-02	7.068079E-02	-1.419918E-02	2.388563E-02	-1.715380E-01	2.556337E+00
9.620567E-05	1.181825E-03	-4.992423E-04	7.198829E-02	-1.167962E-02	1.068345E-02	-2.346579E-03	2.297827E+00
9.620567E-05	1.187883E-03	-5.797082E-04	7.198437E-02	-1.175374E-02	1.049189E-02	2.090435E-03	2.284992E+00
9.620567E-05	1.193942E-03	-9.466465E-04	7.198131E-02	-1.175648E-02	1.038762E-02	3.667590E-03	2.284049E+00
9.620567E-05	1.200000E-03	-9.648878E-04	7.197638E-02	-1.183489E-02	1.018117E-02	8.321405E-03	2.263979E+00
1.010160E-04	1.092063E-03	5.406734E-03	7.010248E-02	-1.136319E-02	1.252129E-02	-7.457631E-03	2.378620E+00
1.010160E-04	9.803169E-04	9.270481E-03	6.999339E-02	-1.123721E-02	1.382480E-02	-5.000735E-03	2.444165E+00
1.010160E-04	8.685710E-04	1.507971E-02	7.003510E-02	-1.204196E-02	1.609953E-02	2.553200E-02	2.468210E+00
1.010160E-04	7.568251E-04	1.918677E-02	7.005644E-02	-1.255681E-02	1.812507E-02	4.042641E-02	2.488073E+00
1.010160E-04	6.450792E-04	2.314293E-02	7.007052E-02	-1.384986E-02	2.102172E-02	5.302364E-02	2.501674E+00
1.010160E-04	5.333333E-04	2.691589E-02	7.007092E-02	-1.519843E-02	2.443959E-02	6.384618E-02	2.513347E+00
1.010160E-04	1.202857E-03	-1.314427E-04	7.017532E-02	-1.129947E-02	1.076706E-02	-3.605001E-03	2.312559E+00
1.010160E-04	1.201904E-03	-6.928937E-06	7.017487E-02	-1.140635E-02	1.066374E-02	-2.382564E-03	2.312021E+00
1.010160E-04	1.200952E-03	1.377381E-04	7.017457E-02	-1.141891E-02	1.066482E-02	-9.089235E-04	2.311129E+00
1.010160E-04	1.200000E-03	2.060856E-04	7.017382E-02	-1.144716E-02	1.064593E-02	-1.667837E-03	2.305764E+00

Table 31. *f*17p1 low energy transfers

Inner Radius	Outer Radius	Delta V1x	Delta V1y	Delta V2x	Delta V2y	Initial *f*	Time of flight
5.101992E-05	9.606048E-04	-4.499049E-04	9.987550E-02	-1.605026E-02	-9.125962E-03	6.838127E-04	1.171755E+00
5.101992E-05	9.217619E-04	-2.924042E-04	9.987211E-02	-1.643713E-02	-9.572253E-03	7.881283E-04	1.157706E+00
5.101992E-05	8.829191E-04	-1.284521E-03	9.986402E-02	-1.682137E-02	-1.009007E-02	3.457058E-03	1.148883E+00
5.101992E-05	8.440762E-04	-1.584286E-03	9.985711E-02	-1.723361E-02	-1.062487E-02	4.804237E-03	1.136287E+00
5.101992E-05	8.052333E-04	-2.244463E-03	9.984346E-02	-1.764398E-02	-1.123782E-02	8.335206E-03	1.125382E+00
5.101992E-05	7.663905E-04	-3.860689E-03	9.980898E-02	-1.806293E-02	-1.192061E-02	1.192775E-02	1.118096E+00
5.101992E-05	7.275476E-04	-5.011398E-03	9.975628E-02	-1.849930E-02	-1.266370E-02	2.249624E-02	1.110435E+00
5.101992E-05	6.887048E-04	-6.196568E-03	9.968666E-02	-1.895173E-02	-1.347972E-02	3.417341E-02	1.102722E+00
5.101992E-05	6.498619E-04	-6.293787E-03	9.967627E-02	-1.954064E-02	-1.420350E-02	3.462350E-02	1.086024E+00
5.101992E-05	6.110190E-04	-6.289039E-03	9.967110E-02	-2.006713E-02	-1.513362E-02	3.431251E-02	1.067799E+00
5.101992E-05	5.721762E-04	-5.047327E-03	9.971877E-02	-2.074758E-02	-1.600474E-02	3.031788E-02	1.042060E+00
5.101992E-05	5.333333E-04	-5.712211E-03	9.967788E-02	-2.137160E-02	-1.712790E-02	3.615331E-02	1.026700E+00
5.101992E-05	1.024517E-03	1.647217E-04	9.988116E-02	-1.545725E-02	-8.433906E-03	1.917734E-04	1.190271E+00
5.101992E-05	1.049586E-03	1.987592E-03	9.988063E-02	-1.525272E-02	-8.144751E-03	2.451570E-03	1.191331E+00
5.101992E-05	1.074655E-03	2.444255E-03	9.987576E-02	-1.501298E-02	-7.947064E-03	-3.654363E-05	1.196543E+00
5.101992E-05	1.099724E-03	3.193108E-03	9.986891E-02	-1.486445E-02	-7.588190E-03	-1.123784E-03	1.201056E+00
5.101992E-05	1.124793E-03	3.643301E-03	9.986220E-02	-1.465445E-02	-7.368591E-03	-2.607560E-03	1.206413E+00
5.101992E-05	1.149862E-03	4.498593E-03	9.984437E-02	-1.449110E-02	-7.071732E-03	-5.278326E-03	1.209734E+00
5.101992E-05	1.174931E-03	5.241252E-03	9.983302E-02	-1.430153E-02	-6.844101E-03	-5.199166E-03	1.214154E+00
5.101992E-05	1.200000E-03	5.821811E-03	9.982130E-02	-1.411855E-02	-6.619013E-03	-5.709503E-03	1.218980E+00
5.357092E-05	9.731446E-04	-5.912187E-04	9.740809E-02	-1.587700E-02	-9.061684E-03	5.346009E-04	1.174434E+00
5.357092E-05	9.331618E-04	-1.404804E-03	9.739862E-02	-1.624189E-02	-9.562852E-03	4.914411E-03	1.165552E+00
5.357092E-05	8.931789E-04	-1.679039E-03	9.739229E-02	-1.664135E-02	-1.006442E-02	5.349646E-03	1.152686E+00
5.357092E-05	8.531961E-04	-2.039915E-03	9.738475E-02	-1.705578E-02	-1.061038E-02	5.551236E-03	1.139666E+00
5.357092E-05	8.132132E-04	-3.071512E-03	9.736022E-02	-1.746687E-02	-1.124115E-02	1.120443E-02	1.130759E+00
5.357092E-05	7.732304E-04	-3.530535E-03	9.734671E-02	-1.792436E-02	-1.187729E-02	1.176566E-02	1.117227E+00
5.357092E-05	7.332475E-04	-5.178071E-03	9.728099E-02	-1.835841E-02	-1.264927E-02	2.280215E-02	1.111649E+00
5.357092E-05	6.932647E-04	-6.214475E-03	9.723539E-02	-1.885156E-02	-1.342884E-02	2.531086E-02	1.100072E+00
5.357092E-05	6.532819E-04	-6.861174E-03	9.720982E-02	-1.932861E-02	-1.434056E-02	2.407280E-02	1.084751E+00
5.357092E-05	6.132990E-04	-4.841684E-03	9.730931E-02	-2.015016E-02	-1.489651E-02	7.327164E-03	1.051883E+00
5.357092E-05	5.733162E-04	-4.287694E-03	9.733617E-02	-2.083693E-02	-1.580619E-02	-1.480826E-03	1.027165E+00
5.357092E-05	5.333333E-04	-6.081426E-03	9.724438E-02	-2.136041E-02	-1.712267E-02	1.431380E-02	1.019101E+00
5.357092E-05	1.036487E-03	1.242323E-03	9.741187E-02	-1.531506E-02	-8.355149E-03	5.907436E-04	1.187455E+00
5.357092E-05	1.059846E-03	1.801360E-03	9.740904E-02	-1.514189E-02	-8.064695E-03	-1.199425E-03	1.192066E+00
5.357092E-05	1.083205E-03	2.063142E-03	9.740816E-02	-1.493831E-02	-7.848403E-03	-1.882935E-03	1.198116E+00
5.357092E-05	1.106564E-03	2.783294E-03	9.739680E-02	-1.475515E-02	-7.607684E-03	-5.856861E-03	1.201114E+00
5.357092E-05	1.129923E-03	3.591013E-03	9.737832E-02	-1.458006E-02	-7.366239E-03	-1.039632E-02	1.203492E+00
5.357092E-05	1.153282E-03	4.489123E-03	9.735947E-02	-1.441482E-02	-7.119737E-03	-1.163272E-02	1.206405E+00
5.357092E-05	1.176641E-03	5.590728E-03	9.732345E-02	-1.426057E-02	-6.863558E-03	-1.557524E-02	1.207605E+00
5.357092E-05	1.200000E-03	6.210694E-03	9.728742E-02	-1.409782E-02	-6.638298E-03	-2.196453E-02	1.210032E+00
5.624946E-05	9.861126E-04	4.851481E-05	9.499929E-02	-1.571142E-02	-8.974611E-03	-6.968061E-04	1.173390E+00
5.624946E-05	9.449508E-04	-1.202613E-03	9.499070E-02	-1.607566E-02	-9.491308E-03	4.681279E-03	1.166544E+00
5.624946E-05	9.037891E-04	-1.472250E-03	9.498509E-02	-1.647984E-02	-9.998476E-03	4.199076E-03	1.153040E+00
5.624946E-05	8.626273E-04	-1.605522E-03	9.497917E-02	-1.691390E-02	-1.052748E-02	4.933229E-03	1.138823E+00
5.624946E-05	8.214656E-04	-1.763505E-03	9.497400E-02	-1.735389E-02	-1.112463E-02	4.050415E-03	1.123678E+00
5.624946E-05	7.803038E-04	-2.061234E-03	9.496432E-02	-1.782383E-02	-1.176286E-02	6.106688E-03	1.109489E+00
5.624946E-05	7.391421E-04	-2.275139E-03	9.495705E-02	-1.835440E-02	-1.240604E-02	5.759954E-03	1.093573E+00
5.624946E-05	6.979803E-04	-6.099032E-03	9.491445E-02	-1.874772E-02	-1.337136E-02	-2.289416E-04	1.091570E+00
5.624946E-05	6.568186E-04	-6.150165E-03	9.489353E-02	-1.931302E-02	-1.419118E-02	3.037336E-03	1.074458E+00
5.624946E-05	6.156568E-04	-5.158847E-03	9.499507E-02	-2.000434E-02	-1.496366E-02	-1.885648E-02	1.043986E+00
5.624946E-05	5.744951E-04	-4.420962E-03	9.511589E-02	-2.124927E-02	-1.585328E-02	-4.374014E-02	1.012992E+00
5.624946E-05	5.333333E-04	-4.868180E-03	9.522893E-02	-2.125612E-02	-1.706370E-02	-6.027484E-02	9.894460E-01
5.624946E-05	1.048865E-03	1.149054E-03	9.500036E-02	-1.517393E-02	-8.271117E-03	-2.771920E-03	1.188593E+00
5.624946E-05	1.070456E-03	1.847722E-03	9.499347E-02	-1.500023E-02	-8.035469E-03	-6.628520E-03	1.191249E+00
5.624946E-05	1.092046E-03	2.739296E-03	9.497828E-02	-1.483340E-02	-7.800140E-03	-1.126495E-02	1.192744E+00
5.624946E-05	1.113637E-03	3.435724E-03	9.496840E-02	-1.469474E-02	-7.525018E-03	-1.087108E-02	1.196611E+00
5.624946E-05	1.135228E-03	4.052150E-03	9.495150E-02	-1.450581E-02	-7.359045E-03	-1.413750E-02	1.199513E+00
5.624946E-05	1.156819E-03	5.011900E-03	9.491614E-02	-1.436800E-02	-7.103884E-03	-1.991283E-02	1.200236E+00
5.624946E-05	1.178409E-03	5.514721E-03	9.489988E-02	-1.421253E-02	-6.896764E-03	-2.095654E-02	1.204231E+00
5.624946E-05	1.200000E-03	6.480991E-03	9.485925E-02	-1.406922E-02	-6.674968E-03	-2.422906E-02	1.205521E+00
5.906194E-05	9.995273E-04	-1.136324E-03	9.264354E-02	-1.551108E-02	-8.950504E-03	2.345643E-03	1.182066E+00
5.906194E-05	9.571461E-04	-7.123889E-04	9.264156E-02	-1.589990E-02	-9.427747E-03	1.443143E-03	1.165279E+00
5.906194E-05	9.147648E-04	2.012784E-05	9.263830E-02	-1.634538E-02	-9.875327E-03	9.188732E-04	1.146827E+00
5.906194E-05	8.723835E-04	-1.294068E-04	9.263357E-02	-1.678145E-02	-1.041718E-02	8.153941E-04	1.132061E+00
5.906194E-05	8.300022E-04	-1.615650E-03	9.261770E-02	-1.718759E-02	-1.109557E-02	1.013379E-02	1.125739E+00
5.906194E-05	7.876210E-04	-2.494105E-03	9.259880E-02	-1.763213E-02	-1.179473E-02	1.246890E-02	1.113910E+00
5.906194E-05	7.452397E-04	-3.076914E-03	9.258295E-02	-1.813553E-02	-1.250098E-02	1.276330E-02	1.099427E+00
5.906194E-05	7.028584E-04	-3.391269E-03	9.257177E-02	-1.866421E-02	-1.327970E-02	1.225780E-02	1.082672E+00
5.906194E-05	6.604772E-04	-3.520720E-03	9.256265E-02	-1.921505E-02	-1.415117E-02	1.220391E-02	1.064327E+00
5.906194E-05	6.180959E-04	-4.227710E-03	9.253362E-02	-1.982035E-02	-1.509645E-02	1.601933E-02	1.048937E+00
5.906194E-05	5.757146E-04	-4.865528E-03	9.250645E-02	-2.042951E-02	-1.619907E-02	1.750185E-02	1.031381E+00
5.906194E-05	5.333333E-04	-4.854422E-03	9.250700E-02	-2.101115E-02	-1.751299E-02	1.345177E-02	1.007752E+00
5.906194E-05	1.061670E-03	5.025747E-04	9.265132E-02	-1.497943E-02	-8.281371E-03	-8.860955E-04	1.193902E+00

5.906194E-05	1.081431E-03	1.439907E-03	9.264522E-02	-1.485245E-02	-8.014201E-03	-5.755788E-03	1.194475E+00
5.906194E-05	1.101193E-03	1.562696E-03	9.264530E-02	-1.466983E-02	-7.864430E-03	-6.364369E-03	1.199926E+00
5.906194E-05	1.120954E-03	2.779031E-03	9.263442E-02	-1.453105E-02	-7.640838E-03	-5.372071E-03	1.200834E+00
5.906194E-05	1.140716E-03	3.301006E-03	9.262839E-02	-1.438977E-02	-7.433293E-03	-5.642438E-03	1.204493E+00
5.906194E-05	1.160477E-03	3.955343E-03	9.261133E-02	-1.426244E-02	-7.207331E-03	-9.262434E-03	1.206504E+00
5.906194E-05	1.180239E-03	4.414922E-03	9.259876E-02	-1.410835E-02	-7.044320E-03	-1.127621E-02	1.209742E+00
5.906194E-05	1.200000E-03	5.280804E-03	9.256514E-02	-1.398417E-02	-6.829695E-03	-1.653083E-02	1.210196E+00
6.201503E-05	1.013409E-03	2.062103E-04	9.034903E-02	-1.535279E-02	-8.835195E-03	-1.489067E-04	1.177335E+00
6.201503E-05	9.697658E-04	-5.999551E-05	9.034498E-02	-1.575144E-02	-9.311641E-03	4.255981E-04	1.163932E+00
6.201503E-05	9.261226E-04	-1.040393E-03	9.033685E-02	-1.614312E-02	-9.871453E-03	4.263330E-03	1.154446E+00
6.201503E-05	8.824793E-04	-7.342752E-04	9.033364E-02	-1.658386E-02	-1.041768E-02	4.058986E-03	1.137045E+00
6.201503E-05	8.388361E-04	-9.732479E-04	9.032735E-02	-1.705434E-02	-1.100238E-02	3.721543E-03	1.121659E+00
6.201503E-05	7.951928E-04	-1.255411E-03	9.031988E-02	-1.755281E-02	-1.163743E-02	4.161304E-03	1.106077E+00
6.201503E-05	7.515496E-04	-8.939124E-04	9.031630E-02	-1.806308E-02	-1.235312E-02	2.516404E-03	1.086082E+00
6.201503E-05	7.079063E-04	-1.726997E-03	9.030291E-02	-1.857614E-02	-1.318759E-02	5.120888E-03	1.072209E+00
6.201503E-05	6.642631E-04	-2.144463E-03	9.028937E-02	-1.914516E-02	-1.406972E-02	7.476253E-03	1.055415E+00
6.201503E-05	6.206198E-04	-2.398713E-03	9.027758E-02	-1.973052E-02	-1.507665E-02	8.118335E-03	1.036284E+00
6.201503E-05	5.769766E-04	-2.526850E-03	9.026492E-02	-2.041354E-02	-1.612724E-02	9.864073E-03	1.015742E+00
6.201503E-05	5.333333E-04	-2.851463E-03	9.025075E-02	-2.111990E-02	-1.735255E-02	9.358325E-03	9.940603E-01
6.201503E-05	1.074921E-03	2.056358E-03	9.034150E-02	-1.487374E-02	-8.115007E-03	-4.263088E-03	1.187535E+00
6.201503E-05	1.092789E-03	2.519784E-03	9.033404E-02	-1.471412E-02	-7.964129E-03	-7.084004E-03	1.190157E+00
6.201503E-05	1.110658E-03	2.826782E-03	9.033084E-02	-1.458405E-02	-7.768445E-03	-7.388256E-03	1.194293E+00
6.201503E-05	1.128526E-03	3.468316E-03	9.031715E-02	-1.444990E-02	-7.587672E-03	-1.009179E-02	1.196047E+00
6.201503E-05	1.146395E-03	3.780740E-03	9.031058E-02	-1.431862E-02	-7.410737E-03	-1.114209E-02	1.199801E+00
6.201503E-05	1.164263E-03	4.040350E-03	9.029993E-02	-1.420669E-02	-7.203970E-03	-1.441122E-02	1.203130E+00
6.201503E-05	1.182132E-03	4.470826E-03	9.028583E-02	-1.402726E-02	-7.138904E-03	-1.677949E-02	1.205653E+00
6.201503E-05	1.200000E-03	5.335302E-03	9.024967E-02	-1.395007E-02	-6.874885E-03	-2.209256E-02	1.205561E+00
6.511579E-05	1.027779E-03	1.230656E-03	8.810183E-02	-1.519214E-02	-8.719655E-03	-7.675550E-04	1.174531E+00
6.511579E-05	9.828293E-04	5.608604E-04	8.809970E-02	-1.557767E-02	-9.232077E-03	-1.543402E-03	1.162322E+00
6.511579E-05	9.378797E-04	4.419963E-04	8.809527E-02	-1.601785E-02	-9.720152E-03	-1.560377E-03	1.147207E+00
6.511579E-05	8.929301E-04	-8.049333E-04	8.809185E-02	-1.643480E-02	-1.032880E-02	-2.045363E-03	1.136859E+00
6.511579E-05	8.479805E-04	-9.595118E-04	8.808557E-02	-1.689495E-02	-1.094742E-02	-1.174369E-03	1.120987E+00
6.511579E-05	8.030309E-04	-1.589249E-03	8.807467E-02	-1.735726E-02	-1.165928E-02	1.982473E-03	1.107456E+00
6.511579E-05	7.580813E-04	-1.468462E-03	8.806732E-02	-1.789768E-02	-1.235536E-02	3.877468E-03	1.089152E+00
6.511579E-05	7.131317E-04	-1.792042E-03	8.805636E-02	-1.845625E-02	-1.314870E-02	5.567899E-03	1.072107E+00
6.511579E-05	6.681821E-04	-2.628769E-03	8.804119E-02	-1.903261E-02	-1.405545E-02	4.825545E-03	1.055862E+00
6.511579E-05	6.232325E-04	-2.497015E-03	8.803426E-02	-1.962561E-02	-1.508657E-02	4.599676E-03	1.033928E+00
6.511579E-05	5.782829E-04	-2.518191E-03	8.802507E-02	-2.030587E-02	-1.618506E-02	4.084376E-03	1.011435E+00
6.511579E-05	5.333333E-04	-3.190065E-03	8.799445E-02	-2.095671E-02	-1.753814E-02	1.145901E-02	9.930090E-01
6.511579E-05	1.088637E-03	3.684877E-04	8.811057E-02	-1.464554E-02	-8.177146E-03	-6.853069E-04	1.198684E+00
6.511579E-05	1.104546E-03	9.631875E-04	8.810919E-02	-1.451995E-02	-8.019244E-03	-2.230994E-03	1.200331E+00
6.511579E-05	1.120455E-03	9.317766E-04	8.811076E-02	-1.442012E-02	-7.823576E-03	-2.136467E-03	1.205628E+00
6.511579E-05	1.136364E-03	1.346207E-03	8.810900E-02	-1.428838E-02	-7.690593E-03	-3.089645E-03	1.208239E+00
6.511579E-05	1.152273E-03	1.832534E-03	8.810166E-02	-1.417535E-02	-7.528769E-03	-8.162554E-03	1.209217E+00
6.511579E-05	1.168182E-03	2.334177E-03	8.809475E-02	-1.407099E-02	-7.357329E-03	-9.714331E-03	1.211199E+00
6.511579E-05	1.184091E-03	2.857083E-03	8.808610E-02	-1.397154E-02	-7.182236E-03	-1.096371E-02	1.213144E+00
6.511579E-05	1.200000E-03	3.138219E-03	8.807982E-02	-1.384551E-02	-7.064494E-03	-1.264417E-02	1.216019E+00
6.837158E-05	1.042659E-03	-6.379882E-04	8.591345E-02	-1.495748E-02	-8.738529E-03	1.512598E-03	1.187071E+00
6.837158E-05	9.963571E-04	-1.014695E-03	8.590702E-02	-1.536054E-02	-9.230371E-03	2.564407E-03	1.173664E+00
6.837158E-05	9.500547E-04	-9.714761E-04	8.590221E-02	-1.578537E-02	-9.754904E-03	2.732700E-03	1.157298E+00
6.837158E-05	9.037523E-04	-6.078857E-04	8.589861E-02	-1.624983E-02	-1.029233E-02	1.511544E-03	1.138337E+00
6.837158E-05	8.574500E-04	-1.397611E-03	8.588988E-02	-1.671596E-02	-1.092157E-02	1.556844E-03	1.124991E+00
6.837158E-05	8.111476E-04	-1.189405E-03	8.588458E-02	-1.722283E-02	-1.158018E-02	1.535819E-03	1.105863E+00
6.837158E-05	7.648452E-04	-3.909752E-04	8.588007E-02	-1.777118E-02	-1.228344E-02	5.462086E-04	1.082738E+00
6.837158E-05	7.185428E-04	-1.818735E-04	8.587246E-02	-1.834318E-02	-1.308146E-02	4.701166E-04	1.061989E+00
6.837158E-05	6.722405E-04	-1.738909E-03	8.585212E-02	-1.887492E-02	-1.408994E-02	1.021356E-02	1.051942E+00
6.837158E-05	6.259381E-04	-2.138985E-03	8.583841E-02	-1.950184E-02	-1.511759E-02	9.604878E-03	1.031861E+00
6.837158E-05	5.796357E-04	-3.069389E-03	8.579958E-02	-2.010906E-02	-1.636569E-02	2.410633E-02	1.017764E+00
6.837158E-05	5.333333E-04	-3.337397E-03	8.577707E-02	-2.087384E-02	-1.763221E-02	3.083144E-02	9.963981E-01
6.837158E-05	1.102842E-03	1.008145E-03	8.591717E-02	-1.449212E-02	-8.088422E-03	-9.370117E-04	1.197559E+00
6.837158E-05	1.116721E-03	1.358974E-03	8.591478E-02	-1.439110E-02	-7.940954E-03	-3.081605E-03	1.199542E+00
6.837158E-05	1.130601E-03	1.635020E-03	8.591288E-02	-1.428092E-02	-7.815645E-03	-4.150353E-03	1.202182E+00
6.837158E-05	1.144481E-03	1.968377E-03	8.591064E-02	-1.417242E-02	-7.692403E-03	-4.432217E-03	1.204739E+00
6.837158E-05	1.158361E-03	2.240939E-03	8.590674E-02	-1.410738E-02	-7.493263E-03	-6.154799E-03	1.207246E+00
6.837158E-05	1.172240E-03	2.458793E-03	8.590418E-02	-1.398101E-02	-7.412606E-03	-6.874536E-03	1.210129E+00
6.837158E-05	1.186120E-03	2.765169E-03	8.590011E-02	-1.388289E-02	-7.283940E-03	-7.496240E-03	1.212638E+00
6.837158E-05	1.200000E-03	3.171108E-03	8.588994E-02	-1.378634E-02	-7.156159E-03	-1.039757E-02	1.213905E+00
7.179015E-05	1.058075E-03	-2.130785E-04	8.377446E-02	-1.476704E-02	-8.664910E-03	4.316102E-03	1.188469E+00
7.179015E-05	1.010371E-03	-1.644613E-03	8.375660E-02	-1.514473E-02	-9.216882E-03	1.195178E-02	1.182424E+00
7.179015E-05	9.626675E-04	-1.843473E-03	8.374952E-02	-1.556971E-02	-9.753602E-03	1.072428E-02	1.166424E+00
7.179015E-05	9.149637E-04	-2.719311E-03	8.373289E-02	-1.600130E-02	-1.036265E-02	9.401498E-03	1.153426E+00
7.179015E-05	8.672599E-04	-3.916208E-03	8.367521E-02	-1.640326E-02	-1.111163E-02	2.911386E-02	1.148286E+00
7.179015E-05	8.195561E-04	-5.829073E-03	8.359847E-02	-1.689981E-02	-1.181420E-02	2.944788E-02	1.140269E+00
7.179015E-05	7.718523E-04	-6.844897E-03	8.349880E-02	-1.736453E-02	-1.267257E-02	5.040141E-02	1.133364E+00
7.179015E-05	7.241485E-04	-7.757907E-03	8.341789E-02	-1.788174E-02	-1.357376E-02	5.924572E-02	1.121017E+00
7.179015E-05	6.764447E-04	-7.066579E-03	8.345803E-02	-1.849092E-02	-1.447831E-02	5.790233E-02	1.095013E+00
7.179015E-05	6.287409E-04	-5.665542E-03	8.353453E-02	-1.915228E-02	-1.547902E-02	6.074264E-02	1.065267E+00
7.179015E-05	5.810371E-04	-5.991172E-03	8.350358E-02	-1.980310E-02	-1.669902E-02	6.243130E-02	1.043428E+00
7.179015E-05	5.333333E-04	-4.507760E-03	8.358227E-02	-2.054900E-02	-1.801963E-02	6.259563E-02	1.009374E+00
7.179015E-05	1.117557E-03	8.202384E-04	8.377844E-02	-1.430257E-02	-8.061007E-03	-3.364710E-05	1.201006E+00
7.179015E-05	1.129334E-03	8.716406E-04	8.378002E-02	-1.420402E-02	-7.965490E-03	9.902556E-04	1.204791E+00
7.179015E-05	1.141112E-03	8.228596E-04	8.378061E-02	-1.411938E-02	-7.848401E-03	8.724161E-05	1.208510E+00

7.179015E-05	1.152889E-03	9.102978E-04	8.378063E-02	-1.402732E-02	-7.747700E-03	-1.038975E-03	1.211382E+00
7.179015E-05	1.164667E-03	7.781698E-04	8.378192E-02	-1.393735E-02	-7.647468E-03	-1.556101E-03	1.215598E+00
7.179015E-05	1.176445E-03	7.312402E-04	8.378297E-02	-1.385038E-02	-7.544892E-03	-1.865285E-03	1.219409E+00
7.179015E-05	1.188222E-03	1.119551E-03	8.378113E-02	-1.376052E-02	-7.449302E-03	-3.278967E-03	1.220502E+00
7.179015E-05	1.200000E-03	1.331439E-03	8.378044E-02	-1.367236E-02	-7.354735E-03	-3.541602E-03	1.222884E+00
7.537966E-05	1.074052E-03	-3.316063E-04	8.168405E-02	-1.460945E-02	-8.530559E-03	1.551100E-03	1.191160E+00
7.537966E-05	1.024896E-03	-2.192820E-04	8.167922E-02	-1.500127E-02	-9.060902E-03	7.182563E-04	1.173709E+00
7.537966E-05	9.757393E-04	-1.604662E-03	8.166360E-02	-1.540239E-02	-9.658927E-03	7.750173E-03	1.166430E+00
7.537966E-05	9.265831E-04	-2.450356E-03	8.164688E-02	-1.582646E-02	-1.029504E-02	8.525471E-03	1.153581E+00
7.537966E-05	8.774269E-04	-2.602542E-03	8.163791E-02	-1.630939E-02	-1.092630E-02	9.019372E-03	1.136157E+00
7.537966E-05	8.282707E-04	-2.966736E-03	8.162586E-02	-1.683966E-02	-1.159067E-02	8.854657E-03	1.118949E+00
7.537966E-05	7.791144E-04	-3.180029E-03	8.161674E-02	-1.732475E-02	-1.243373E-02	7.867289E-03	1.099687E+00
7.537966E-05	7.299582E-04	-4.327077E-03	8.155882E-02	-1.785271E-02	-1.335530E-02	2.168358E-02	1.089405E+00
7.537966E-05	6.808020E-04	-4.317331E-03	8.155420E-02	-1.843395E-02	-1.433701E-02	1.947361E-02	1.066415E+00
7.537966E-05	6.316458E-04	-5.746959E-03	8.145832E-02	-1.904110E-02	-1.547510E-02	3.795387E-02	1.056959E+00
7.537966E-05	5.824896E-04	-7.246099E-03	8.137174E-02	-1.976061E-02	-1.667250E-02	3.918886E-02	1.040919E+00
7.537966E-05	5.333333E-04	-8.635203E-03	8.129335E-02	-2.047935E-02	-1.810879E-02	3.689342E-02	1.021327E+00
7.537966E-05	1.132807E-03	1.443201E-03	8.168662E-02	-1.414714E-02	-7.962939E-03	9.684017E-04	1.200326E+00
7.537966E-05	1.142406E-03	1.571388E-03	8.168350E-02	-1.407431E-02	-7.874262E-03	-2.067759E-03	1.201715E+00
7.537966E-05	1.152005E-03	1.546926E-03	8.168382E-02	-1.398345E-02	-7.821648E-03	-2.812368E-03	1.204591E+00
7.537966E-05	1.161604E-03	1.633610E-03	8.168191E-02	-1.393548E-02	-7.692729E-03	-4.998137E-03	1.206510E+00
7.537966E-05	1.171203E-03	2.146637E-03	8.167643E-02	-1.387509E-02	-7.587317E-03	-5.730074E-03	1.206516E+00
7.537966E-05	1.180802E-03	2.415561E-03	8.167346E-02	-1.381450E-02	-7.485155E-03	-6.063876E-03	1.207961E+00
7.537966E-05	1.190401E-03	2.539602E-03	8.167218E-02	-1.374383E-02	-7.404043E-03	-6.384293E-03	1.210148E+00
7.537966E-05	1.200000E-03	2.662498E-03	8.167069E-02	-1.368580E-02	-7.301666E-03	-6.780043E-03	1.212339E+00
7.914864E-05	1.090618E-03	-1.180618E-03	7.964010E-02	-1.436770E-02	-8.535386E-03	1.261886E-03	1.198828E+00
7.914864E-05	1.039956E-03	-5.521237E-04	7.963720E-02	-1.481038E-02	-8.988902E-03	6.430325E-04	1.178101E+00
7.914864E-05	9.892935E-04	-3.402266E-04	7.963186E-02	-1.523689E-02	-9.547626E-03	3.349404E-04	1.159240E+00
7.914864E-05	9.386313E-04	-1.087426E-03	7.962368E-02	-1.570680E-02	-1.012702E-02	1.086303E-03	1.145516E+00
7.914864E-05	8.879690E-04	-1.397122E-03	7.961568E-02	-1.617497E-02	-1.079989E-02	1.071373E-03	1.128315E+00
7.914864E-05	8.373068E-04	-1.238598E-03	7.960826E-02	-1.668472E-02	-1.151225E-02	2.233607E-03	1.108083E+00
7.914864E-05	7.866445E-04	-9.727784E-04	7.960170E-02	-1.725549E-02	-1.225428E-02	1.630875E-03	1.085847E+00
7.914864E-05	7.359823E-04	-1.183904E-03	7.959166E-02	-1.783657E-02	-1.312242E-02	2.048155E-03	1.065513E+00
7.914864E-05	6.853201E-04	-1.321699E-03	7.958062E-02	-1.845045E-02	-1.410158E-02	2.618219E-03	1.043629E+00
7.914864E-05	6.346578E-04	-1.862870E-03	7.956119E-02	-1.910520E-02	-1.521379E-02	7.896617E-03	1.024024E+00
7.914864E-05	5.839956E-04	-2.307265E-03	7.954187E-02	-1.977782E-02	-1.651279E-02	9.423933E-03	1.001083E+00
7.914864E-05	5.333333E-04	-3.443800E-03	7.948861E-02	-2.045843E-02	-1.806422E-02	2.559928E-02	9.846400E-01
7.914864E-05	1.148620E-03	5.524574E-04	7.964767E-02	-1.394362E-02	-7.948593E-03	-1.910566E-05	1.207363E+00
7.914864E-05	1.155960E-03	6.498586E-04	7.964749E-02	-1.389111E-02	-7.878447E-03	-1.374581E-03	1.208710E+00
7.914864E-05	1.163300E-03	6.754925E-04	7.964785E-02	-1.383379E-02	-7.818581E-03	-1.894044E-03	1.210711E+00
7.914864E-05	1.170640E-03	7.050803E-04	7.964833E-02	-1.378838E-02	-7.738940E-03	-2.064244E-03	1.212825E+00
7.914864E-05	1.177980E-03	3.374154E-04	7.965031E-02	-1.371051E-02	-7.719700E-03	-2.605902E-03	1.216984E+00
7.914864E-05	1.185320E-03	4.259505E-04	7.965093E-02	-1.365620E-02	-7.658503E-03	-1.252347E-03	1.219208E+00
7.914864E-05	1.192660E-03	6.426641E-04	7.965117E-02	-1.360741E-02	-7.588025E-03	-5.726792E-04	1.220487E+00
7.914864E-05	1.200000E-03	6.477998E-04	7.965180E-02	-1.355195E-02	-7.531385E-03	-5.638193E-04	1.222729E+00
8.310608E-05	1.107803E-03	-5.713309E-04	7.764831E-02	-1.417137E-02	-8.449914E-03	1.167285E-03	1.198501E+00
8.310608E-05	1.055579E-03	-2.953764E-04	7.764333E-02	-1.460658E-02	-8.928201E-03	1.033648E-03	1.179428E+00
8.310608E-05	1.003354E-03	-1.349830E-03	7.763214E-02	-1.502740E-02	-9.515530E-03	3.680758E-03	1.168391E+00
8.310608E-05	9.511297E-04	-1.078144E-03	7.762860E-02	-1.550134E-02	-1.009528E-02	1.675672E-03	1.147445E+00
8.310608E-05	8.989051E-04	-6.927620E-04	7.762258E-02	-1.601709E-02	-1.070853E-02	2.835570E-03	1.126172E+00
8.310608E-05	8.466806E-04	-7.133408E-04	7.761475E-02	-1.653164E-02	-1.143460E-02	2.959623E-03	1.106048E+00
8.310608E-05	7.944561E-04	-8.625558E-04	7.760737E-02	-1.711269E-02	-1.218777E-02	-6.407900E-04	1.084578E+00
8.310608E-05	7.422315E-04	-5.015914E-04	7.759889E-02	-1.770246E-02	-1.306678E-02	-2.774701E-03	1.059590E+00
8.310608E-05	6.900070E-04	-1.377006E-03	7.758329E-02	-1.830455E-02	-1.410264E-02	2.666964E-03	1.042631E+00
8.310608E-05	6.377824E-04	-1.990390E-03	7.756652E-02	-1.890901E-02	-1.531583E-02	3.422566E-03	1.021253E+00
8.310608E-05	5.855579E-04	-2.637286E-03	7.753410E-02	-1.961244E-02	-1.662906E-02	1.340461E-02	1.001260E+00
8.310608E-05	5.333333E-04	-5.224786E-03	7.741687E-02	-2.026645E-02	-1.828343E-02	3.297022E-02	9.936929E-01
8.310608E-05	1.165024E-03	7.186859E-05	7.765451E-02	-1.374567E-02	-7.915681E-03	-2.641841E-05	1.212824E+00
8.310608E-05	1.170021E-03	3.803776E-04	7.765457E-02	-1.371543E-02	-7.859642E-03	2.902160E-05	1.212622E+00
8.310608E-05	1.175017E-03	5.629247E-04	7.765445E-02	-1.367773E-02	-7.817639E-03	-4.186170E-04	1.212975E+00
8.310608E-05	1.180014E-03	6.509462E-04	7.765481E-02	-1.364790E-02	-7.762903E-03	-7.667965E-05	1.214163E+00
8.310608E-05	1.185010E-03	6.264280E-04	7.765497E-02	-1.360838E-02	-7.725968E-03	-9.123807E-04	1.215597E+00
8.310608E-05	1.190007E-03	6.337308E-04	7.765526E-02	-1.357402E-02	-7.680490E-03	-1.258122E-03	1.217013E+00
8.310608E-05	1.195003E-03	8.316959E-04	7.765463E-02	-1.353613E-02	-7.641071E-03	-1.796938E-03	1.217226E+00
8.310608E-05	1.200000E-03	1.034891E-03	7.765410E-02	-1.351689E-02	-7.569076E-03	-1.701959E-03	1.217669E+00
8.726138E-05	1.125640E-03	5.241607E-04	7.570021E-02	-1.400022E-02	-8.312592E-03	-6.682501E-04	1.194669E+00
8.726138E-05	1.071794E-03	5.643175E-04	7.569422E-02	-1.443026E-02	-8.811900E-03	-7.553411E-04	1.176536E+00
8.726138E-05	1.017948E-03	1.662374E-04	7.568892E-02	-1.488045E-02	-9.360340E-03	-1.443478E-03	1.160204E+00
8.726138E-05	9.641018E-04	-1.172734E-03	7.567594E-02	-1.529493E-02	-1.006091E-02	9.467510E-03	1.152459E+00
8.726138E-05	9.102557E-04	-1.550398E-03	7.566521E-02	-1.578105E-02	-1.073693E-02	8.324402E-03	1.134322E+00
8.726138E-05	8.564097E-04	-1.964795E-03	7.565173E-02	-1.630602E-02	-1.146897E-02	9.235339E-03	1.116177E+00
8.726138E-05	8.025636E-04	-2.153117E-03	7.564077E-02	-1.685515E-02	-1.228886E-02	8.945119E-03	1.095271E+00
8.726138E-05	7.487176E-04	-2.833688E-03	7.561411E-02	-1.740501E-02	-1.325535E-02	1.442558E-02	1.078007E+00
8.726138E-05	6.948715E-04	-3.769000E-03	7.557350E-02	-1.805238E-02	-1.426215E-02	2.052174E-02	1.061224E+00
8.726138E-05	6.410254E-04	-4.127119E-03	7.555209E-02	-1.866253E-02	-1.549871E-02	2.018356E-02	1.037223E+00
8.726138E-05	5.871794E-04	-4.407384E-03	7.552906E-02	-1.936354E-02	-1.685228E-02	2.082611E-02	1.011246E+00
8.726138E-05	5.333333E-04	-5.052202E-03	7.548650E-02	-2.014382E-02	-1.839639E-02	2.389352E-02	9.860612E-01
8.726138E-05	1.182050E-03	2.284316E-04	7.570660E-02	-1.357431E-02	-7.826745E-03	-3.719221E-04	1.214704E+00
8.726138E-05	1.184615E-03	4.879447E-04	7.570582E-02	-1.354776E-02	-7.817204E-03	-1.886747E-03	1.213410E+00
8.726138E-05	1.187179E-03	5.296502E-04	7.570605E-02	-1.353370E-02	-7.787694E-03	-1.413768E-03	1.214135E+00
8.726138E-05	1.189743E-03	7.084311E-04	7.570555E-02	-1.351976E-02	-7.757378E-03	-1.540560E-03	1.213826E+00
8.726138E-05	1.192307E-03	8.713026E-04	7.570474E-02	-1.350754E-02	-7.724163E-03	-2.092515E-03	1.213481E+00

8.726138E-05	1.194872E-03	9.976353E-04	7.570396E-02	-1.348408E-02	-7.710941E-03	-2.634677E-03	1.213327E+00
8.726138E-05	1.197436E-03	8.987995E-04	7.570482E-02	-1.347687E-02	-7.670397E-03	-2.582104E-03	1.214784E+00
8.726138E-05	1.200000E-03	9.884855E-04	7.570378E-02	-1.345601E-02	-7.652782E-03	-3.946075E-03	1.214589E+00
9.162445E-05	1.144164E-03	-4.742024E-04	7.379838E-02	-1.376227E-02	-8.291423E-03	1.027675E-03	1.204725E+00
9.162445E-05	1.088634E-03	-2.853656E-04	7.379262E-02	-1.420877E-02	-8.772524E-03	1.665519E-03	1.185370E+00
9.162445E-05	1.033104E-03	-1.182398E-03	7.378126E-02	-1.464053E-02	-9.365589E-03	4.909082E-03	1.172960E+00
9.162445E-05	9.775741E-04	-2.381735E-03	7.375852E-02	-1.505690E-02	-1.007110E-02	8.365998E-03	1.161749E+00
9.162445E-05	9.220440E-04	-2.819666E-03	7.374122E-02	-1.555207E-02	-1.075062E-02	1.062799E-02	1.144593E+00
9.162445E-05	8.665139E-04	-2.989735E-03	7.373139E-02	-1.609757E-02	-1.146793E-02	9.737049E-03	1.123782E+00
9.162445E-05	8.109838E-04	-3.042042E-03	7.372081E-02	-1.665206E-02	-1.230079E-02	1.011011E-02	1.101566E+00
9.162445E-05	7.554537E-04	-3.743382E-03	7.368435E-02	-1.719951E-02	-1.329652E-02	1.716662E-02	1.084389E+00
9.162445E-05	6.999236E-04	-4.224199E-03	7.365497E-02	-1.782244E-02	-1.435905E-02	2.008726E-02	1.063084E+00
9.162445E-05	6.443935E-04	-5.962585E-03	7.352955E-02	-1.830546E-02	-1.580731E-02	4.308949E-02	1.054913E+00
9.162445E-05	5.888634E-04	-7.646893E-03	7.342031E-02	-1.909072E-02	-1.711779E-02	4.402851E-02	1.037379E+00
9.162445E-05	5.333333E-04	-7.676501E-03	7.343177E-02	-1.987902E-02	-1.869886E-02	3.506517E-02	1.003012E+00
9.162445E-05	1.199733E-03	1.943891E-05	7.380434E-02	-1.336336E-02	-7.800388E-03	-2.717530E-05	1.219119E+00
9.162445E-05	1.199771E-03	5.121238E-05	7.380433E-02	-1.336321E-02	-7.799741E-03	7.279749E-05	1.218963E+00
9.162445E-05	1.199809E-03	6.645985E-05	7.380435E-02	-1.336409E-02	-7.797467E-03	3.981471E-04	1.218990E+00
9.162445E-05	1.199847E-03	3.497000E-05	7.380435E-02	-1.336219E-02	-7.800031E-03	1.807820E-04	1.219125E+00
9.162445E-05	1.199885E-03	5.853479E-05	7.380436E-02	-1.336085E-02	-7.801502E-03	4.461442E-04	1.219073E+00
9.162445E-05	1.199924E-03	3.837796E-05	7.380439E-02	-1.336168E-02	-7.799418E-03	5.638869E-04	1.219255E+00
9.162445E-05	1.199962E-03	6.480850E-05	7.380438E-02	-1.336182E-02	-7.798260E-03	5.439145E-04	1.219094E+00
9.162445E-05	1.200000E-03	3.470057E-05	7.380439E-02	-1.335844E-02	-7.803412E-03	5.569320E-04	1.219293E+00
9.620567E-05	1.163415E-03	-1.398697E-03	7.192907E-02	-1.350103E-02	-8.303365E-03	1.353830E-02	1.218647E+00
9.620567E-05	1.106134E-03	-1.602266E-03	7.192041E-02	-1.393642E-02	-8.812342E-03	1.257253E-02	1.200666E+00
9.620567E-05	1.048854E-03	-2.144295E-03	7.190427E-02	-1.439167E-02	-9.375205E-03	1.404219E-02	1.185066E+00
9.620567E-05	9.915742E-04	-2.055089E-03	7.189967E-02	-1.486882E-02	-9.990403E-03	1.267743E-02	1.163561E+00
9.620567E-05	9.342941E-04	-2.277594E-03	7.188771E-02	-1.537547E-02	-1.066712E-02	1.315346E-02	1.143887E+00
9.620567E-05	8.770140E-04	-2.668097E-03	7.187068E-02	-1.588935E-02	-1.144996E-02	1.438878E-02	1.124559E+00
9.620567E-05	8.197339E-04	-3.181520E-03	7.184773E-02	-1.644211E-02	-1.231078E-02	1.647583E-02	1.105213E+00
9.620567E-05	7.624538E-04	-3.835464E-03	7.181562E-02	-1.702111E-02	-1.328874E-02	1.984897E-02	1.085924E+00
9.620567E-05	7.051737E-04	-4.948521E-03	7.174217E-02	-1.759296E-02	-1.445720E-02	3.376449E-02	1.071739E+00
9.620567E-05	6.478936E-04	-7.512016E-03	7.163398E-02	-1.822692E-02	-1.575229E-02	2.705105E-02	1.058719E+00
9.620567E-05	5.906134E-04	-7.545979E-03	7.156937E-02	-1.891873E-02	-1.721816E-02	4.079882E-02	1.033566E+00
9.620567E-05	5.333333E-04	-9.629364E-03	7.131086E-02	-1.958104E-02	-1.903666E-02	7.055039E-02	1.026107E+00
9.620567E-05	1.218108E-03	1.080397E-04	7.194563E-02	-1.317055E-02	-7.733541E-03	-1.423275E-03	1.221325E+00
9.620567E-05	1.215521E-03	2.931517E-05	7.194552E-02	-1.318736E-02	-7.757094E-03	-1.070478E-03	1.221140E+00
9.620567E-05	1.212934E-03	-6.608457E-06	7.194531E-02	-1.320977E-02	-7.771042E-03	-9.190741E-04	1.220622E+00
9.620567E-05	1.210347E-03	-4.526172E-05	7.194510E-02	-1.322316E-02	-7.800376E-03	-9.840693E-04	1.220020E+00
9.620567E-05	1.207761E-03	-2.013453E-05	7.194481E-02	-1.324399E-02	-7.817056E-03	-6.143137E-04	1.219167E+00
9.620567E-05	1.205174E-03	1.379248E-05	7.194452E-02	-1.326079E-02	-7.840540E-03	-6.584555E-04	1.218103E+00
9.620567E-05	1.202587E-03	-3.551839E-05	7.194431E-02	-1.327708E-02	-7.865419E-03	-6.917020E-04	1.217587E+00
9.620567E-05	1.200000E-03	-1.241051E-04	7.194409E-02	-1.329763E-02	-7.883575E-03	-3.029439E-04	1.217482E+00
1.010160E-04	1.183433E-03	-5.681516E-04	7.012409E-02	-1.333534E-02	-8.134683E-03	9.287581E-04	1.212913E+00
1.010160E-04	1.124333E-03	-5.953848E-04	7.011722E-02	-1.377271E-02	-8.651108E-03	1.995621E-03	1.193972E+00
1.010160E-04	1.065233E-03	-6.950912E-05	7.011104E-02	-1.426101E-02	-9.167400E-03	8.833230E-04	1.169980E+00
1.010160E-04	1.006133E-03	-1.740743E-03	7.008744E-02	-1.467003E-02	-9.917060E-03	1.348953E-02	1.164293E+00
1.010160E-04	9.470328E-04	-2.266585E-03	7.006915E-02	-1.515818E-02	-1.063827E-02	1.506652E-02	1.146426E+00
1.010160E-04	8.879329E-04	-2.692333E-03	7.005080E-02	-1.569125E-02	-1.141191E-02	1.599740E-02	1.126660E+00
1.010160E-04	8.288330E-04	-3.199054E-03	7.002743E-02	-1.626597E-02	-1.226505E-02	1.765415E-02	1.106520E+00
1.010160E-04	7.697331E-04	-3.621631E-03	7.000167E-02	-1.684491E-02	-1.326744E-02	2.031019E-02	1.084756E+00
1.010160E-04	7.106331E-04	-4.811115E-03	6.992642E-02	-1.743661E-02	-1.444483E-02	3.429878E-02	1.070497E+00
1.010160E-04	6.515332E-04	-5.684802E-03	6.985726E-02	-1.807735E-02	-1.576800E-02	4.279570E-02	1.050437E+00
1.010160E-04	5.924333E-04	-7.840068E-03	6.973342E-02	-1.874876E-02	-1.731648E-02	3.933171E-02	1.033088E+00
1.010160E-04	5.333333E-04	-9.819657E-03	6.954688E-02	-1.945844E-02	-1.913877E-02	4.951008E-02	1.016963E+00
1.010160E-04	1.237216E-03	2.426151E-05	7.013018E-02	-1.296846E-02	-7.676349E-03	3.535912E-04	1.225902E+00
1.010160E-04	1.231899E-03	3.142201E-05	7.012962E-02	-1.300750E-02	-7.713908E-03	-1.824592E-04	1.224002E+00
1.010160E-04	1.226583E-03	-1.270663E-05	7.012912E-02	-1.304357E-02	-7.757306E-03	-5.696956E-04	1.222488E+00
1.010160E-04	1.221266E-03	-4.455935E-05	7.012862E-02	-1.308199E-02	-7.797352E-03	-6.794546E-04	1.220986E+00
1.010160E-04	1.215950E-03	-4.111939E-05	7.012807E-02	-1.311910E-02	-7.840063E-03	-4.913844E-04	1.219336E+00
1.010160E-04	1.210633E-03	-4.752799E-06	7.012750E-02	-1.315738E-02	-7.881232E-03	-3.126928E-04	1.217458E+00
1.010160E-04	1.205317E-03	2.635302E-05	7.012692E-02	-1.319751E-02	-7.919851E-03	-4.937914E-04	1.215494E+00
1.010160E-04	1.200000E-03	-2.196477E-05	7.012642E-02	-1.323333E-02	-7.966615E-03	-7.309641E-04	1.214032E+00
1.060668E-04	1.204265E-03	2.080509E-04	6.835059E-02	-1.314003E-02	-8.010524E-03	-3.496865E-04	1.211223E+00
1.060668E-04	1.143271E-03	-1.118663E-03	6.834133E-02	-1.353791E-02	-8.607447E-03	1.518591E-03	1.201014E+00
1.060668E-04	1.082277E-03	-3.695328E-04	6.833710E-02	-1.404033E-02	-9.110262E-03	-1.676427E-03	1.174030E+00
1.060668E-04	1.021284E-03	-1.007732E-04	6.832849E-02	-1.454600E-02	-9.710226E-03	-5.730200E-04	1.151047E+00
1.060668E-04	9.602898E-04	-9.371527E-04	6.831685E-02	-1.502261E-02	-1.046958E-02	3.582787E-03	1.135586E+00
1.060668E-04	8.992960E-04	-1.567690E-03	6.830148E-02	-1.556190E-02	-1.125714E-02	5.413056E-03	1.116924E+00
1.060668E-04	8.383022E-04	-1.681784E-03	6.829033E-02	-1.615011E-02	-1.211198E-02	4.459817E-03	1.092587E+00
1.060668E-04	7.773084E-04	-2.162448E-03	6.827120E-02	-1.675118E-02	-1.311599E-02	6.411171E-03	1.070247E+00
1.060668E-04	7.163147E-04	-2.359243E-03	6.825306E-02	-1.739104E-02	-1.425573E-02	7.889019E-03	1.044138E+00
1.060668E-04	6.553209E-04	-2.684557E-03	6.823063E-02	-1.808829E-02	-1.555091E-02	9.235998E-03	1.016859E+00
1.060668E-04	5.943271E-04	-3.532387E-03	6.818457E-02	-1.880258E-02	-1.710491E-02	1.672474E-02	9.926891E-01
1.060668E-04	5.333333E-04	-4.647296E-03	6.810910E-02	-1.953542E-02	-1.897336E-02	2.883580E-02	9.689556E-01
1.060668E-04	1.257101E-03	1.100202E-03	6.835220E-02	-1.279811E-02	-7.551320E-03	-1.215034E-03	1.221561E+00
1.060668E-04	1.248944E-03	8.240678E-04	6.835237E-02	-1.284778E-02	-7.624357E-03	-2.185607E-03	1.220579E+00
1.060668E-04	1.240787E-03	5.587359E-04	6.835274E-02	-1.290551E-02	-7.685212E-03	-2.633848E-03	1.219714E+00
1.060668E-04	1.232629E-03	4.812297E-04	6.835205E-02	-1.295463E-02	-7.760704E-03	-3.532119E-03	1.217344E+00
1.060668E-04	1.224472E-03	4.479855E-04	6.835137E-02	-1.301050E-02	-7.826233E-03	-3.525853E-03	1.214982E+00
1.060668E-04	1.216315E-03	4.300579E-04	6.835081E-02	-1.306671E-02	-7.892708E-03	-2.406016E-03	1.212880E+00
1.060668E-04	1.208157E-03	2.599506E-04	6.835051E-02	-1.312004E-02	-7.965523E-03	-3.589010E-03	1.211035E+00

1.060668E-04	1.200000E-03	6.181345E-05	6.835033E-02	-1.317046E-02	-8.045199E-03	-2.354585E-03	1.210187E+00
1.113701E-04	1.225963E-03	3.844606E-05	6.661796E-02	-1.291079E-02	-7.939923E-03	8.516337E-04	1.217012E+00
1.113701E-04	1.162996E-03	3.440147E-04	6.661062E-02	-1.337376E-02	-8.432147E-03	1.235574E-03	1.194399E+00
1.113701E-04	1.100030E-03	-1.356414E-03	6.659271E-02	-1.377033E-02	-9.130343E-03	1.265054E-02	1.189062E+00
1.113701E-04	1.037064E-03	-2.472586E-03	6.656377E-02	-1.423773E-02	-9.808518E-03	1.539252E-02	1.175814E+00
1.113701E-04	9.740974E-04	-2.860231E-03	6.654401E-02	-1.473047E-02	-1.055128E-02	1.764788E-02	1.156065E+00
1.113701E-04	9.111311E-04	-3.175974E-03	6.652453E-02	-1.526679E-02	-1.135474E-02	1.919164E-02	1.134443E+00
1.113701E-04	8.481648E-04	-3.699251E-03	6.649445E-02	-1.582682E-02	-1.226993E-02	2.248731E-02	1.113640E+00
1.113701E-04	7.851985E-04	-4.862992E-03	6.643556E-02	-1.638900E-02	-1.335066E-02	2.654983E-02	1.096294E+00
1.113701E-04	7.222322E-04	-5.924690E-03	6.635071E-02	-1.697920E-02	-1.458748E-02	3.774292E-02	1.079022E+00
1.113701E-04	6.592659E-04	-6.604006E-03	6.629369E-02	-1.764567E-02	-1.595262E-02	4.020668E-02	1.053746E+00
1.113701E-04	5.962996E-04	-8.091338E-03	6.612275E-02	-1.837375E-02	-1.754549E-02	6.129822E-02	1.038957E+00
1.113701E-04	5.333333E-04	-1.033843E-02	6.590449E-02	-1.910229E-02	-1.948217E-02	6.559088E-02	1.021223E+00
1.113701E-04	1.277813E-03	1.575892E-05	6.662369E-02	-1.255498E-02	-7.552488E-03	3.075854E-03	1.234402E+00
1.113701E-04	1.266697E-03	-6.465674E-05	6.662237E-02	-1.262658E-02	-7.639463E-03	2.568577E-03	1.231311E+00
1.113701E-04	1.255581E-03	-6.089432E-04	6.661995E-02	-1.268454E-02	-7.753603E-03	2.191446E-03	1.231610E+00
1.113701E-04	1.244464E-03	-6.998168E-04	6.661813E-02	-1.276097E-02	-7.837613E-03	3.207198E-03	1.229097E+00
1.113701E-04	1.233348E-03	-7.045214E-04	6.661686E-02	-1.284200E-02	-7.915915E-03	3.477775E-03	1.225679E+00
1.113701E-04	1.222232E-03	-5.640454E-04	6.661626E-02	-1.291241E-02	-8.012841E-03	3.646618E-03	1.221105E+00
1.113701E-04	1.211116E-03	-2.753273E-04	6.661610E-02	-1.300246E-02	-8.079692E-03	3.689760E-03	1.215447E+00
1.113701E-04	1.200000E-03	-5.116918E-04	6.661411E-02	-1.307432E-02	-8.181278E-03	3.288231E-03	1.213394E+00
1.169386E-04	1.248582E-03	-2.217108E-04	6.492476E-02	-1.268467E-02	-7.856127E-03	2.240284E-04	1.223221E+00
1.169386E-04	1.183559E-03	-1.427755E-03	6.490863E-02	-1.312941E-02	-8.399359E-03	6.804835E-03	1.213401E+00
1.169386E-04	1.118537E-03	-1.863973E-03	6.489567E-02	-1.354628E-02	-9.062228E-03	7.125060E-03	1.194589E+00
1.169386E-04	1.053514E-03	-1.970698E-03	6.488391E-02	-1.403834E-02	-9.708177E-03	9.013944E-03	1.173050E+00
1.169386E-04	9.884916E-04	-2.348234E-03	6.486505E-02	-1.452347E-02	-1.047691E-02	1.253158E-02	1.153037E+00
1.169386E-04	9.234690E-04	-2.998715E-03	6.484065E-02	-1.507698E-02	-1.127710E-02	1.357038E-02	1.133103E+00
1.169386E-04	8.584464E-04	-3.755193E-03	6.479751E-02	-1.562735E-02	-1.223084E-02	2.168150E-02	1.115026E+00
1.169386E-04	7.934238E-04	-4.510879E-03	6.474942E-02	-1.620654E-02	-1.331319E-02	2.740611E-02	1.094540E+00
1.169386E-04	7.284012E-04	-6.067483E-03	6.462384E-02	-1.680708E-02	-1.458002E-02	4.675515E-02	1.083324E+00
1.169386E-04	6.633786E-04	-1.005672E-02	6.438888E-02	-1.733008E-02	-1.617743E-02	3.800008E-02	1.079536E+00
1.169386E-04	5.983559E-04	-1.026455E-02	6.434457E-02	-1.816266E-02	-1.768363E-02	4.024265E-02	1.046820E+00
1.169386E-04	5.333333E-04	-1.085609E-02	6.417071E-02	-1.903242E-02	-1.954240E-02	6.122718E-02	1.020996E+00
1.169386E-04	1.299404E-03	-2.337718E-04	6.493006E-02	-1.234232E-02	-7.488194E-03	5.626843E-04	1.239528E+00
1.169386E-04	1.285204E-03	-3.857297E-04	6.492825E-02	-1.242435E-02	-7.609720E-03	1.609338E-03	1.236556E+00
1.169386E-04	1.271003E-03	-1.214367E-04	6.492716E-02	-1.253068E-02	-7.692499E-03	1.450293E-03	1.230035E+00
1.169386E-04	1.256802E-03	-3.214024E-04	6.492544E-02	-1.261997E-02	-7.809295E-03	1.217843E-03	1.226930E+00
1.169386E-04	1.242602E-03	-6.156718E-04	6.492312E-02	-1.270220E-02	-7.941815E-03	1.686303E-03	1.224729E+00
1.169386E-04	1.228401E-03	-7.924402E-04	6.492004E-02	-1.279394E-02	-8.062617E-03	4.020790E-03	1.222284E+00
1.169386E-04	1.214201E-03	-1.304461E-03	6.491476E-02	-1.287327E-02	-8.208400E-03	4.518009E-03	1.221678E+00
1.169386E-04	1.200000E-03	-1.258368E-03	6.491351E-02	-1.297554E-02	-8.318301E-03	4.534930E-03	1.216667E+00
1.227855E-04	1.272187E-03	1.126848E-04	6.327002E-02	-1.246567E-02	-7.748248E-03	-3.266946E-03	1.224210E+00
1.227855E-04	1.205018E-03	-4.567263E-04	6.326222E-02	-1.289659E-02	-8.314966E-03	1.188644E-03	1.208300E+00
1.227855E-04	1.137850E-03	-8.106469E-04	6.325248E-02	-1.336083E-02	-8.916641E-03	2.088938E-03	1.188645E+00
1.227855E-04	1.070681E-03	-3.117360E-03	6.320375E-02	-1.376474E-02	-9.723032E-03	1.237643E-02	1.186504E+00
1.227855E-04	1.003513E-03	-3.225590E-03	6.318780E-02	-1.430164E-02	-1.042922E-02	1.478463E-02	1.163392E+00
1.227855E-04	9.363443E-04	-3.381902E-03	6.316976E-02	-1.486217E-02	-1.123375E-02	1.694283E-02	1.139329E+00
1.227855E-04	8.691758E-04	-3.782747E-03	6.314120E-02	-1.544722E-02	-1.215862E-02	2.025802E-02	1.116156E+00
1.227855E-04	8.020073E-04	-4.537486E-03	6.309768E-02	-1.603813E-02	-1.325239E-02	2.326285E-02	1.093947E+00
1.227855E-04	7.348388E-04	-5.043524E-03	6.305597E-02	-1.667414E-02	-1.449255E-02	2.669646E-02	1.067942E+00
1.227855E-04	6.676703E-04	-6.481577E-03	6.291091E-02	-1.732206E-02	-1.598479E-02	5.112990E-02	1.054637E+00
1.227855E-04	6.005018E-04	-7.650725E-03	6.276475E-02	-1.796697E-02	-1.776985E-02	7.060690E-02	1.034777E+00
1.227855E-04	5.333333E-04	-1.116596E-02	6.246214E-02	-1.875061E-02	-1.980320E-02	6.351370E-02	1.021699E+00
1.227855E-04	1.321936E-03	2.292017E-04	6.327564E-02	-1.213657E-02	-7.400809E-03	1.455109E-03	1.240657E+00
1.227855E-04	1.304516E-03	-1.475494E-04	6.327378E-02	-1.223575E-02	-7.545916E-03	1.274257E-03	1.238059E+00
1.227855E-04	1.287097E-03	-5.221334E-05	6.327195E-02	-1.235667E-02	-7.657922E-03	-1.122527E-04	1.231330E+00
1.227855E-04	1.269678E-03	-7.092586E-05	6.327006E-02	-1.246584E-02	-7.795184E-03	-3.116592E-04	1.225834E+00
1.227855E-04	1.252258E-03	-1.610216E-04	6.326809E-02	-1.258070E-02	-7.929596E-03	1.309574E-03	1.221507E+00
1.227855E-04	1.234839E-03	-8.291839E-05	6.326614E-02	-1.270825E-02	-8.047635E-03	-3.842782E-04	1.214679E+00
1.227855E-04	1.217419E-03	-4.087220E-04	6.326422E-02	-1.282637E-02	-8.189502E-03	-6.940209E-04	1.211403E+00
1.227855E-04	1.200000E-03	-4.056944E-04	6.326212E-02	-1.293914E-02	-8.343423E-03	-6.184355E-04	1.205624E+00
1.289248E-04	1.296848E-03	3.878542E-04	6.165220E-02	-1.226263E-02	-7.605395E-03	-1.102535E-02	1.224327E+00
1.289248E-04	1.227437E-03	-8.889428E-05	6.164873E-02	-1.270417E-02	-8.162800E-03	-1.139637E-02	1.205508E+00
1.289248E-04	1.158027E-03	-1.992557E-04	6.164061E-02	-1.318256E-02	-8.752142E-03	-1.122290E-02	1.183133E+00
1.289248E-04	1.088616E-03	6.762200E-04	6.162158E-02	-1.373239E-02	-9.330521E-03	-1.429976E-02	1.151136E+00
1.289248E-04	1.019206E-03	9.831540E-04	6.160540E-02	-1.430050E-02	-1.000770E-02	-1.512966E-02	1.123349E+00
1.289248E-04	9.497957E-04	4.016731E-04	6.160189E-02	-1.485329E-02	-1.085066E-02	-1.536712E-02	1.101131E+00
1.289248E-04	8.803853E-04	1.389269E-04	6.159113E-02	-1.546082E-02	-1.177237E-02	-1.498790E-02	1.075227E+00
1.289248E-04	8.109749E-04	3.422226E-04	6.157226E-02	-1.614264E-02	-1.277668E-02	-1.618143E-02	1.043630E+00
1.289248E-04	7.415645E-04	-1.823897E-04	6.155635E-02	-1.680058E-02	-1.404151E-02	-7.721172E-03	1.018322E+00
1.289248E-04	6.721541E-04	-9.813272E-04	6.153708E-02	-1.749524E-02	-1.552153E-02	-5.444959E-03	9.904244E-01
1.289248E-04	6.027437E-04	-2.694553E-03	6.148269E-02	-1.817523E-02	-1.735140E-02	1.098880E-02	9.708341E-01
1.289248E-04	5.333333E-04	-3.376030E-03	6.143259E-02	-1.902509E-02	-1.938525E-02	1.547668E-02	9.354856E-01
1.289248E-04	1.345476E-03	3.713983E-04	6.165914E-02	-1.192040E-02	-7.323184E-03	2.396734E-03	1.244437E+00
1.289248E-04	1.324694E-03	-6.646384E-05	6.165686E-02	-1.203337E-02	-7.496619E-03	2.607213E-03	1.241526E+00
1.289248E-04	1.303911E-03	5.435369E-05	6.165477E-02	-1.217581E-02	-7.626405E-03	2.777576E-03	1.234056E+00
1.289248E-04	1.283129E-03	-2.768098E-04	6.165192E-02	-1.229122E-02	-7.809256E-03	3.136854E-03	1.230182E+00
1.289248E-04	1.262347E-03	-5.789187E-04	6.164828E-02	-1.242125E-02	-7.976394E-03	4.659619E-03	1.226461E+00
1.289248E-04	1.241565E-03	-7.291537E-04	6.164507E-02	-1.254995E-02	-8.151853E-03	4.636860E-03	1.220858E+00
1.289248E-04	1.220782E-03	-1.035905E-03	6.163983E-02	-1.268120E-02	-8.332802E-03	6.512658E-03	1.217142E+00
1.289248E-04	1.200000E-03	-9.929786E-04	6.163755E-02	-1.283536E-02	-8.484305E-03	6.804556E-03	1.209977E+00
1.353710E-04	1.322645E-03	-5.044507E-05	6.007414E-02	-1.197652E-02	-7.595869E-03	2.984157E-03	1.238115E+00

1.353710E-04	1.250890E-03	-2.576740E-04	6.006539E-02	-1.243639E-02	-8.129518E-03	2.860163E-03	1.216571E+00
1.353710E-04	1.179134E-03	-8.551926E-04	6.005373E-02	-1.290014E-02	-8.752887E-03	3.883946E-03	1.197764E+00
1.353710E-04	1.107378E-03	-4.816886E-04	6.004532E-02	-1.343222E-02	-9.372316E-03	3.591214E-03	1.169492E+00
1.353710E-04	1.035623E-03	-8.935258E-04	6.003167E-02	-1.395713E-02	-1.012900E-02	5.158514E-03	1.147147E+00
1.353710E-04	9.638672E-04	-2.323475E-03	5.999903E-02	-1.446982E-02	-1.104770E-02	1.059346E-02	1.133109E+00
1.353710E-04	8.921115E-04	-3.028845E-03	5.996959E-02	-1.506896E-02	-1.200046E-02	1.241791E-02	1.110316E+00
1.353710E-04	8.203559E-04	-3.279427E-03	5.994517E-02	-1.569196E-02	-1.310281E-02	1.442244E-02	1.081905E+00
1.353710E-04	7.486003E-04	-4.358242E-03	5.986505E-02	-1.629819E-02	-1.445913E-02	3.183915E-02	1.063606E+00
1.353710E-04	6.768446E-04	-5.766609E-03	5.975254E-02	-1.694389E-02	-1.603269E-02	4.521318E-02	1.044150E+00
1.353710E-04	6.050890E-04	-7.566181E-03	5.957081E-02	-1.765734E-02	-1.786616E-02	6.232168E-02	1.026541E+00
1.353710E-04	5.333333E-04	-1.183195E-02	5.924788E-02	-1.835619E-02	-2.014364E-02	5.012657E-02	1.016743E+00
1.353710E-04	1.370101E-03	1.389413E-03	6.006599E-02	-1.175291E-02	-7.149251E-03	-8.937805E-03	1.236845E+00
1.353710E-04	1.345801E-03	9.725585E-04	6.006832E-02	-1.188451E-02	-7.342963E-03	-9.850784E-03	1.232368E+00
1.353710E-04	1.321501E-03	5.251370E-04	6.007068E-02	-1.202248E-02	-7.535799E-03	-9.151975E-03	1.228652E+00
1.353710E-04	1.297201E-03	2.925646E-05	6.007226E-02	-1.216587E-02	-7.729555E-03	-9.308054E-03	1.224954E+00
1.353710E-04	1.272900E-03	4.077646E-04	6.006607E-02	-1.233490E-02	-7.887631E-03	-8.772652E-03	1.214180E+00
1.353710E-04	1.248600E-03	6.263631E-04	6.006082E-02	-1.251443E-02	-8.040423E-03	-1.034975E-02	1.203946E+00
1.353710E-04	1.224300E-03	1.202266E-04	6.006250E-02	-1.264998E-02	-8.277754E-03	-8.941628E-03	1.200504E+00
1.353710E-04	1.200000E-03	9.021416E-04	6.005161E-02	-1.284689E-02	-8.423048E-03	-8.635404E-03	1.186162E+00
1.421396E-04	1.349671E-03	-7.564899E-05	5.853022E-02	-1.174172E-02	-7.487364E-03	-1.792100E-03	1.242373E+00
1.421396E-04	1.275459E-03	2.631978E-04	5.852051E-02	-1.222485E-02	-7.989636E-03	-1.679419E-03	1.215570E+00
1.421396E-04	1.201246E-03	-1.455449E-04	5.851172E-02	-1.270152E-02	-8.602971E-03	1.026528E-03	1.195206E+00
1.421396E-04	1.127034E-03	-1.482937E-03	5.848858E-02	-1.315502E-02	-9.365693E-03	1.281961E-02	1.184951E+00
1.421396E-04	1.052821E-03	-1.717549E-03	5.847281E-02	-1.368320E-02	-1.012793E-02	1.393950E-02	1.160136E+00
1.421396E-04	9.786085E-04	-2.711635E-03	5.843860E-02	-1.424189E-02	-1.099571E-02	1.623321E-02	1.140907E+00
1.421396E-04	9.043960E-04	-2.855990E-03	5.842012E-02	-1.483641E-02	-1.196770E-02	1.679107E-02	1.111999E+00
1.421396E-04	8.301835E-04	-3.386550E-03	5.838386E-02	-1.547644E-02	-1.307963E-02	2.122981E-02	1.085856E+00
1.421396E-04	7.559709E-04	-3.977308E-03	5.834522E-02	-1.616189E-02	-1.436491E-02	2.221068E-02	1.056582E+00
1.421396E-04	6.817584E-04	-4.404917E-03	5.830763E-02	-1.685571E-02	-1.591350E-02	2.268470E-02	1.022727E+00
1.421396E-04	6.075459E-04	-4.859876E-03	5.825171E-02	-1.767028E-02	-1.768401E-02	2.846215E-02	9.873081E-01
1.421396E-04	5.333333E-04	-5.798304E-03	5.817246E-02	-1.843322E-02	-1.995241E-02	3.095816E-02	9.499836E-01
1.421396E-04	1.395898E-03	-1.225968E-04	5.853524E-02	-1.145720E-02	-7.191726E-03	-7.924798E-04	1.257675E+00
1.421396E-04	1.367913E-03	-2.171559E-04	5.853229E-02	-1.162619E-02	-7.372945E-03	-8.796603E-04	1.249717E+00
1.421396E-04	1.339927E-03	-3.563750E-05	5.852899E-02	-1.180020E-02	-7.556286E-03	-6.771341E-04	1.239325E+00
1.421396E-04	1.311942E-03	-5.579700E-05	5.852578E-02	-1.197662E-02	-7.749273E-03	-8.831948E-04	1.230455E+00
1.421396E-04	1.283956E-03	-1.727169E-04	5.852242E-02	-1.215059E-02	-7.959638E-03	-3.274511E-04	1.222571E+00
1.421396E-04	1.255971E-03	-2.130647E-04	5.851889E-02	-1.232951E-02	-8.175390E-03	1.007204E-03	1.214178E+00
1.421396E-04	1.227985E-03	-1.238101E-04	5.851532E-02	-1.251637E-02	-8.391896E-03	1.197604E-03	1.204127E+00
1.421396E-04	1.200000E-03	-2.838862E-04	5.851155E-02	-1.270337E-02	-8.624058E-03	4.845800E-04	1.195758E+00
1.492466E-04	1.378031E-03	-1.963609E-05	5.702130E-02	-1.149111E-02	-7.394635E-03	2.300482E-04	1.248761E+00
1.492466E-04	1.301241E-03	-4.457480E-04	5.701156E-02	-1.194170E-02	-7.959763E-03	2.924826E-03	1.228825E+00
1.492466E-04	1.224450E-03	-7.088874E-04	5.700002E-02	-1.242953E-02	-8.563044E-03	5.108507E-03	1.206280E+00
1.492466E-04	1.147659E-03	-2.195996E-03	5.696382E-02	-1.287682E-02	-9.344286E-03	1.587352E-02	1.196757E+00
1.492466E-04	1.070868E-03	-3.311002E-03	5.691373E-02	-1.336522E-02	-1.018286E-02	2.447532E-02	1.181880E+00
1.492466E-04	9.940776E-04	-4.470572E-03	5.683297E-02	-1.385729E-02	-1.115576E-02	4.005069E-02	1.168643E+00
1.492466E-04	9.172869E-04	-5.291893E-03	5.676235E-02	-1.446538E-02	-1.213691E-02	4.808753E-02	1.147826E+00
1.492466E-04	8.404962E-04	-6.224866E-03	5.666826E-02	-1.507720E-02	-1.330967E-02	5.829538E-02	1.126850E+00
1.492466E-04	7.637055E-04	-8.051175E-03	5.650110E-02	-1.570100E-02	-1.470648E-02	6.278761E-02	1.109936E+00
1.492466E-04	6.869148E-04	-9.025460E-03	5.637807E-02	-1.644454E-02	-1.624774E-02	6.850316E-02	1.082323E+00
1.492466E-04	6.101241E-04	-1.113037E-02	5.605244E-02	-1.714339E-02	-1.821592E-02	9.077839E-02	1.069447E+00
1.492466E-04	5.333333E-04	-1.308633E-02	5.565998E-02	-1.790151E-02	-2.058474E-02	1.164768E-01	1.052811E+00
1.492466E-04	1.422969E-03	2.648340E-04	5.702591E-02	-1.123357E-02	-7.094620E-03	4.072013E-04	1.260338E+00
1.492466E-04	1.391116E-03	-3.232164E-04	5.702273E-02	-1.140555E-02	-7.323331E-03	2.897130E-04	1.255693E+00
1.492466E-04	1.359264E-03	-5.477882E-04	5.701837E-02	-1.158181E-02	-7.557551E-03	1.815832E-03	1.248161E+00
1.492466E-04	1.327411E-03	-4.388553E-04	5.701502E-02	-1.178199E-02	-7.767225E-03	1.443789E-03	1.236768E+00
1.492466E-04	1.295558E-03	-9.172078E-05	5.701152E-02	-1.199742E-02	-7.967976E-03	6.113707E-04	1.222947E+00
1.492466E-04	1.263705E-03	-3.861055E-05	5.700738E-02	-1.220656E-02	-8.197307E-03	-3.542451E-05	1.211698E+00
1.492466E-04	1.231853E-03	-5.556155E-05	5.700312E-02	-1.241455E-02	-8.446759E-03	-1.726161E-05	1.201143E+00
1.492466E-04	1.200000E-03	1.715216E-05	5.699862E-02	-1.263060E-02	-8.702508E-03	8.342858E-04	1.189924E+00

Table 32. *f*17p2 low energy transfers

Inner Radius	Outer Radius	Delta V1x	Delta V1y	Delta V2x	Delta V2y	Initial *f*	Time of flight
5.101992E-05	9.080334E-04	6.154258E-03	9.972626E-02	-1.585806E-02	1.103931E-02	-3.561160E-02	2.049182E+00
5.101992E-05	8.330934E-04	8.278767E-03	9.957711E-02	-1.569329E-02	1.336649E-02	-6.289949E-02	2.127913E+00
5.101992E-05	7.581534E-04	1.110114E-02	9.930623E-02	-1.601967E-02	1.512630E-02	-9.373921E-02	2.157412E+00
5.101992E-05	6.832134E-04	1.189771E-02	9.921395E-02	-1.623654E-02	1.720475E-02	-1.274406E-01	2.210048E+00
5.101992E-05	6.082733E-04	1.253004E-02	9.916973E-02	-1.665741E-02	1.890255E-02	-1.539081E-01	2.262380E+00
5.101992E-05	5.333333E-04	1.295808E-02	9.913864E-02	-1.669597E-02	2.218898E-02	-1.648993E-01	2.347658E+00
5.101992E-05	1.037230E-03	-2.941792E-03	9.986573E-02	-2.002215E-02	5.206595E-03	9.618118E-04	2.014279E+00
5.101992E-05	1.091487E-03	-5.608073E-03	9.983086E-02	-2.216840E-02	4.353726E-03	4.143383E-04	2.007035E+00
5.101992E-05	1.145743E-03	-7.280330E-03	9.980975E-02	-2.398397E-02	4.350370E-03	-1.789069E-03	2.003991E+00
5.101992E-05	1.200000E-03	-7.949515E-03	9.980408E-02	-2.551509E-02	4.630457E-03	-3.545316E-03	1.992453E+00

5.357092E-05	9.193431E-04	8.064013E-03	9.715449E-02	-1.608375E-02	1.031073E-02	-3.710554E-02	1.994282E+00
5.357092E-05	8.421411E-04	1.216447E-02	9.669246E-02	-1.655816E-02	1.188095E-02	-9.456008E-02	1.992533E+00
5.357092E-05	7.649392E-04	1.352141E-02	9.653249E-02	-1.637264E-02	1.449959E-02	-1.079653E-01	2.093493E+00
5.357092E-05	6.877372E-04	1.466049E-02	9.635766E-02	-1.657931E-02	1.670573E-02	-1.308161E-01	2.158245E+00
5.357092E-05	6.105353E-04	1.571146E-02	9.618913E-02	-1.683412E-02	1.910510E-02	-1.537767E-01	2.214402E+00
5.357092E-05	5.333333E-04	1.724243E-02	9.593414E-02	-1.722151E-02	2.176405E-02	-1.701911E-01	2.264639E+00
5.357092E-05	1.047409E-03	-2.710471E-03	9.739845E-02	-1.956917E-02	5.158183E-03	1.514996E-03	2.011861E+00
5.357092E-05	1.098273E-03	-4.897743E-03	9.737288E-02	-2.172996E-02	4.375906E-03	6.841927E-04	2.008176E+00
5.357092E-05	1.149136E-03	-6.361594E-03	9.734566E-02	-2.344443E-02	4.351490E-03	1.039568E-03	2.007656E+00
5.357092E-05	1.200000E-03	-7.790494E-03	9.731706E-02	-2.474144E-02	4.600558E-03	4.791065E-04	2.006301E+00
5.624946E-05	9.310397E-04	4.676828E-03	9.494862E-02	-1.493832E-02	1.175697E-02	-1.602987E-02	2.131692E+00
5.624946E-05	8.514984E-04	7.684367E-03	9.478870E-02	-1.484879E-02	1.387174E-02	-3.814437E-02	2.194764E+00
5.624946E-05	7.719572E-04	1.006428E-02	9.459974E-02	-1.501363E-02	1.579098E-02	-5.547616E-02	2.254202E+00
5.624946E-05	6.924159E-04	1.215258E-02	9.444663E-02	-1.504035E-02	1.802221E-02	-5.734272E-02	2.331233E+00
5.624946E-05	6.128746E-04	1.317248E-02	9.427165E-02	-1.523560E-02	2.036750E-02	-7.842889E-02	2.395839E+00
5.624946E-05	5.333333E-04	1.610407E-02	9.405235E-02	-1.547929E-02	2.306935E-02	-6.784675E-02	2.461448E+00
5.624946E-05	1.057936E-03	-3.379951E-03	9.499277E-02	-1.887448E-02	5.727762E-03	-1.000599E-03	2.042326E+00
5.624946E-05	1.105290E-03	-6.894655E-03	9.494866E-02	-2.051269E-02	4.203648E-03	-3.942044E-03	2.014739E+00
5.624946E-05	1.152645E-03	-6.842349E-03	9.495709E-02	-2.268667E-02	4.304920E-03	-5.239042E-03	2.015600E+00
5.624946E-05	1.200000E-03	-8.125787E-03	9.493561E-02	-2.399268E-02	4.473228E-03	-5.814018E-03	2.014293E+00
5.906194E-05	9.431403E-04	5.199468E-03	9.259711E-02	-1.477711E-02	1.166836E-02	-1.059531E-02	2.135610E+00
5.906194E-05	8.611789E-04	7.800261E-03	9.242722E-02	-1.472136E-02	1.376872E-02	-3.845344E-02	2.201455E+00
5.906194E-05	7.792175E-04	8.807204E-03	9.231429E-02	-1.472004E-02	1.588632E-02	-6.249856E-02	2.285578E+00
5.906194E-05	6.972561E-04	1.098696E-02	9.205446E-02	-1.529338E-02	1.765328E-02	-1.074011E-01	2.292950E+00
5.906194E-05	6.152947E-04	1.393894E-02	9.167500E-02	-1.574859E-02	1.987146E-02	-1.267383E-01	2.313477E+00
5.906194E-05	5.333333E-04	1.469488E-02	9.154915E-02	-1.616057E-02	2.258227E-02	-1.485890E-01	2.377495E+00
5.906194E-05	1.068826E-03	-3.943963E-03	9.262212E-02	-1.454494E-02	1.169863E-02	-3.132676E-04	2.006473E+00
5.906194E-05	1.112551E-03	-6.379515E-03	9.260790E-02	-1.441182E-02	1.456337E-02	-3.609204E-03	2.064110E+00
5.906194E-05	1.156275E-03	-9.320481E-03	9.261104E-02	-1.555498E-02	1.519177E-02	-1.375201E-02	2.044983E+00
5.906194E-05	1.200000E-03	-1.135192E-02	9.269719E-02	-1.655546E-02	1.615965E-02	-2.774860E-02	2.050581E+00
6.201503E-05	9.556629E-04	7.301439E-03	9.012538E-02	-1.538135E-02	1.041154E-02	-3.815431E-02	2.026433E+00
6.201503E-05	8.711970E-04	1.336028E-02	8.963508E-02	-1.572143E-02	1.221315E-02	-5.879871E-02	2.031440E+00
6.201503E-05	7.867311E-04	1.572841E-02	8.927033E-02	-1.592101E-02	1.436355E-02	-8.429504E-02	2.092460E+00
6.201503E-05	7.022651E-04	1.668159E-02	8.915554E-02	-1.589678E-02	1.692139E-02	-8.803399E-02	2.196777E+00
6.201503E-05	6.177992E-04	1.879660E-02	8.874247E-02	-1.630725E-02	1.930438E-02	-1.117212E-01	2.232104E+00
6.201503E-05	5.333333E-04	1.944420E-02	8.854177E-02	-1.664548E-02	2.221139E-02	-1.309933E-01	2.302652E+00
6.201503E-05	1.080097E-03	-1.145510E-03	9.035590E-02	-1.770402E-02	5.423001E-03	3.141428E-02	2.017434E+00
6.201503E-05	1.120064E-03	-5.019788E-03	9.086936E-02	-1.479404E-02	7.059983E-03	1.976023E-01	2.069028E+00
6.201503E-05	1.160032E-03	1.719946E-02	1.289982E-01	-1.592170E-02	1.361108E-02	8.653688E-01	2.073733E+00
6.201503E-05	1.200000E-03	1.735463E-02	1.293160E-01	-1.712589E-02	1.476734E-02	8.679814E-01	2.070897E+00
6.511579E-05	9.686267E-04	4.350847E-03	8.802801E-02	-1.451246E-02	1.138257E-02	-4.045083E-02	2.122812E+00
6.511579E-05	8.815680E-04	8.125131E-03	8.779024E-02	-1.465941E-02	1.331260E-02	-6.622299E-02	2.167030E+00
6.511579E-05	7.945093E-04	1.044813E-02	8.755125E-02	-1.483330E-02	1.534491E-02	-9.480272E-02	2.215338E+00
6.511579E-05	7.074507E-04	1.153311E-02	8.742162E-02	-1.480137E-02	1.777390E-02	-1.109821E-01	2.300118E+00
6.511579E-05	6.203920E-04	1.272066E-02	8.725235E-02	-1.525830E-02	2.008684E-02	-1.413290E-01	2.348651E+00
6.511579E-05	5.333333E-04	1.459743E-02	8.696448E-02	-1.604592E-02	2.266278E-02	-1.669962E-01	2.380293E+00
6.511579E-05	1.091764E-03	-1.575662E-03	8.809768E-02	-1.693925E-02	5.647970E-03	2.937544E-02	2.019196E+00
6.511579E-05	1.127843E-03	-2.304975E-03	8.811543E-02	-1.839055E-02	4.414561E-03	6.139870E-02	2.020359E+00
6.511579E-05	1.163921E-03	-2.178263E-03	8.827110E-02	-1.949900E-02	4.295260E-03	1.018011E-01	2.049943E+00
6.511579E-05	1.200000E-03	-3.103066E-03	8.833973E-02	-2.057533E-02	3.487042E-03	1.306487E-01	2.036640E+00
6.837158E-05	9.820523E-04	7.201588E-03	8.571824E-02	-1.481082E-02	1.059286E-02	-3.014798E-02	2.056051E+00
6.837158E-05	8.923085E-04	9.899719E-03	8.547033E-02	-1.468535E-02	1.300013E-02	-6.056016E-02	2.135282E+00
6.837158E-05	8.025647E-04	1.362851E-02	8.494258E-02	-1.545118E-02	1.444587E-02	-1.095811E-01	2.115065E+00
6.837158E-05	7.128209E-04	1.701557E-02	8.448370E-02	-1.587258E-02	1.660373E-02	-1.149123E-01	2.159975E+00
6.837158E-05	6.230771E-04	1.795362E-02	8.414853E-02	-1.651598E-02	1.892438E-02	-1.629368E-01	2.190473E+00
6.837158E-05	5.333333E-04	1.962848E-02	8.387547E-02	-1.673915E-02	2.213137E-02	-1.586545E-01	2.274886E+00
6.837158E-05	1.103847E-03	-5.511196E-03	8.590797E-02	-1.448458E-02	8.073010E-03	-9.595087E-03	2.042712E+00
6.837158E-05	1.135898E-03	-1.149299E-02	8.594516E-02	-1.547057E-02	6.580880E-03	-2.670160E-02	2.103642E+00
6.837158E-05	1.167949E-03	-1.251021E-02	8.595745E-02	-1.769180E-02	5.161159E-03	-2.992268E-02	2.101938E+00
6.837158E-05	1.200000E-03	-1.177650E-02	8.601180E-02	-1.972083E-02	4.426191E-03	-3.416790E-02	2.074555E+00
7.179015E-05	9.959616E-04	6.040585E-03	8.367614E-02	-1.418592E-02	1.117610E-02	-1.705221E-02	2.131422E+00
7.179015E-05	9.034359E-04	9.255003E-03	8.344796E-02	-1.414191E-02	1.336942E-02	-4.154871E-02	2.197153E+00
7.179015E-05	8.109103E-04	1.157032E-02	8.318649E-02	-1.428670E-02	1.545889E-02	-6.549303E-02	2.256047E+00
7.179015E-05	7.183846E-04	1.250683E-02	8.303757E-02	-1.423650E-02	1.793825E-02	-8.301111E-02	2.351210E+00
7.179015E-05	6.258590E-04	1.537360E-02	8.276599E-02	-1.462317E-02	2.039073E-02	-7.849706E-02	2.416055E+00
7.179015E-05	5.333333E-04	1.586530E-02	8.265060E-02	-1.466377E-02	2.359962E-02	-8.978309E-02	2.526847E+00
7.179015E-05	1.116365E-03	-4.480864E-03	8.376890E-02	-1.425126E-02	8.198111E-03	-6.468663E-03	2.058243E+00
7.179015E-05	1.144244E-03	-9.982299E-03	8.373656E-02	-1.406754E-02	7.879644E-03	-1.658687E-02	2.079230E+00
7.179015E-05	1.172122E-03	-1.000919E-02	8.373569E-02	-1.406422E-02	7.880451E-03	-1.655642E-02	2.079257E+00
7.179015E-05	1.200000E-03	-1.010781E-02	8.373046E-02	-1.405026E-02	7.881563E-03	-1.620084E-02	2.079323E+00
7.537966E-05	1.010378E-03	7.160489E-03	8.151138E-02	-1.436626E-02	1.055908E-02	-2.371562E-02	2.082139E+00
7.537966E-05	9.149692E-04	1.033471E-02	8.122944E-02	-1.429328E-02	1.290789E-02	-5.223835E-02	2.151722E+00
7.537966E-05	8.195603E-04	1.216808E-02	8.090379E-02	-1.455656E-02	1.495586E-02	-9.717763E-02	2.191539E+00

7.537966E-05	7.241513E-04	1.587719E-02	8.046322E-02	-1.502209E-02	1.707987E-02	-9.855772E-02	2.231043E+00
7.537966E-05	6.287423E-04	1.796065E-02	8.012008E-02	-1.537322E-02	1.970501E-02	-1.104871E-01	2.288990E+00
7.537966E-05	5.333333E-04	1.815907E-02	8.008138E-02	-1.544636E-02	2.307597E-02	-1.141343E-01	2.413251E+00
7.537966E-05	1.129340E-03	-3.799202E-03	8.167368E-02	-1.443249E-02	7.452777E-03	-5.960231E-03	2.014529E+00
7.537966E-05	1.152894E-03	-6.300090E-03	8.165525E-02	-1.549059E-02	5.892085E-03	-7.986472E-03	2.046175E+00
7.537966E-05	1.176447E-03	-7.066334E-03	8.164745E-02	-1.704064E-02	4.720107E-03	-9.026157E-03	2.046829E+00
7.537966E-05	1.200000E-03	-7.846404E-03	8.163951E-02	-1.816618E-02	4.204776E-03	-1.022714E-02	2.048495E+00
7.914864E-05	1.025327E-03	6.097133E-03	7.947309E-02	-1.405595E-02	1.064606E-02	-3.938763E-02	2.095604E+00
7.914864E-05	9.269286E-04	8.089057E-03	7.931645E-02	-1.366777E-02	1.334276E-02	-6.524091E-02	2.212998E+00
7.914864E-05	8.285298E-04	9.602912E-03	7.915621E-02	-1.367734E-02	1.560034E-02	-8.772107E-02	2.307266E+00
7.914864E-05	7.301310E-04	1.110434E-02	7.896505E-02	-1.388287E-02	1.790627E-02	-1.100994E-01	2.381460E+00
7.914864E-05	6.317321E-04	1.308402E-02	7.866550E-02	-1.441000E-02	2.035333E-02	-1.373713E-01	2.412820E+00
7.914864E-05	5.333333E-04	1.501353E-02	7.830477E-02	-1.573089E-02	2.288988E-02	-1.835643E-01	2.402045E+00
7.914864E-05	1.142795E-03	-2.745861E-03	7.964092E-02	-1.404582E-02	7.883833E-03	-4.142191E-03	2.039537E+00
7.914864E-05	1.161863E-03	-5.366895E-03	7.960335E-02	-1.374449E-02	8.023851E-03	-1.832993E-03	2.061594E+00
7.914864E-05	1.180932E-03	-6.385804E-03	7.960258E-02	-1.581711E-02	5.494017E-03	-5.810202E-03	2.063058E+00
7.914864E-05	1.200000E-03	-8.779869E-03	7.958511E-02	-1.611451E-02	5.313617E-03	-1.093626E-02	2.089637E+00
8.310608E-05	1.040836E-03	5.463459E-03	7.751848E-02	-1.357937E-02	1.095973E-02	-3.708335E-02	2.137212E+00
8.310608E-05	9.393358E-04	9.140992E-03	7.718833E-02	-1.395018E-02	1.272917E-02	-8.100924E-02	2.153313E+00
8.310608E-05	8.378352E-04	1.234795E-02	7.682226E-02	-1.424908E-02	1.480407E-02	-9.775127E-02	2.202485E+00
8.310608E-05	7.363346E-04	1.521362E-02	7.636785E-02	-1.483756E-02	1.689661E-02	-1.216474E-01	2.227980E+00
8.310608E-05	6.348339E-04	1.557073E-02	7.622921E-02	-1.499275E-02	1.980417E-02	-1.490311E-01	2.317185E+00
8.310608E-05	5.333333E-04	1.925752E-02	7.563237E-02	-1.590902E-02	2.274618E-02	-1.435910E-01	2.348552E+00
8.310608E-05	1.156753E-03	-1.892400E-03	7.764858E-02	-1.384850E-02	7.908408E-03	-1.715819E-03	2.044282E+00
8.310608E-05	1.171169E-03	-3.097701E-03	7.760982E-02	-1.392253E-02	7.434004E-03	1.279702E-02	2.044859E+00
8.310608E-05	1.185584E-03	-4.220621E-03	7.758141E-02	-1.444608E-02	6.718781E-03	1.756824E-02	2.083270E+00
8.310608E-05	1.200000E-03	-4.535438E-03	7.757745E-02	-1.560828E-02	5.903017E-03	1.674122E-02	2.098235E+00
8.726138E-05	1.056935E-03	7.314114E-03	7.542402E-02	-1.429978E-02	9.484889E-03	-4.711761E-02	2.020191E+00
8.726138E-05	9.522145E-04	1.012971E-02	7.508890E-02	-1.428968E-02	1.199489E-02	-9.776123E-02	2.087562E+00
8.726138E-05	8.474942E-04	1.453088E-02	7.443255E-02	-1.503619E-02	1.366771E-02	-1.323972E-01	2.069858E+00
8.726138E-05	7.427739E-04	1.645331E-02	7.399519E-02	-1.572788E-02	1.581997E-02	-1.758162E-01	2.102372E+00
8.726138E-05	6.380536E-04	1.811618E-02	7.357193E-02	-1.645202E-02	1.842850E-02	-2.260925E-01	2.119894E+00
8.726138E-05	5.333333E-04	1.916282E-02	7.332541E-02	-1.754611E-02	2.145230E-02	-2.776643E-01	2.148789E+00
8.726138E-05	1.171241E-03	-1.167913E-03	7.569964E-02	-1.376971E-02	7.688366E-03	5.058597E-04	2.034715E+00
8.726138E-05	1.180828E-03	-2.293904E-03	7.566824E-02	-1.397652E-02	6.998992E-03	1.243203E-02	2.006989E+00
8.726138E-05	1.190414E-03	-3.136690E-03	7.563895E-02	-1.395158E-02	6.817400E-03	2.297164E-02	2.013476E+00
8.726138E-05	1.200000E-03	-4.124273E-03	7.560499E-02	-1.397744E-02	6.583204E-03	2.722842E-02	2.028086E+00
9.162445E-05	1.073654E-03	6.012936E-03	7.362556E-02	-1.333846E-02	1.056250E-02	-4.105488E-02	2.119736E+00
9.162445E-05	9.655902E-04	9.245748E-03	7.331218E-02	-1.358379E-02	1.256148E-02	-8.199561E-02	2.165517E+00
9.162445E-05	8.575260E-04	1.250216E-02	7.289281E-02	-1.404677E-02	1.452295E-02	-1.075536E-01	2.195586E+00
9.162445E-05	7.494618E-04	1.398367E-02	7.259739E-02	-1.419951E-02	1.709702E-02	-1.413445E-01	2.253092E+00
9.162445E-05	6.413975E-04	1.397145E-02	7.256658E-02	-1.437628E-02	2.006235E-02	-1.673827E-01	2.367875E+00
9.162445E-05	5.333333E-04	1.496650E-02	7.236518E-02	-1.546839E-02	2.308073E-02	-1.993637E-01	2.423441E+00
9.162445E-05	1.186289E-03	-5.006949E-04	7.380278E-02	-1.348951E-02	7.864227E-03	3.545096E-04	2.050394E+00
9.162445E-05	1.190859E-03	-9.464309E-04	7.379765E-02	-1.352957E-02	7.676470E-03	2.840930E-03	2.042958E+00
9.162445E-05	1.195430E-03	-1.630967E-03	7.379175E-02	-1.357065E-02	7.484252E-03	1.824298E-03	2.036553E+00
9.162445E-05	1.200000E-03	-1.945219E-03	7.377802E-02	-1.364032E-02	7.229289E-03	9.663664E-03	2.028401E+00

Table 33. *f*18p1 low energy transfers

Inner Radius	Outer Radius	Delta V1x	Delta V1y	Delta V2x	Delta V2y	Initial *f*	Time of flight
5.101992E-05	5.458158E-04	2.203695E-04	9.981882E-02	-2.143390E-02	-1.636030E-02	-1.240581E-04	9.975437E-01
5.101992E-05	5.446811E-04	8.535863E-05	9.981874E-02	-2.143417E-02	-1.641835E-02	-5.919812E-04	9.973567E-01
5.101992E-05	5.435463E-04	2.065225E-04	9.981807E-02	-2.149467E-02	-1.639592E-02	-2.271483E-03	9.958167E-01
5.101992E-05	5.424115E-04	1.951223E-04	9.981786E-02	-2.149216E-02	-1.645728E-02	-2.247241E-03	9.952688E-01
5.101992E-05	5.412768E-04	1.995688E-04	9.981759E-02	-2.151009E-02	-1.649196E-02	-2.552428E-03	9.945710E-01
5.101992E-05	5.401420E-04	1.586840E-04	9.981748E-02	-2.152938E-02	-1.652537E-02	-2.658040E-03	9.941121E-01
5.101992E-05	5.390072E-04	8.103535E-05	9.981749E-02	-2.154491E-02	-1.656408E-02	-2.695353E-03	9.938180E-01
5.101992E-05	5.378724E-04	5.871834E-04	9.981539E-02	-2.165628E-02	-1.647607E-02	-2.264321E-03	9.913504E-01
5.101992E-05	5.367377E-04	5.403752E-04	9.981530E-02	-2.162944E-02	-1.656915E-02	-2.414616E-03	9.908811E-01
5.101992E-05	5.356029E-04	5.419065E-04	9.981509E-02	-2.162282E-02	-1.663669E-02	-2.235372E-03	9.903178E-01
5.101992E-05	5.344681E-04	3.416103E-04	9.981565E-02	-2.162281E-02	-1.669707E-02	-2.089311E-03	9.905557E-01
5.101992E-05	5.333333E-04	4.395119E-04	9.981491E-02	-2.166482E-02	-1.670095E-02	-2.646886E-03	9.894149E-01
5.101992E-05	6.285818E-04	2.141538E-03	9.981996E-02	-2.015410E-02	-1.418674E-02	-4.654593E-03	1.029124E+00
5.101992E-05	7.102130E-04	4.380603E-03	9.977358E-02	-1.928896E-02	-1.208007E-02	-1.696885E-02	1.053083E+00
5.101992E-05	7.918441E-04	7.266789E-03	9.967082E-02	-1.856563E-02	-1.021269E-02	-2.479911E-02	1.072975E+00
5.101992E-05	8.734753E-04	1.035819E-02	9.952342E-02	-1.728934E-02	-9.527935E-03	-2.897469E-02	1.090401E+00
5.101992E-05	9.551065E-04	1.187549E-02	9.940517E-02	-1.646418E-02	-8.484246E-03	-3.659102E-02	1.111066E+00
5.101992E-05	1.036738E-03	1.330634E-02	9.930693E-02	-1.579742E-02	-7.394620E-03	-3.903201E-02	1.131927E+00
5.101992E-05	1.118369E-03	1.764034E-02	9.904554E-02	-1.510617E-02	-6.569314E-03	-3.434355E-02	1.142601E+00

5.101992E-05	1.200000E-03	1.893345E-02	9.886463E-02	-1.451447E-02	-5.716716E-03	-4.247522E-02	1.159909E+00
5.357092E-05	5.539769E-04	5.566400E-05	9.735036E-02	-2.118636E-02	-1.625940E-02	-6.058621E-05	9.992127E-01
5.357092E-05	5.521002E-04	4.558151E-04	9.734935E-02	-2.123700E-02	-1.628463E-02	3.324238E-04	9.967032E-01
5.357092E-05	5.502235E-04	5.141781E-04	9.734829E-02	-2.127361E-02	-1.632998E-02	-8.984551E-04	9.951395E-01
5.357092E-05	5.483468E-04	5.384689E-04	9.734750E-02	-2.123739E-02	-1.647224E-02	-1.709896E-03	9.937994E-01
5.357092E-05	5.464701E-04	4.388563E-04	9.734735E-02	-2.134000E-02	-1.643253E-02	-2.110376E-03	9.931532E-01
5.357092E-05	5.445934E-04	5.676850E-04	9.734648E-02	-2.138092E-02	-1.647378E-02	-1.878730E-03	9.917111E-01
5.357092E-05	5.427168E-04	6.416541E-04	9.734549E-02	-2.143567E-02	-1.649764E-02	-2.570138E-03	9.902359E-01
5.357092E-05	5.408401E-04	6.356377E-04	9.734493E-02	-2.145267E-02	-1.657122E-02	-3.039898E-03	9.891349E-01
5.357092E-05	5.389634E-04	6.339277E-04	9.734471E-02	-2.157080E-02	-1.651708E-02	-2.437973E-03	9.883577E-01
5.357092E-05	5.370867E-04	7.379435E-04	9.734392E-02	-2.152209E-02	-1.667416E-02	-2.104163E-03	9.870025E-01
5.357092E-05	5.352100E-04	8.758441E-04	9.734289E-02	-2.157221E-02	-1.670588E-02	-1.950350E-03	9.854929E-01
5.357092E-05	5.333333E-04	6.394171E-04	9.734395E-02	-2.161368E-02	-1.675205E-02	-1.080877E-03	9.857185E-01
5.357092E-05	6.363719E-04	3.223645E-03	9.734134E-02	-2.005102E-02	-1.397385E-02	-3.059717E-03	1.025610E+00
5.357092E-05	7.168902E-04	5.551257E-03	9.727312E-02	-1.886222E-02	-1.244433E-02	-1.651395E-02	1.047882E+00
5.357092E-05	7.974085E-04	7.923541E-03	9.718476E-02	-1.803982E-02	-1.079437E-02	-2.191401E-02	1.069674E+00
5.357092E-05	8.779268E-04	9.551080E-03	9.710399E-02	-1.676924E-02	-1.042680E-02	-2.632219E-02	1.091229E+00
5.357092E-05	9.584451E-04	1.134397E-02	9.697791E-02	-1.639712E-02	-8.491094E-03	-3.391145E-02	1.111340E+00
5.357092E-05	1.038963E-03	1.271941E-02	9.676817E-02	-1.570252E-02	-7.500914E-03	-5.676666E-02	1.126527E+00
5.357092E-05	1.119482E-03	1.447528E-02	9.655807E-02	-1.507836E-02	-6.576619E-03	-6.945873E-02	1.141697E+00
5.357092E-05	1.200000E-03	1.655248E-02	9.632192E-02	-1.444087E-02	-5.871894E-03	-7.600528E-02	1.155939E+00
5.624946E-05	5.624068E-04	-1.497533E-04	9.493981E-02	-2.094938E-02	-1.613993E-02	1.165430E-04	1.001180E+00
5.624946E-05	5.597638E-04	2.025557E-05	9.493911E-02	-2.099664E-02	-1.620625E-02	3.915625E-04	9.991719E-01
5.624946E-05	5.571207E-04	4.689683E-05	9.493851E-02	-2.105958E-02	-1.625399E-02	5.161434E-04	9.977374E-01
5.624946E-05	5.544777E-04	-4.905347E-05	9.493816E-02	-2.107945E-02	-1.635939E-02	1.895921E-03	9.971667E-01
5.624946E-05	5.518347E-04	-1.979028E-04	9.493752E-02	-2.112206E-02	-1.643620E-02	2.311698E-03	9.965503E-01
5.624946E-05	5.491916E-04	-5.275031E-04	9.493645E-02	-2.114816E-02	-1.653581E-02	2.003375E-03	9.964912E-01
5.624946E-05	5.465486E-04	-7.602302E-04	9.493512E-02	-2.122076E-02	-1.657688E-02	2.695786E-03	9.963232E-01
5.624946E-05	5.439055E-04	-1.087944E-03	9.493359E-02	-2.124001E-02	-1.668657E-02	2.270753E-03	9.962158E-01
5.624946E-05	5.412625E-04	-1.263692E-03	9.493104E-02	-2.126319E-02	-1.679215E-02	4.369282E-03	9.961867E-01
5.624946E-05	5.386194E-04	-1.433349E-03	9.492900E-02	-2.133362E-02	-1.683862E-02	5.028288E-03	9.957252E-01
5.624946E-05	5.359764E-04	-1.565156E-03	9.492722E-02	-2.134318E-02	-1.696161E-02	5.414216E-03	9.949896E-01
5.624946E-05	5.333333E-04	-1.729293E-03	9.492708E-02	-2.139225E-02	-1.703521E-02	3.367644E-03	9.936896E-01
5.624946E-05	6.444186E-04	9.218023E-04	9.495273E-02	-1.964077E-02	-1.420720E-02	-1.546656E-03	1.035913E+00
5.624946E-05	7.237874E-04	3.078002E-03	9.493345E-02	-1.870985E-02	-1.240431E-02	-9.742344E-03	1.059419E+00
5.624946E-05	8.031562E-04	5.480263E-03	9.488021E-02	-1.781450E-02	-1.095867E-02	-1.464726E-02	1.080116E+00
5.624946E-05	8.825249E-04	7.661833E-03	9.479307E-02	-1.698555E-02	-9.744700E-03	-2.167630E-02	1.098955E+00
5.624946E-05	9.618937E-04	9.863318E-03	9.463395E-02	-1.627881E-02	-8.593742E-03	-3.642485E-02	1.113945E+00
5.624946E-05	1.041262E-03	1.136681E-02	9.447840E-02	-1.556670E-02	-7.692085E-03	-5.119254E-02	1.130111E+00
5.624946E-05	1.120631E-03	1.342702E-02	9.423543E-02	-1.497692E-02	-6.744739E-03	-6.885124E-02	1.142288E+00
5.624946E-05	1.200000E-03	1.483247E-02	9.401111E-02	-1.439184E-02	-5.960949E-03	-8.815079E-02	1.155409E+00
5.906194E-05	5.711160E-04	-3.098685E-04	9.258552E-02	-2.070181E-02	-1.602975E-02	2.366645E-03	1.003650E+00
5.906194E-05	5.676812E-04	-5.115598E-04	9.258440E-02	-2.074982E-02	-1.613214E-02	2.394210E-03	1.002792E+00
5.906194E-05	5.642464E-04	-5.495624E-04	9.258369E-02	-2.080705E-02	-1.622255E-02	1.588532E-03	1.000958E+00
5.906194E-05	5.608116E-04	-6.583339E-04	9.258259E-02	-2.085972E-02	-1.632113E-02	2.058032E-03	9.998011E-01
5.906194E-05	5.573768E-04	-7.321995E-04	9.258157E-02	-2.091449E-02	-1.641802E-02	2.231250E-03	9.983941E-01
5.906194E-05	5.539420E-04	-7.816465E-04	9.258048E-02	-2.096092E-02	-1.652684E-02	2.778883E-03	9.969752E-01
5.906194E-05	5.505073E-04	-8.949420E-04	9.257917E-02	-2.102120E-02	-1.661978E-02	3.127811E-03	9.957803E-01
5.906194E-05	5.470725E-04	-1.015541E-03	9.257762E-02	-2.107905E-02	-1.671742E-02	3.786329E-03	9.946990E-01
5.906194E-05	5.436377E-04	-1.423117E-03	9.257531E-02	-2.111529E-02	-1.684480E-02	2.901130E-03	9.944323E-01
5.906194E-05	5.402029E-04	-1.475588E-03	9.257235E-02	-2.116645E-02	-1.695402E-02	5.629165E-03	9.936374E-01
5.906194E-05	5.367681E-04	-1.548156E-03	9.257121E-02	-2.118806E-02	-1.710044E-02	5.301737E-03	9.920081E-01
5.906194E-05	5.333333E-04	-1.681452E-03	9.256868E-02	-2.125327E-02	-1.719455E-02	6.262124E-03	9.910430E-01
5.906194E-05	6.527319E-04	2.588986E-03	9.259053E-02	-1.956792E-02	-1.393662E-02	8.472385E-04	1.029999E+00
5.906194E-05	7.309131E-04	5.066646E-03	9.250764E-02	-1.864797E-02	-1.220027E-02	-2.144824E-02	1.047471E+00
5.906194E-05	8.090942E-04	9.873547E-03	9.235443E-02	-1.782213E-02	-1.070500E-02	-1.744443E-02	1.060307E+00
5.906194E-05	8.872754E-04	1.147952E-02	9.222691E-02	-1.698644E-02	-9.557072E-03	-2.697963E-02	1.080433E+00
5.906194E-05	9.654565E-04	1.337791E-02	9.207788E-02	-1.629810E-02	-8.422114E-03	-3.332834E-02	1.098576E+00
5.906194E-05	1.043638E-03	1.554132E-02	9.180778E-02	-1.581376E-02	-7.107683E-03	-4.877511E-02	1.112264E+00
5.906194E-05	1.121819E-03	1.730608E-02	9.148860E-02	-1.506012E-02	-6.504267E-03	-6.950054E-02	1.124356E+00
5.906194E-05	1.200000E-03	1.955865E-02	9.121467E-02	-1.446391E-02	-5.771249E-03	-7.379732E-02	1.137883E+00
6.201503E-05	5.801149E-04	6.026910E-05	9.028635E-02	-2.048473E-02	-1.587002E-02	5.077075E-04	1.002607E+00
6.201503E-05	5.758620E-04	-3.260843E-04	9.028568E-02	-2.053682E-02	-1.600252E-02	-2.989715E-04	1.001959E+00
6.201503E-05	5.716092E-04	-4.084443E-05	9.028447E-02	-2.061673E-02	-1.609735E-02	3.047570E-04	9.986599E-01
6.201503E-05	5.673563E-04	2.132606E-04	9.028317E-02	-2.068763E-02	-1.620578E-02	4.539745E-04	9.953562E-01
6.201503E-05	5.631034E-04	-4.470534E-05	9.028252E-02	-2.074950E-02	-1.633121E-02	8.407909E-04	9.944319E-01
6.201503E-05	5.588506E-04	1.822189E-04	9.028131E-02	-2.082911E-02	-1.643279E-02	9.456039E-04	9.912159E-01
6.201503E-05	5.545977E-04	2.090591E-04	9.027994E-02	-2.090319E-02	-1.654430E-02	-2.410262E-04	9.885177E-01
6.201503E-05	5.503448E-04	2.844931E-04	9.027848E-02	-2.097245E-02	-1.666379E-02	-1.236423E-03	9.856400E-01
6.201503E-05	5.460919E-04	-3.875462E-05	9.027828E-02	-2.099855E-02	-1.684271E-02	-1.797117E-03	9.846583E-01
6.201503E-05	5.418391E-04	5.224778E-05	9.027712E-02	-2.111577E-02	-1.690722E-02	2.231729E-04	9.825770E-01
6.201503E-05	5.375862E-04	6.054202E-05	9.027594E-02	-2.118201E-02	-1.703756E-02	-4.136599E-04	9.800609E-01
6.201503E-05	5.333333E-04	4.289970E-04	9.027422E-02	-2.131067E-02	-1.709118E-02	3.308802E-04	9.763293E-01
6.201503E-05	6.613218E-04	3.552870E-03	9.027290E-02	-1.940553E-02	-1.378894E-02	-2.501257E-03	1.025222E+00
6.201503E-05	7.382758E-04	7.669549E-03	9.010333E-02	-1.857234E-02	-1.201080E-02	-2.369673E-02	1.035377E+00
6.201503E-05	8.152299E-04	9.558294E-03	8.998000E-02	-1.773976E-02	-1.060893E-02	-3.386326E-02	1.055707E+00
6.201503E-05	8.921839E-04	1.145441E-02	8.985013E-02	-1.701726E-02	-9.326640E-03	-3.910323E-02	1.075339E+00
6.201503E-05	9.691379E-04	1.318707E-02	8.969641E-02	-1.623158E-02	-8.409772E-03	-4.606827E-02	1.093301E+00
6.201503E-05	1.046092E-03	1.457176E-02	8.948742E-02	-1.554423E-02	-7.545141E-03	-6.205666E-02	1.108762E+00
6.201503E-05	1.123046E-03	1.658907E-02	8.919635E-02	-1.503057E-02	-6.511191E-03	-7.683346E-02	1.121508E+00
6.201503E-05	1.200000E-03	1.831112E-02	8.892463E-02	-1.443522E-02	-5.812036E-03	-8.820189E-02	1.134773E+00
6.511579E-05	5.894149E-04	-9.051363E-05	8.804119E-02	-2.024261E-02	-1.574205E-02	9.063794E-04	1.004611E+00

6.511579E-05	5.843165E-04	-9.276935E-05	8.803998E-02	-2.031713E-02	-1.587724E-02	4.034594E-04	1.001886E+00
6.511579E-05	5.792182E-04	1.405879E-04	8.803861E-02	-2.040892E-02	-1.599186E-02	8.521263E-04	9.983251E-01
6.511579E-05	5.741199E-04	2.403819E-04	8.803684E-02	-2.050397E-02	-1.610525E-02	-7.029936E-04	9.947885E-01
6.511579E-05	5.690216E-04	5.209748E-04	8.803464E-02	-2.058528E-02	-1.623824E-02	-9.022577E-04	9.907829E-01
6.511579E-05	5.639233E-04	5.969569E-04	8.803298E-02	-2.067314E-02	-1.636725E-02	-1.119675E-03	9.877108E-01
6.511579E-05	5.588249E-04	5.601521E-04	8.803190E-02	-2.073958E-02	-1.652712E-02	-9.896944E-04	9.852337E-01
6.511579E-05	5.537266E-04	1.531500E-04	8.803193E-02	-2.084404E-02	-1.664480E-02	-1.126410E-03	9.843973E-01
6.511579E-05	5.486283E-04	-4.506992E-05	8.803113E-02	-2.091102E-02	-1.681032E-02	-2.086812E-03	9.823140E-01
6.511579E-05	5.435300E-04	-1.335906E-04	8.803005E-02	-2.097510E-02	-1.698193E-02	-2.239828E-03	9.799322E-01
6.511579E-05	5.384317E-04	-9.893010E-05	8.802851E-02	-2.103972E-02	-1.715563E-02	-1.098314E-03	9.773317E-01
6.511579E-05	5.333333E-04	-1.919637E-04	8.802722E-02	-2.113199E-02	-1.729959E-02	1.054685E-03	9.756131E-01
6.511579E-05	6.701990E-04	2.683269E-03	8.804031E-02	-1.914207E-02	-1.378973E-02	-1.473654E-03	1.029763E+00
6.511579E-05	7.458849E-04	4.762415E-03	8.799702E-02	-1.827138E-02	-1.219068E-02	-1.043890E-02	1.050805E+00
6.511579E-05	8.215707E-04	6.904484E-03	8.789617E-02	-1.750133E-02	-1.078059E-02	-2.457433E-02	1.067859E+00
6.511579E-05	8.972566E-04	8.635610E-03	8.777541E-02	-1.674671E-02	-9.631738E-03	-3.790923E-02	1.085014E+00
6.511579E-05	9.729424E-04	1.040429E-02	8.759847E-02	-1.609102E-02	-8.546250E-03	-5.648602E-02	1.099121E+00
6.511579E-05	1.048628E-03	1.231971E-02	8.739913E-02	-1.548852E-02	-7.568254E-03	-6.865742E-02	1.113027E+00
6.511579E-05	1.124314E-03	1.426824E-02	8.712965E-02	-1.487898E-02	-6.779769E-03	-8.692069E-02	1.123882E+00
6.511579E-05	1.200000E-03	1.605723E-02	8.685846E-02	-1.437508E-02	-5.923902E-03	-1.009975E-01	1.135996E+00
6.837158E-05	5.990275E-04	-1.226469E-04	8.584834E-02	-1.998810E-02	-1.562417E-02	1.234476E-03	1.006113E+00
6.837158E-05	5.930553E-04	-8.604480E-04	8.584650E-02	-2.005909E-02	-1.580223E-02	4.257235E-04	1.006445E+00
6.837158E-05	5.870831E-04	-8.724342E-04	8.584458E-02	-2.013000E-02	-1.597926E-02	1.444094E-03	1.003756E+00
6.837158E-05	5.811109E-04	-1.157861E-03	8.583953E-02	-2.019977E-02	-1.616513E-02	7.419348E-03	1.003870E+00
6.837158E-05	5.751387E-04	-1.547517E-03	8.583439E-02	-2.025609E-02	-1.636973E-02	8.260521E-03	1.002937E+00
6.837158E-05	5.691665E-04	-2.121297E-03	8.582659E-02	-2.034898E-02	-1.653335E-02	8.602870E-03	1.002772E+00
6.837158E-05	5.631943E-04	-2.174956E-03	8.582455E-02	-2.042467E-02	-1.671808E-02	8.525457E-03	9.998562E-01
6.837158E-05	5.572221E-04	-2.660251E-03	8.581962E-02	-2.053752E-02	-1.686357E-02	6.504764E-03	9.984958E-01
6.837158E-05	5.512499E-04	-2.797437E-03	8.581067E-02	-2.061815E-02	-1.705258E-02	1.072298E-02	9.972587E-01
6.837158E-05	5.452777E-04	-2.855709E-03	8.580586E-02	-2.067799E-02	-1.726874E-02	1.266957E-02	9.948946E-01
6.837158E-05	5.393055E-04	-2.367222E-03	8.581565E-02	-2.082766E-02	-1.737455E-02	8.900211E-03	9.880475E-01
6.837158E-05	5.333333E-04	-2.389353E-03	8.581357E-02	-2.091081E-02	-1.757014E-02	9.087049E-03	9.849117E-01
6.837158E-05	6.793747E-04	3.312829E-03	8.580524E-02	-1.908823E-02	-1.347501E-02	-2.129572E-02	1.021744E+00
6.837158E-05	7.537498E-04	6.001685E-03	8.571240E-02	-1.818776E-02	-1.200437E-02	-2.906008E-02	1.039577E+00
6.837158E-05	8.281248E-04	7.933838E-03	8.565625E-02	-1.732757E-02	-1.082367E-02	-2.461151E-02	1.061882E+00
6.837158E-05	9.024998E-04	1.028681E-02	8.545370E-02	-1.671250E-02	-9.501584E-03	-4.339333E-02	1.074469E+00
6.837158E-05	9.768749E-04	1.224774E-02	8.526691E-02	-1.604857E-02	-8.488569E-03	-5.400521E-02	1.089362E+00
6.837158E-05	1.051250E-03	1.408966E-02	8.505701E-02	-1.540491E-02	-7.635209E-03	-6.427388E-02	1.103383E+00
6.837158E-05	1.125625E-03	1.608823E-02	8.477390E-02	-1.490295E-02	-6.670363E-03	-7.801854E-02	1.115312E+00
6.837158E-05	1.200000E-03	1.772955E-02	8.450627E-02	-1.438510E-02	-5.874505E-03	-9.002516E-02	1.127971E+00
7.179015E-05	6.089650E-04	-2.777996E-04	8.370665E-02	-1.973284E-02	-1.550258E-02	1.579940E-04	1.007879E+00
7.179015E-05	6.020894E-04	-5.450491E-04	8.370442E-02	-1.981216E-02	-1.570100E-02	2.970571E-03	1.006598E+00
7.179015E-05	5.952138E-04	-9.142804E-04	8.370143E-02	-1.990353E-02	-1.588816E-02	3.456761E-03	1.005115E+00
7.179015E-05	5.883382E-04	-1.092339E-03	8.369868E-02	-1.999113E-02	-1.608318E-02	4.093620E-03	1.002678E+00
7.179015E-05	5.814626E-04	-1.270768E-03	8.369493E-02	-2.008587E-02	-1.627448E-02	6.219009E-03	1.000664E+00
7.179015E-05	5.745870E-04	-1.664764E-03	8.368894E-02	-2.018468E-02	-1.646701E-02	7.952035E-03	9.996026E-01
7.179015E-05	5.677114E-04	-2.266078E-03	8.368072E-02	-2.027865E-02	-1.667124E-02	7.936145E-03	9.990470E-01
7.179015E-05	5.608358E-04	-2.608932E-03	8.366969E-02	-2.031584E-02	-1.694841E-02	1.228571E-02	9.984546E-01
7.179015E-05	5.539602E-04	-2.479437E-03	8.367012E-02	-2.046747E-02	-1.708749E-02	1.208704E-02	9.940320E-01
7.179015E-05	5.470846E-04	-2.658089E-03	8.366356E-02	-2.055545E-02	-1.731139E-02	1.397912E-02	9.917836E-01
7.179015E-05	5.402089E-04	-2.383456E-03	8.366711E-02	-2.068110E-02	-1.749105E-02	1.296662E-02	9.862704E-01
7.179015E-05	5.333333E-04	-2.563814E-03	8.365903E-02	-2.077668E-02	-1.771785E-02	1.693733E-02	9.845905E-01
7.179015E-05	6.888606E-04	3.828014E-03	8.368817E-02	-1.884638E-02	-1.343309E-02	-2.053860E-03	1.025135E+00
7.179015E-05	7.618805E-04	7.022473E-03	8.355455E-02	-1.813779E-02	-1.177262E-02	-2.034049E-02	1.036745E+00
7.179015E-05	8.349004E-04	9.537738E-03	8.341206E-02	-1.731247E-02	-1.059636E-02	-2.874790E-02	1.052234E+00
7.179015E-05	9.079203E-04	1.150061E-02	8.327872E-02	-1.672681E-02	-9.290082E-03	-3.383599E-02	1.069605E+00
7.179015E-05	9.809402E-04	1.327355E-02	8.306555E-02	-1.595553E-02	-8.522722E-03	-4.870352E-02	1.083409E+00
7.179015E-05	1.053960E-03	1.510143E-02	8.279704E-02	-1.541908E-02	-7.509350E-03	-6.558647E-02	1.095646E+00
7.179015E-05	1.126980E-03	1.686446E-02	8.250159E-02	-1.489030E-02	-6.635205E-03	-8.179090E-02	1.107294E+00
7.179015E-05	1.200000E-03	1.849229E-02	8.221862E-02	-1.439414E-02	-5.826546E-03	-9.385608E-02	1.119652E+00
7.537966E-05	6.192403E-04	-3.157783E-05	8.161452E-02	-1.948655E-02	-1.536201E-02	-3.608848E-04	1.007850E+00
7.537966E-05	6.114306E-04	-9.428856E-05	8.161273E-02	-1.960723E-02	-1.553795E-02	2.724447E-04	1.004466E+00
7.537966E-05	6.036209E-04	-1.890433E-04	8.161091E-02	-1.972892E-02	-1.571808E-02	-4.655496E-04	1.000794E+00
7.537966E-05	5.958111E-04	-6.572384E-05	8.160874E-02	-1.984257E-02	-1.591224E-02	-1.506302E-03	9.958634E-01
7.537966E-05	5.880014E-04	-6.949506E-05	8.160671E-02	-1.997082E-02	-1.609585E-02	-1.137515E-03	9.919708E-01
7.537966E-05	5.801917E-04	1.693041E-04	8.160402E-02	-2.007109E-02	-1.631816E-02	-9.945113E-04	9.867102E-01
7.537966E-05	5.723820E-04	4.967696E-05	8.160230E-02	-2.017966E-02	-1.653936E-02	-1.887139E-04	9.834392E-01
7.537966E-05	5.645722E-04	-2.265618E-04	8.160053E-02	-2.030727E-02	-1.674473E-02	3.390339E-04	9.808503E-01
7.537966E-05	5.567625E-04	-2.300865E-04	8.159831E-02	-2.042520E-02	-1.696669E-02	1.394548E-03	9.769665E-01
7.537966E-05	5.489528E-04	-6.390860E-04	8.159566E-02	-2.052367E-02	-1.722157E-02	1.788776E-03	9.749119E-01
7.537966E-05	5.411431E-04	-2.669299E-04	8.159373E-02	-2.066759E-02	-1.742275E-02	1.760065E-03	9.686816E-01
7.537966E-05	5.333333E-04	-5.337113E-04	8.159110E-02	-2.080476E-02	-1.764486E-02	2.078218E-03	9.657799E-01
7.537966E-05	6.986688E-04	2.587026E-03	8.161085E-02	-1.850215E-02	-1.352979E-02	-3.257231E-03	1.031246E+00
7.537966E-05	7.702875E-04	3.837699E-03	8.158914E-02	-1.772790E-02	-1.209523E-02	-1.035841E-02	1.053976E+00
7.537966E-05	8.419063E-04	6.274393E-03	8.145540E-02	-1.706689E-02	-1.075759E-02	-3.650330E-02	1.063547E+00
7.537966E-05	9.135250E-04	7.265508E-03	8.141411E-02	-1.630043E-02	-9.844108E-03	-3.724942E-02	1.085352E+00
7.537966E-05	9.851438E-04	9.352486E-03	8.126164E-02	-1.579102E-02	-8.686953E-03	-4.789005E-02	1.098121E+00
7.537966E-05	1.056763E-03	1.075587E-02	8.109215E-02	-1.519937E-02	-7.844304E-03	-6.633150E-02	1.110598E+00
7.537966E-05	1.128381E-03	1.265146E-02	8.087851E-02	-1.469578E-02	-6.983478E-03	-7.738006E-02	1.121940E+00
7.537966E-05	1.200000E-03	1.401475E-02	8.063798E-02	-1.416195E-02	-6.320848E-03	-9.973432E-02	1.131667E+00
7.914864E-05	6.298667E-04	-1.840347E-04	7.957146E-02	-1.924470E-02	-1.521421E-02	-1.291644E-04	1.010157E+00
7.914864E-05	6.210910E-04	-2.701548E-04	7.956928E-02	-1.935424E-02	-1.543703E-02	2.263184E-03	1.006966E+00
7.914864E-05	6.123152E-04	-6.758838E-04	7.956615E-02	-1.946924E-02	-1.566163E-02	3.431826E-03	1.005071E+00

7.914864E-05	6.035394E-04	-7.189859E-04	7.956426E-02	-1.958930E-02	-1.588246E-02	1.591827E-03	1.000270E+00
7.914864E-05	5.947637E-04	-1.027779E-03	7.956004E-02	-1.969392E-02	-1.613334E-02	3.982175E-03	9.981184E-01
7.914864E-05	5.859879E-04	-1.460501E-03	7.955238E-02	-1.978388E-02	-1.641122E-02	9.205441E-03	9.974537E-01
7.914864E-05	5.772122E-04	-2.037169E-03	7.954092E-02	-1.989956E-02	-1.666580E-02	1.304176E-02	9.971343E-01
7.914864E-05	5.684364E-04	-2.335473E-03	7.953278E-02	-2.001967E-02	-1.691939E-02	1.456095E-02	9.945329E-01
7.914864E-05	5.596606E-04	-2.428328E-03	7.952839E-02	-2.011681E-02	-1.720543E-02	1.500891E-02	9.903973E-01
7.914864E-05	5.508849E-04	-2.716328E-03	7.951703E-02	-2.023535E-02	-1.747785E-02	1.933408E-02	9.884997E-01
7.914864E-05	5.421091E-04	-2.873977E-03	7.951177E-02	-2.039368E-02	-1.770939E-02	1.845507E-02	9.842049E-01
7.914864E-05	5.333333E-04	-3.246230E-03	7.949760E-02	-2.049812E-02	-1.801544E-02	2.158762E-02	9.822782E-01
7.914864E-05	7.088122E-04	8.797322E-04	7.958187E-02	-1.827195E-02	-1.346273E-02	-1.135683E-02	1.038359E+00
7.914864E-05	7.789819E-04	3.059701E-03	7.953920E-02	-1.755134E-02	-1.204042E-02	-3.126019E-02	1.051591E+00
7.914864E-05	8.491516E-04	4.386303E-03	7.949983E-02	-1.672545E-02	-1.106176E-02	-3.347857E-02	1.072250E+00
7.914864E-05	9.193213E-04	6.198571E-03	7.942892E-02	-1.617053E-02	-9.871137E-03	-3.450952E-02	1.089442E+00
7.914864E-05	9.894909E-04	8.015619E-03	7.931473E-02	-1.567669E-02	-8.755326E-03	-4.350184E-02	1.103064E+00
7.914864E-05	1.059661E-03	9.551028E-03	7.916936E-02	-1.506632E-02	-7.998188E-03	-5.806788E-02	1.115117E+00
7.914864E-05	1.129830E-03	1.167013E-02	7.896723E-02	-1.448838E-02	-7.349783E-03	-6.659231E-02	1.124983E+00
7.914864E-05	1.200000E-03	1.370423E-02	7.865785E-02	-1.408694E-02	-6.459408E-03	-9.035451E-02	1.130978E+00
8.310608E-05	6.408585E-04	-1.907468E-04	7.757561E-02	-1.898211E-02	-1.508679E-02	1.543033E-06	1.011739E+00
8.310608E-05	6.310835E-04	-2.017043E-04	7.757316E-02	-1.911823E-02	-1.530574E-02	3.638225E-05	1.006975E+00
8.310608E-05	6.213085E-04	-2.184448E-04	7.757068E-02	-1.925427E-02	-1.553274E-02	-3.229182E-04	1.002060E+00
8.310608E-05	6.115334E-04	-4.290118E-04	7.756817E-02	-1.942261E-02	-1.573084E-02	2.069910E-04	9.984307E-01
8.310608E-05	6.017584E-04	-2.880938E-04	7.756544E-02	-1.952791E-02	-1.601109E-02	5.673798E-04	9.927189E-01
8.310608E-05	5.919834E-04	1.292200E-04	7.756235E-02	-1.968284E-02	-1.623776E-02	1.170856E-03	9.855317E-01
8.310608E-05	5.822084E-04	8.139404E-04	7.755519E-02	-1.988306E-02	-1.641499E-02	-2.496396E-03	9.756027E-01
8.310608E-05	5.724334E-04	8.658548E-04	7.755174E-02	-2.002137E-02	-1.668376E-02	-2.769668E-03	9.700091E-01
8.310608E-05	5.626584E-04	8.773058E-04	7.754837E-02	-2.013376E-02	-1.699373E-02	-3.231585E-03	9.644887E-01
8.310608E-05	5.528834E-04	5.367240E-04	7.754728E-02	-2.034274E-02	-1.720182E-02	-4.478597E-03	9.605821E-01
8.310608E-05	5.431083E-04	4.370908E-04	7.754470E-02	-2.045032E-02	-1.753819E-02	-5.025804E-03	9.554666E-01
8.310608E-05	5.333333E-04	-3.769251E-04	7.754503E-02	-2.055530E-02	-1.789609E-02	-1.489827E-03	9.553276E-01
8.310608E-05	7.193043E-04	1.023468E-03	7.758873E-02	-1.802749E-02	-1.340446E-02	-1.219515E-03	1.041516E+00
8.310608E-05	7.879751E-04	2.182444E-03	7.758415E-02	-1.729664E-02	-1.210877E-02	-7.450833E-03	1.063017E+00
8.310608E-05	8.566459E-04	3.236791E-03	7.756574E-02	-1.657351E-02	-1.104973E-02	-1.491320E-02	1.082756E+00
8.310608E-05	9.253167E-04	4.582566E-03	7.752197E-02	-1.598266E-02	-9.993672E-03	-2.361340E-02	1.099091E+00
8.310608E-05	9.939876E-04	5.311616E-03	7.749455E-02	-1.537127E-02	-9.149804E-03	-2.821619E-02	1.118441E+00
8.310608E-05	1.062658E-03	7.371549E-03	7.737991E-02	-1.482438E-02	-8.348178E-03	-3.765164E-02	1.128255E+00
8.310608E-05	1.131329E-03	8.827718E-03	7.723817E-02	-1.438246E-02	-7.484345E-03	-5.552338E-02	1.138104E+00
8.310608E-05	1.200000E-03	1.038784E-02	7.707632E-02	-1.397438E-02	-6.663626E-03	-6.854287E-02	1.148071E+00
8.726138E-05	6.522303E-04	-3.193457E-04	7.562601E-02	-1.872520E-02	-1.494867E-02	6.592060E-04	1.014254E+00
8.726138E-05	6.414215E-04	-6.311398E-04	7.562237E-02	-1.884008E-02	-1.522860E-02	4.155435E-03	1.011768E+00
8.726138E-05	6.306126E-04	-6.932886E-04	7.561937E-02	-1.898945E-02	-1.547119E-02	3.995119E-03	1.006676E+00
8.726138E-05	6.198038E-04	-1.137698E-03	7.561423E-02	-1.911109E-02	-1.576098E-02	4.657458E-03	1.003938E+00
8.726138E-05	6.089950E-04	-1.648455E-03	7.560402E-02	-1.922450E-02	-1.607325E-02	1.064084E-02	1.003172E+00
8.726138E-05	5.981862E-04	-1.829054E-03	7.559902E-02	-1.941593E-02	-1.629743E-02	1.035262E-02	9.985012E-01
8.726138E-05	5.873774E-04	-2.348338E-03	7.558933E-02	-1.950248E-02	-1.665836E-02	1.011916E-02	9.956898E-01
8.726138E-05	5.765686E-04	-2.975564E-03	7.556855E-02	-1.956254E-02	-1.706448E-02	1.647583E-02	9.955226E-01
8.726138E-05	5.657598E-04	-3.307661E-03	7.555813E-02	-1.974891E-02	-1.732930E-02	1.654987E-02	9.916033E-01
8.726138E-05	5.549510E-04	-3.643388E-03	7.554574E-02	-2.001726E-02	-1.751866E-02	1.750883E-02	9.879220E-01
8.726138E-05	5.441421E-04	-4.418152E-03	7.552635E-02	-2.006106E-02	-1.797083E-02	1.578196E-02	9.858408E-01
8.726138E-05	5.333333E-04	-4.304711E-03	7.552677E-02	-2.015581E-02	-1.837194E-02	1.530560E-02	9.787329E-01
8.726138E-05	7.301592E-04	1.999144E-03	7.562446E-02	-1.787111E-02	-1.320922E-02	-6.425777E-03	1.035499E+00
8.726138E-05	7.972793E-04	3.113069E-03	7.561115E-02	-1.712577E-02	-1.202867E-02	-1.174490E-02	1.056599E+00
8.726138E-05	8.643994E-04	4.974030E-03	7.553557E-02	-1.655369E-02	-1.081365E-02	-2.826036E-02	1.068748E+00
8.726138E-05	9.315195E-04	7.362919E-03	7.542008E-02	-1.602462E-02	-9.713277E-03	-3.395702E-02	1.079964E+00
8.726138E-05	9.986397E-04	9.131406E-03	7.525003E-02	-1.545979E-02	-8.832295E-03	-5.330023E-02	1.089410E+00
8.726138E-05	1.065760E-03	1.084778E-02	7.506253E-02	-1.499521E-02	-7.916893E-03	-6.796634E-02	1.099639E+00
8.726138E-05	1.132880E-03	1.314455E-02	7.473971E-02	-1.457269E-02	-7.036552E-03	-9.031983E-02	1.104383E+00
8.726138E-05	1.200000E-03	1.465093E-02	7.448198E-02	-1.414881E-02	-6.250646E-03	-1.075844E-01	1.113627E+00
9.162445E-05	6.639977E-04	-2.599161E-04	7.372145E-02	-1.847081E-02	-1.480082E-02	5.501282E-04	1.015556E+00
9.162445E-05	6.521191E-04	-2.845232E-04	7.371835E-02	-1.861817E-02	-1.506685E-02	1.533424E-03	1.010179E+00
9.162445E-05	6.402405E-04	-4.394947E-04	7.371507E-02	-1.876219E-02	-1.534852E-02	1.868573E-03	1.005277E+00
9.162445E-05	6.283620E-04	-6.336675E-04	7.371137E-02	-1.891398E-02	-1.563240E-02	2.669400E-03	1.000661E+00
9.162445E-05	6.164834E-04	-1.076380E-03	7.370619E-02	-1.905253E-02	-1.594607E-02	3.611739E-03	9.974563E-01
9.162445E-05	6.046048E-04	-1.234734E-03	7.370183E-02	-1.920271E-02	-1.625476E-02	3.884087E-03	9.922705E-01
9.162445E-05	5.927262E-04	-1.604517E-03	7.369366E-02	-1.933682E-02	-1.659808E-02	7.169776E-03	9.891713E-01
9.162445E-05	5.808476E-04	-1.770508E-03	7.368726E-02	-1.949007E-02	-1.692915E-02	8.862124E-03	9.842665E-01
9.162445E-05	5.689691E-04	-1.660155E-03	7.368465E-02	-1.967622E-02	-1.723280E-02	8.930408E-03	9.771025E-01
9.162445E-05	5.570905E-04	-2.121720E-03	7.367422E-02	-1.981749E-02	-1.760812E-02	1.047675E-02	9.736827E-01
9.162445E-05	5.452119E-04	-3.560254E-03	7.365026E-02	-1.991476E-02	-1.805448E-02	8.886063E-03	9.750843E-01
9.162445E-05	5.333333E-04	-4.205600E-03	7.363318E-02	-1.995398E-02	-1.857330E-02	9.255337E-03	9.721914E-01
9.162445E-05	7.413917E-04	1.980972E-03	7.372476E-02	-1.762745E-02	-1.313718E-02	-2.544598E-03	1.037590E+00
9.162445E-05	8.069072E-04	3.488813E-03	7.368935E-02	-1.696266E-02	-1.193471E-02	-1.616398E-02	1.053063E+00
9.162445E-05	8.724227E-04	5.315413E-03	7.361561E-02	-1.650759E-02	-1.062773E-02	-2.794241E-02	1.065981E+00
9.162445E-05	9.379381E-04	6.778537E-03	7.350900E-02	-1.587491E-02	-9.754235E-03	-4.601597E-02	1.077575E+00
9.162445E-05	1.003454E-03	8.431810E-03	7.337382E-02	-1.534738E-02	-8.873025E-03	-5.851590E-02	1.088741E+00
9.162445E-05	1.068969E-03	1.008829E-02	7.320558E-02	-1.488400E-02	-8.015596E-03	-7.113023E-02	1.099003E+00
9.162445E-05	1.134485E-03	1.183558E-02	7.298682E-02	-1.446519E-02	-7.184005E-03	-8.580749E-02	1.107557E+00
9.162445E-05	1.200000E-03	1.315118E-02	7.276120E-02	-1.402626E-02	-6.494359E-03	-1.061980E-01	1.116000E+00
9.620567E-05	6.761771E-04	1.141076E-05	7.186072E-02	-1.821723E-02	-1.464599E-02	1.658825E-03	1.016010E+00
9.620567E-05	6.631913E-04	-3.060816E-04	7.185741E-02	-1.837707E-02	-1.492804E-02	1.309849E-03	1.011493E+00
9.620567E-05	6.502055E-04	-7.798337E-04	7.185314E-02	-1.850474E-02	-1.526344E-02	2.111589E-03	1.008167E+00
9.620567E-05	6.372197E-04	-3.465312E-04	7.185022E-02	-1.866764E-02	-1.555978E-02	1.689598E-03	9.988523E-01
9.620567E-05	6.242339E-04	-2.526051E-04	7.184648E-02	-1.886906E-02	-1.582671E-02	2.113768E-03	9.917802E-01

9.620567E-05	6.112481E-04	-1.335671E-04	7.184255E-02	-1.904507E-02	-1.613807E-02	1.926563E-03	9.842374E-01
9.620567E-05	5.982623E-04	-2.954539E-04	7.183856E-02	-1.922170E-02	-1.646544E-02	3.171965E-04	9.778237E-01
9.620567E-05	5.852765E-04	-6.117820E-05	7.183419E-02	-1.941679E-02	-1.678461E-02	1.711837E-03	9.698355E-01
9.620567E-05	5.722907E-04	-1.461864E-04	7.182991E-02	-1.955348E-02	-1.718983E-02	1.974648E-03	9.632481E-01
9.620567E-05	5.593049E-04	2.575478E-05	7.182531E-02	-1.978610E-02	-1.749955E-02	2.442178E-03	9.550929E-01
9.620567E-05	5.463191E-04	-2.100922E-04	7.182067E-02	-1.997007E-02	-1.788657E-02	2.173734E-03	9.489681E-01
9.620567E-05	5.333333E-04	-1.533427E-04	7.181573E-02	-2.015964E-02	-1.828335E-02	2.373319E-03	9.410884E-01
9.620567E-05	7.530175E-04	2.769777E-03	7.187002E-02	-1.742117E-02	-1.300441E-02	5.148796E-03	1.036181E+00
9.620567E-05	8.168722E-04	4.430881E-03	7.182696E-02	-1.682065E-02	-1.180532E-02	-6.239699E-03	1.050446E+00
9.620567E-05	8.807268E-04	5.790241E-03	7.171948E-02	-1.626227E-02	-1.073035E-02	-3.276602E-02	1.060632E+00
9.620567E-05	9.445814E-04	7.831437E-03	7.157350E-02	-1.580221E-02	-9.659638E-03	-4.741281E-02	1.069494E+00
9.620567E-05	1.008436E-03	9.145704E-03	7.141704E-02	-1.529833E-02	-8.796568E-03	-6.898521E-02	1.079338E+00
9.620567E-05	1.072291E-03	1.084611E-02	7.126939E-02	-1.484644E-02	-7.971131E-03	-7.017758E-02	1.091846E+00
9.620567E-05	1.136145E-03	1.173619E-02	7.115029E-02	-1.439245E-02	-7.257501E-03	-7.964284E-02	1.105416E+00
9.620567E-05	1.200000E-03	1.246738E-02	7.099962E-02	-1.396267E-02	-6.595738E-03	-9.846572E-02	1.116552E+00
1.010160E-04	6.887857E-04	-1.963739E-04	7.004303E-02	-1.794466E-02	-1.451312E-02	1.552809E-05	1.018574E+00
1.010160E-04	6.746537E-04	-2.678677E-04	7.003939E-02	-1.811070E-02	-1.481373E-02	-4.631157E-04	1.012012E+00
1.010160E-04	6.605217E-04	4.246927E-05	7.003471E-02	-1.830575E-02	-1.508895E-02	-1.964744E-03	1.002652E+00
1.010160E-04	6.463896E-04	1.451662E-05	7.003074E-02	-1.847594E-02	-1.541309E-02	-1.883900E-03	9.957509E-01
1.010160E-04	6.322576E-04	1.676939E-04	7.002591E-02	-1.867637E-02	-1.571475E-02	-1.655881E-03	9.876614E-01
1.010160E-04	6.181256E-04	1.093083E-03	7.001490E-02	-1.888514E-02	-1.601527E-02	-2.761003E-03	9.743299E-01
1.010160E-04	6.039935E-04	1.051335E-03	7.000953E-02	-1.909544E-02	-1.634010E-02	-4.311357E-03	9.666367E-01
1.010160E-04	5.898615E-04	7.543410E-04	7.000675E-02	-1.928219E-02	-1.671312E-02	-6.001908E-03	9.602750E-01
1.010160E-04	5.757294E-04	9.756376E-04	6.999880E-02	-1.947712E-02	-1.709071E-02	-7.164951E-03	9.507915E-01
1.010160E-04	5.615974E-04	1.310760E-03	6.998928E-02	-1.972899E-02	-1.742282E-02	-7.211263E-03	9.408346E-01
1.010160E-04	5.474654E-04	2.539139E-03	6.996153E-02	-2.010865E-02	-1.762473E-02	-8.986446E-03	9.250454E-01
1.010160E-04	5.333333E-04	2.466404E-03	6.995555E-02	-2.021868E-02	-1.816371E-02	-1.000341E-02	9.168256E-01
1.010160E-04	7.650531E-04	2.386065E-03	7.004098E-02	-1.718516E-02	-1.290885E-02	-2.432278E-03	1.037028E+00
1.010160E-04	8.271883E-04	5.676783E-03	6.991250E-02	-1.667937E-02	-1.165878E-02	-2.489739E-02	1.037487E+00
1.010160E-04	8.893236E-04	7.285986E-03	6.978748E-02	-1.624173E-02	-1.047875E-02	-4.308244E-02	1.048170E+00
1.010160E-04	9.514589E-04	9.666032E-03	6.951002E-02	-1.579149E-02	-9.449249E-03	-7.622585E-02	1.049708E+00
1.010160E-04	1.013594E-03	1.151018E-02	6.926259E-02	-1.532903E-02	-8.575615E-03	-9.561134E-02	1.057189E+00
1.010160E-04	1.075729E-03	1.301299E-02	6.902286E-02	-1.497229E-02	-7.607783E-03	-1.128241E-01	1.066510E+00
1.010160E-04	1.137865E-03	1.511639E-02	6.862576E-02	-1.461542E-02	-6.728262E-03	-1.377563E-01	1.070522E+00
1.010160E-04	1.200000E-03	1.676244E-02	6.826476E-02	-1.422669E-02	-6.001445E-03	-1.591206E-01	1.077112E+00
1.060668E-04	7.018418E-04	4.549051E-05	6.826647E-02	-1.770302E-02	-1.433166E-02	-5.050220E-03	1.017348E+00
1.060668E-04	6.865228E-04	1.916365E-04	6.826135E-02	-1.788500E-02	-1.463924E-02	-4.881732E-03	1.009077E+00
1.060668E-04	6.712039E-04	4.135410E-04	6.825551E-02	-1.809021E-02	-1.493380E-02	-4.399152E-03	1.000305E+00
1.060668E-04	6.558849E-04	7.646649E-04	6.824820E-02	-1.831692E-02	-1.521826E-02	-4.270594E-03	9.904849E-01
1.060668E-04	6.405660E-04	3.851136E-04	6.824627E-02	-1.848960E-02	-1.559168E-02	-5.414265E-03	9.847084E-01
1.060668E-04	6.252470E-04	3.321179E-04	6.824178E-02	-1.867550E-02	-1.596476E-02	-5.111531E-03	9.771579E-01
1.060668E-04	6.099281E-04	1.976059E-03	6.821830E-02	-1.890514E-02	-1.628972E-02	-5.121912E-03	9.588210E-01
1.060668E-04	5.946091E-04	2.232051E-03	6.820740E-02	-1.918951E-02	-1.658424E-02	-6.136749E-03	9.487158E-01
1.060668E-04	5.792902E-04	3.232684E-03	6.816813E-02	-1.948151E-02	-1.688280E-02	-1.370679E-02	9.321047E-01
1.060668E-04	5.639712E-04	4.175063E-03	6.812195E-02	-1.984436E-02	-1.712694E-02	-1.952074E-02	9.164007E-01
1.060668E-04	5.486523E-04	4.024694E-03	6.812220E-02	-1.999679E-02	-1.765094E-02	-1.865243E-02	9.086393E-01
1.060668E-04	5.333333E-04	3.023808E-03	6.817047E-02	-2.010170E-02	-1.826611E-02	-4.774030E-03	9.092827E-01
1.060668E-04	7.775156E-04	1.007456E-03	6.828191E-02	-1.688493E-02	-1.290545E-02	9.608078E-04	1.047729E+00
1.060668E-04	8.378705E-04	3.441358E-03	6.822296E-02	-1.643410E-02	-1.168221E-02	-2.243059E-02	1.051090E+00
1.060668E-04	8.982255E-04	4.261055E-03	6.819449E-02	-1.588324E-02	-1.076942E-02	-2.962607E-02	1.068111E+00
1.060668E-04	9.585804E-04	4.737835E-03	6.818153E-02	-1.534566E-02	-9.981256E-03	-3.190440E-02	1.087442E+00
1.060668E-04	1.018935E-03	5.091933E-03	6.816696E-02	-1.485013E-02	-9.251361E-03	-3.633864E-02	1.105829E+00
1.060668E-04	1.079290E-03	6.227833E-03	6.807509E-02	-1.443349E-02	-8.491559E-03	-5.602540E-02	1.114113E+00
1.060668E-04	1.139645E-03	6.206711E-03	6.808683E-02	-1.392791E-02	-7.989905E-03	-5.505898E-02	1.134411E+00
1.060668E-04	1.200000E-03	7.357497E-03	6.799746E-02	-1.355023E-02	-7.346524E-03	-6.293665E-02	1.144524E+00
1.113701E-04	7.153644E-04	1.550175E-04	6.653087E-02	-1.742938E-02	-1.419097E-02	-2.496947E-04	1.020067E+00
1.113701E-04	6.988161E-04	-5.590162E-05	6.652686E-02	-1.760961E-02	-1.453106E-02	-7.627338E-04	1.013360E+00
1.113701E-04	6.822678E-04	-9.120118E-05	6.652222E-02	-1.780308E-02	-1.487121E-02	-3.151597E-04	1.005634E+00
1.113701E-04	6.657196E-04	-3.631183E-04	6.651767E-02	-1.797581E-02	-1.525919E-02	5.173257E-03	1.000898E+00
1.113701E-04	6.491713E-04	-2.823433E-04	6.651243E-02	-1.819218E-02	-1.560713E-02	2.145797E-03	9.909627E-01
1.113701E-04	6.326230E-04	-9.683816E-04	6.650438E-02	-1.833412E-02	-1.607356E-02	7.326542E-03	9.885257E-01
1.113701E-04	6.160747E-04	-1.345925E-03	6.649577E-02	-1.852180E-02	-1.650249E-02	8.057570E-03	9.824553E-01
1.113701E-04	5.995264E-04	-1.645149E-03	6.648668E-02	-1.871349E-02	-1.694844E-02	9.027931E-03	9.757294E-01
1.113701E-04	5.829782E-04	-2.255041E-03	6.646675E-02	-1.886634E-02	-1.746740E-02	1.722096E-02	9.732416E-01
1.113701E-04	5.664299E-04	-2.998721E-03	6.643591E-02	-1.895231E-02	-1.808444E-02	2.772204E-02	9.722786E-01
1.113701E-04	5.498816E-04	-3.425174E-03	6.641474E-02	-1.923913E-02	-1.850469E-02	2.986484E-02	9.662294E-01
1.113701E-04	5.333333E-04	-4.241523E-03	6.636920E-02	-1.936696E-02	-1.912822E-02	3.885287E-02	9.649802E-01
1.113701E-04	7.904236E-04	1.358539E-03	6.654267E-02	-1.667340E-02	-1.276273E-02	-5.715092E-04	1.046248E+00
1.113701E-04	8.489345E-04	2.414385E-03	6.652890E-02	-1.615122E-02	-1.174772E-02	-1.091865E-02	1.061024E+00
1.113701E-04	9.074454E-04	3.295404E-03	6.650830E-02	-1.566218E-02	-1.083030E-02	-1.872553E-02	1.076488E+00
1.113701E-04	9.659563E-04	3.834569E-03	6.649578E-02	-1.514591E-02	-1.008118E-02	-2.267025E-02	1.094141E+00
1.113701E-04	1.024467E-03	4.809025E-03	6.643699E-02	-1.471255E-02	-9.311286E-03	-3.956567E-02	1.104163E+00
1.113701E-04	1.082978E-03	5.078784E-03	6.643522E-02	-1.424112E-02	-8.709004E-03	-3.812860E-02	1.123249E+00
1.113701E-04	1.141489E-03	6.205055E-03	6.635495E-02	-1.387448E-02	-8.014162E-03	-5.094906E-02	1.131983E+00
1.113701E-04	1.200000E-03	7.223543E-03	6.625469E-02	-1.350479E-02	-7.399337E-03	-6.863132E-02	1.139302E+00
1.169386E-04	7.293738E-04	-2.399749E-04	6.483615E-02	-1.713002E-02	-1.408087E-02	1.733278E-03	1.025434E+00
1.169386E-04	7.115520E-04	-2.844259E-04	6.483121E-02	-1.732741E-02	-1.442535E-02	2.519996E-03	1.017474E+00
1.169386E-04	6.937301E-04	-6.998440E-04	6.482542E-02	-1.751212E-02	-1.480789E-02	2.880599E-03	1.011764E+00
1.169386E-04	6.759083E-04	-9.996676E-04	6.481879E-02	-1.769392E-02	-1.521266E-02	3.733749E-03	1.005218E+00
1.169386E-04	6.580864E-04	-1.320176E-03	6.481050E-02	-1.788310E-02	-1.563052E-02	5.702905E-03	9.989843E-01
1.169386E-04	6.402645E-04	-2.083927E-03	6.479244E-02	-1.804385E-02	-1.610961E-02	1.180305E-02	9.970220E-01
1.169386E-04	6.224427E-04	-2.377105E-03	6.477932E-02	-1.823564E-02	-1.656976E-02	1.471852E-02	9.904991E-01

1.169386E-04	6.046208E-04	-2.621697E-03	6.476787E-02	-1.844859E-02	-1.702995E-02	1.546512E-02	9.826847E-01
1.169386E-04	5.867989E-04	-3.607777E-03	6.472072E-02	-1.856791E-02	-1.763271E-02	2.983680E-02	9.845488E-01
1.169386E-04	5.689771E-04	-3.850782E-03	6.470598E-02	-1.879791E-02	-1.812826E-02	2.963984E-02	9.759052E-01
1.169386E-04	5.511552E-04	-5.447640E-03	6.465008E-02	-1.896294E-02	-1.873566E-02	2.490048E-02	9.754949E-01
1.169386E-04	5.333333E-04	-5.706325E-03	6.461690E-02	-1.916856E-02	-1.932248E-02	3.106312E-02	9.685887E-01
1.169386E-04	8.037962E-04	9.299937E-04	6.484950E-02	-1.640678E-02	-1.269357E-02	-1.451679E-03	1.050188E+00
1.169386E-04	8.603968E-04	1.750989E-03	6.484781E-02	-1.590332E-02	-1.175102E-02	-7.750823E-03	1.066647E+00
1.169386E-04	9.169973E-04	3.149028E-03	6.481248E-02	-1.547620E-02	-1.082233E-02	-2.092514E-02	1.075978E+00
1.169386E-04	9.735979E-04	3.970117E-03	6.478994E-02	-1.499365E-02	-1.009240E-02	-2.474376E-02	1.090991E+00
1.169386E-04	1.030198E-03	5.178594E-03	6.471260E-02	-1.462428E-02	-9.284173E-03	-4.327688E-02	1.098250E+00
1.169386E-04	1.086799E-03	5.359453E-03	6.470479E-02	-1.418023E-02	-8.690762E-03	-4.733801E-02	1.115618E+00
1.169386E-04	1.143399E-03	5.461769E-03	6.470495E-02	-1.374761E-02	-8.163489E-03	-4.877305E-02	1.133534E+00
1.169386E-04	1.200000E-03	5.578093E-03	6.470492E-02	-1.331932E-02	-7.705835E-03	-4.935318E-02	1.150922E+00
1.227855E-04	7.438916E-04	8.115501E-05	6.317895E-02	-1.687698E-02	-1.390154E-02	1.874368E-03	1.025417E+00
1.227855E-04	7.247500E-04	7.108238E-05	6.317337E-02	-1.708439E-02	-1.425900E-02	1.152939E-03	1.016159E+00
1.227855E-04	7.056083E-04	-2.043665E-04	6.316816E-02	-1.727983E-02	-1.465527E-02	1.563358E-03	1.008942E+00
1.227855E-04	6.864666E-04	8.515786E-04	6.315830E-02	-1.756169E-02	-1.495726E-02	9.585725E-04	9.918394E-01
1.227855E-04	6.673250E-04	1.359418E-03	6.314541E-02	-1.781764E-02	-1.531979E-02	-1.615140E-03	9.778329E-01
1.227855E-04	6.481833E-04	6.544413E-05	6.314821E-02	-1.797165E-02	-1.584581E-02	-3.522377E-03	9.762055E-01
1.227855E-04	6.290416E-04	6.758585E-04	6.313666E-02	-1.825072E-02	-1.623537E-02	-3.524126E-03	9.617672E-01
1.227855E-04	6.099000E-04	6.316411E-04	6.312987E-02	-1.847074E-02	-1.672831E-02	-2.908270E-03	9.517620E-01
1.227855E-04	5.907583E-04	1.895539E-04	6.312514E-02	-1.868029E-02	-1.726667E-02	-3.922493E-03	9.436862E-01
1.227855E-04	5.716167E-04	1.072509E-04	6.311745E-02	-1.892854E-02	-1.779204E-02	-3.533210E-03	9.332595E-01
1.227855E-04	5.524750E-04	-4.487581E-05	6.310963E-02	-1.920185E-02	-1.832749E-02	-3.507563E-03	9.228615E-01
1.227855E-04	5.333333E-04	8.024815E-05	6.309938E-02	-1.944237E-02	-1.893134E-02	-3.799441E-03	9.100703E-01
1.227855E-04	8.176541E-04	1.071652E-03	6.319395E-02	-1.617075E-02	-1.257541E-02	1.499631E-03	1.051624E+00
1.227855E-04	8.722750E-04	1.852848E-03	6.319073E-02	-1.567848E-02	-1.170907E-02	-5.944190E-03	1.066874E+00
1.227855E-04	9.268958E-04	1.771862E-03	6.320285E-02	-1.517842E-02	-1.098081E-02	-6.272299E-03	1.089354E+00
1.227855E-04	9.815166E-04	3.349912E-03	6.316744E-02	-1.479778E-02	-1.017017E-02	-1.541567E-02	1.096460E+00
1.227855E-04	1.036137E-03	3.735395E-03	6.316111E-02	-1.438179E-02	-9.509883E-03	-1.800310E-02	1.113109E+00
1.227855E-04	1.090758E-03	4.697115E-03	6.309719E-02	-1.405004E-02	-8.790943E-03	-3.710786E-02	1.119959E+00
1.227855E-04	1.145379E-03	4.415277E-03	6.311947E-02	-1.359392E-02	-8.354757E-03	-3.688737E-02	1.140436E+00
1.227855E-04	1.200000E-03	4.720591E-03	6.311006E-02	-1.318243E-02	-7.915074E-03	-3.841658E-02	1.155729E+00
1.289248E-04	7.589405E-04	-7.688794E-04	6.155868E-02	-1.654982E-02	-1.382151E-02	5.111887E-03	1.035046E+00
1.289248E-04	7.384307E-04	-1.204215E-03	6.155008E-02	-1.673726E-02	-1.423258E-02	6.163846E-03	1.028932E+00
1.289248E-04	7.179210E-04	-1.848631E-03	6.153424E-02	-1.690864E-02	-1.468917E-02	1.172139E-02	1.025699E+00
1.289248E-04	6.974113E-04	-2.092754E-03	6.152407E-02	-1.711491E-02	-1.512177E-02	1.228845E-02	1.017528E+00
1.289248E-04	6.769015E-04	-2.676891E-03	6.149868E-02	-1.732262E-02	-1.558722E-02	2.133991E-02	1.014615E+00
1.289248E-04	6.563918E-04	-4.224390E-03	6.145641E-02	-1.745132E-02	-1.617034E-02	1.768754E-02	1.014699E+00
1.289248E-04	6.358820E-04	-5.228733E-03	6.139061E-02	-1.762990E-02	-1.672759E-02	2.863765E-02	1.015517E+00
1.289248E-04	6.153723E-04	-5.804538E-03	6.135611E-02	-1.781475E-02	-1.729667E-02	2.980230E-02	1.009177E+00
1.289248E-04	5.948626E-04	-6.199086E-03	6.132173E-02	-1.803252E-02	-1.786289E-02	3.299957E-02	1.001781E+00
1.289248E-04	5.743528E-04	-6.724979E-03	6.125665E-02	-1.826743E-02	-1.845379E-02	4.368192E-02	9.978938E-01
1.289248E-04	5.538431E-04	-7.427102E-03	6.115908E-02	-1.843978E-02	-1.914993E-02	6.051848E-02	9.975197E-01
1.289248E-04	5.333333E-04	-6.714762E-03	6.122512E-02	-1.870553E-02	-1.976046E-02	4.963767E-02	9.743723E-01
1.289248E-04	8.320189E-04	7.018039E-04	6.157469E-02	-1.591750E-02	-1.247616E-02	-1.676828E-03	1.054872E+00
1.289248E-04	8.845877E-04	1.908902E-03	6.156430E-02	-1.550111E-02	-1.159417E-02	-1.130366E-02	1.065223E+00
1.289248E-04	9.371564E-04	2.763316E-03	6.154646E-02	-1.509423E-02	-1.080339E-02	-1.963299E-02	1.077626E+00
1.289248E-04	9.897251E-04	3.563895E-03	6.151882E-02	-1.470118E-02	-1.008296E-02	-2.860814E-02	1.089358E+00
1.289248E-04	1.042294E-03	4.435348E-03	6.147904E-02	-1.432830E-02	-9.413207E-03	-3.669412E-02	1.100119E+00
1.289248E-04	1.094863E-03	5.382231E-03	6.141262E-02	-1.398487E-02	-8.768069E-03	-5.110109E-02	1.107786E+00
1.289248E-04	1.147431E-03	5.801872E-03	6.139036E-02	-1.360651E-02	-8.251393E-03	-5.342097E-02	1.122068E+00
1.289248E-04	1.200000E-03	6.064233E-03	6.138007E-02	-1.321221E-02	-7.827070E-03	-5.390264E-02	1.137315E+00
1.353710E-04	7.745446E-04	-1.167120E-04	5.997743E-02	-1.632835E-02	-1.358977E-02	1.017564E-03	1.031295E+00
1.353710E-04	7.526163E-04	-7.408998E-04	5.997001E-02	-1.649036E-02	-1.406015E-02	5.924981E-03	1.027360E+00
1.353710E-04	7.306880E-04	-6.180536E-04	5.996403E-02	-1.672403E-02	-1.445866E-02	4.694839E-03	1.015406E+00
1.353710E-04	7.087597E-04	-1.149102E-03	5.995427E-02	-1.692449E-02	-1.493104E-02	5.902011E-03	1.009029E+00
1.353710E-04	6.868314E-04	-1.530050E-03	5.994237E-02	-1.713559E-02	-1.541814E-02	8.980740E-03	1.001839E+00
1.353710E-04	6.649031E-04	-1.797232E-03	5.993091E-02	-1.735728E-02	-1.592086E-02	1.034702E-02	9.928832E-01
1.353710E-04	6.429748E-04	-2.082104E-03	5.991748E-02	-1.757752E-02	-1.645760E-02	1.242012E-02	9.839669E-01
1.353710E-04	6.210465E-04	-2.907037E-03	5.988244E-02	-1.775198E-02	-1.708862E-02	2.381391E-02	9.821537E-01
1.353710E-04	5.991182E-04	-3.232753E-03	5.986410E-02	-1.798685E-02	-1.767965E-02	2.435852E-02	9.723274E-01
1.353710E-04	5.771899E-04	-4.099590E-03	5.980679E-02	-1.817455E-02	-1.837419E-02	4.047779E-02	9.719519E-01
1.353710E-04	5.552616E-04	-5.025941E-03	5.974089E-02	-1.836442E-02	-1.910452E-02	4.994070E-02	9.695157E-01
1.353710E-04	5.333333E-04	-6.144254E-03	5.964736E-02	-1.858113E-02	-1.985817E-02	6.066298E-02	9.689537E-01
1.353710E-04	8.469138E-04	2.274122E-03	5.997588E-02	-1.573143E-02	-1.226721E-02	-1.364021E-03	1.044917E+00
1.353710E-04	8.973547E-04	3.008590E-03	5.996247E-02	-1.533765E-02	-1.144820E-02	-8.300913E-03	1.058535E+00
1.353710E-04	9.477955E-04	3.601153E-03	5.993995E-02	-1.495171E-02	-1.070783E-02	-1.779715E-02	1.071484E+00
1.353710E-04	9.982364E-04	4.333624E-03	5.990515E-02	-1.457960E-02	-1.002704E-02	-2.735042E-02	1.082603E+00
1.353710E-04	1.048677E-03	5.202574E-03	5.984339E-02	-1.423311E-02	-9.378620E-03	-4.211454E-02	1.090474E+00
1.353710E-04	1.099118E-03	6.361390E-03	5.977594E-02	-1.392992E-02	-8.728814E-03	-4.532666E-02	1.099318E+00
1.353710E-04	1.149559E-03	6.081783E-03	5.979110E-02	-1.354585E-02	-8.275402E-03	-5.010787E-02	1.117025E+00
1.353710E-04	1.200000E-03	6.199120E-03	5.978688E-02	-1.318497E-02	-7.841142E-03	-5.214216E-02	1.132250E+00
1.421396E-04	7.907297E-04	-1.587963E-03	5.842368E-02	-1.594354E-02	-1.357770E-02	6.292430E-03	1.047457E+00
1.421396E-04	7.673300E-04	-2.220275E-03	5.840661E-02	-1.614349E-02	-1.402478E-02	9.706774E-03	1.042810E+00
1.421396E-04	7.439304E-04	-2.403079E-03	5.839311E-02	-1.635727E-02	-1.447670E-02	1.329239E-02	1.034233E+00
1.421396E-04	7.205307E-04	-2.686648E-03	5.837738E-02	-1.655988E-02	-1.496999E-02	1.616257E-02	1.025937E+00
1.421396E-04	6.971310E-04	-2.949763E-03	5.836056E-02	-1.678621E-02	-1.546680E-02	1.909423E-02	1.017186E+00
1.421396E-04	6.737314E-04	-3.600194E-03	5.832227E-02	-1.701773E-02	-1.600020E-02	2.879162E-02	1.013763E+00
1.421396E-04	6.503317E-04	-4.500005E-03	5.826299E-02	-1.714065E-02	-1.668491E-02	4.038487E-02	1.012879E+00
1.421396E-04	6.269320E-04	-5.445212E-03	5.818373E-02	-1.728906E-02	-1.737923E-02	5.471090E-02	1.013267E+00
1.421396E-04	6.035323E-04	-6.072851E-03	5.813543E-02	-1.755742E-02	-1.797907E-02	5.477565E-02	1.005294E+00

1.421396E-04	5.801327E-04	-6.970817E-03	5.806933E-02	-1.779161E-02	-1.866050E-02	5.371617E-02	9.989040E-01
1.421396E-04	5.567330E-04	-7.741218E-03	5.796017E-02	-1.803243E-02	-1.939551E-02	7.052002E-02	9.977951E-01
1.421396E-04	5.333333E-04	-8.289403E-03	5.790522E-02	-1.813214E-02	-2.023635E-02	6.996547E-02	9.875617E-01
1.421396E-04	8.623632E-04	3.798244E-04	5.844788E-02	-1.539353E-02	-1.228201E-02	-6.197532E-04	1.061360E+00
1.421396E-04	9.105970E-04	1.204452E-03	5.845230E-02	-1.498799E-02	-1.157156E-02	-2.269957E-03	1.074612E+00
1.421396E-04	9.588309E-04	2.063747E-03	5.844613E-02	-1.468148E-02	-1.080732E-02	-7.205316E-03	1.085824E+00
1.421396E-04	1.007065E-03	1.878444E-03	5.845710E-02	-1.425413E-02	-1.028775E-02	-9.053176E-03	1.105349E+00
1.421396E-04	1.055299E-03	2.697588E-03	5.843657E-02	-1.392655E-02	-9.683869E-03	-1.885385E-02	1.113780E+00
1.421396E-04	1.103532E-03	3.519693E-03	5.840896E-02	-1.361877E-02	-9.109640E-03	-2.589336E-02	1.122564E+00
1.421396E-04	1.151766E-03	4.428154E-03	5.835690E-02	-1.335807E-02	-8.512634E-03	-3.926766E-02	1.128239E+00
1.421396E-04	1.200000E-03	4.673031E-03	5.835536E-02	-1.297253E-02	-8.168503E-03	-3.746714E-02	1.143050E+00
1.492466E-04	8.075233E-04	8.087823E-05	5.691848E-02	-1.574714E-02	-1.329460E-02	3.597175E-04	1.034678E+00
1.492466E-04	7.825969E-04	1.034617E-03	5.690437E-02	-1.606226E-02	-1.360881E-02	-8.446258E-04	1.015006E+00
1.492466E-04	7.576706E-04	-5.782119E-05	5.690339E-02	-1.623993E-02	-1.413917E-02	-2.681712E-03	1.011266E+00
1.492466E-04	7.327442E-04	-4.157066E-04	5.689650E-02	-1.647462E-02	-1.462568E-02	-2.354996E-03	1.002016E+00
1.492466E-04	7.078178E-04	4.083970E-04	5.688613E-02	-1.676524E-02	-1.507085E-02	4.052949E-03	9.848495E-01
1.492466E-04	6.828915E-04	-3.382028E-04	5.687790E-02	-1.697651E-02	-1.565894E-02	6.321077E-03	9.785783E-01
1.492466E-04	6.579651E-04	-2.773519E-04	5.686764E-02	-1.724577E-02	-1.620925E-02	5.029465E-03	9.642022E-01
1.492466E-04	6.330388E-04	-1.002178E-03	5.685433E-02	-1.745791E-02	-1.687530E-02	8.553214E-03	9.572954E-01
1.492466E-04	6.081124E-04	-4.530377E-04	5.684503E-02	-1.778367E-02	-1.744694E-02	5.591964E-03	9.374400E-01
1.492466E-04	5.831861E-04	-1.124174E-03	5.682907E-02	-1.799980E-02	-1.820307E-02	1.189761E-02	9.299062E-01
1.492466E-04	5.582597E-04	-1.353980E-03	5.681378E-02	-1.828232E-02	-1.893558E-02	1.097907E-02	9.158235E-01
1.492466E-04	5.333333E-04	-1.818822E-03	5.679318E-02	-1.851728E-02	-1.977832E-02	1.344656E-02	9.040764E-01
1.492466E-04	8.783934E-04	7.854632E-04	5.693364E-02	-1.515878E-02	-1.213575E-02	-6.334580E-04	1.060104E+00
1.492466E-04	9.243372E-04	1.344166E-03	5.693679E-02	-1.480249E-02	-1.144955E-02	-4.152125E-03	1.073718E+00
1.492466E-04	9.702810E-04	2.123003E-03	5.692303E-02	-1.447267E-02	-1.079164E-02	-1.513942E-02	1.082361E+00
1.492466E-04	1.016225E-03	3.398606E-03	5.688591E-02	-1.419836E-02	-1.011520E-02	-2.265566E-02	1.087681E+00
1.492466E-04	1.062169E-03	4.237716E-03	5.683698E-02	-1.389842E-02	-9.530299E-03	-3.719390E-02	1.093675E+00
1.492466E-04	1.108112E-03	4.794047E-03	5.681575E-02	-1.359623E-02	-9.006274E-03	-3.821970E-02	1.105632E+00
1.492466E-04	1.154056E-03	5.385918E-03	5.677960E-02	-1.333297E-02	-8.468229E-03	-4.387127E-02	1.115422E+00
1.492466E-04	1.200000E-03	6.248120E-03	5.670370E-02	-1.304929E-02	-8.000775E-03	-5.639570E-02	1.120486E+00

Table 34. *f*18p2 low energy transfers

Inner Radius	Outer Radius	Delta V1x	Delta V1y	Delta V2x	Delta V2y	Initial *f*	Time of flight
5.101992E-05	7.896343E-04	8.178547E-03	9.950950E-02	-2.065425E-02	2.776086E-03	-3.705458E-02	1.610921E+00
5.101992E-05	7.255590E-04	1.062893E-02	9.931925E-02	-2.197532E-02	1.506302E-03	-4.380618E-02	1.541823E+00
5.101992E-05	6.614838E-04	1.410179E-02	9.895767E-02	-2.338810E-02	3.610445E-04	-5.958473E-02	1.475161E+00
5.101992E-05	5.974086E-04	1.572924E-02	9.860425E-02	-2.494917E-02	-8.105753E-04	-9.037046E-02	1.408573E+00
5.101992E-05	5.333333E-04	1.572854E-02	9.857158E-02	-2.665189E-02	-2.787784E-03	-8.936743E-02	1.343202E+00
5.101992E-05	9.883386E-04	-3.460164E-03	9.982224E-02	-1.682296E-02	6.277149E-03	5.417953E-03	1.887708E+00
5.101992E-05	1.058892E-03	-4.387067E-03	9.983292E-02	-2.072832E-02	4.335995E-03	5.177611E-03	1.995095E+00
5.101992E-05	1.129446E-03	-4.599004E-03	9.983606E-02	-2.397573E-02	4.474533E-03	3.186506E-03	1.995388E+00
5.101992E-05	1.200000E-03	-8.244149E-03	9.975435E-02	-2.531553E-02	4.761006E-03	4.254062E-03	2.004877E+00
5.357092E-05	8.636725E-04	3.488967E-03	9.728535E-02	-1.934195E-02	3.153000E-03	-1.046405E-02	1.681649E+00
5.357092E-05	7.976047E-04	5.500129E-03	9.718831E-02	-2.057481E-02	1.909790E-03	-2.053701E-02	1.608059E+00
5.357092E-05	7.315368E-04	8.128379E-03	9.700095E-02	-2.186352E-02	8.332084E-04	-3.901768E-02	1.539145E+00
5.357092E-05	6.654690E-04	1.124294E-02	9.670806E-02	-2.327206E-02	-1.330092E-04	-5.778617E-02	1.471475E+00
5.357092E-05	5.994012E-04	9.832063E-03	9.681661E-02	-2.476062E-02	-2.407571E-03	-4.586012E-02	1.409817E+00
5.357092E-05	5.333333E-04	7.703433E-03	9.700491E-02	-2.631958E-02	-4.951068E-03	-1.637356E-02	1.354655E+00
5.357092E-05	9.973052E-04	-4.476108E-03	9.735545E-02	-1.688177E-02	5.672878E-03	-1.679828E-03	1.874433E+00
5.357092E-05	1.064870E-03	-2.249327E-02	9.850721E-02	-1.530528E-02	7.660702E-03	-9.569675E-02	2.077123E+00
5.357092E-05	1.132435E-03	-2.261517E-02	9.861747E-02	-2.076385E-02	4.400295E-03	-1.004461E-01	2.079525E+00
5.357092E-05	1.200000E-03	-2.164908E-02	9.863963E-02	-2.354480E-02	4.794420E-03	-1.016864E-01	2.070611E+00
5.624946E-05	8.739484E-04	-1.270625E-03	9.490361E-02	-1.927258E-02	2.226866E-03	4.504486E-03	1.681667E+00
5.624946E-05	8.058254E-04	-4.009977E-03	9.481676E-02	-2.048589E-02	2.614457E-04	1.858158E-02	1.620480E+00
5.624946E-05	7.377024E-04	-5.569699E-03	9.474181E-02	-2.169957E-02	-1.309683E-03	2.097167E-02	1.564859E+00
5.624946E-05	6.695794E-04	-8.449192E-03	9.452638E-02	-2.291935E-02	-3.499463E-03	4.388508E-02	1.520734E+00
5.624946E-05	6.014564E-04	-1.092080E-02	9.437158E-02	-2.424949E-02	-5.498858E-03	3.961187E-02	1.471529E+00
5.624946E-05	5.333333E-04	-1.209921E-02	9.420305E-02	-2.581836E-02	-7.402153E-03	5.146056E-02	1.420663E+00
5.624946E-05	1.006554E-03	-3.399458E-03	9.494338E-02	-1.673263E-02	5.685707E-03	3.393978E-03	1.871107E+00
5.624946E-05	1.071036E-03	-5.859879E-03	9.493881E-02	-1.869587E-02	4.500550E-03	1.593674E-03	1.996766E+00
5.624946E-05	1.135518E-03	-5.311857E-03	9.495223E-02	-2.237033E-02	3.922154E-03	8.982702E-04	1.995053E+00
5.624946E-05	1.200000E-03	-7.944007E-03	9.488256E-02	-2.401783E-02	3.835494E-03	3.368046E-03	1.993632E+00
5.906194E-05	8.845488E-04	4.834396E-03	9.248142E-02	-1.895261E-02	3.360671E-03	-1.444545E-02	1.683792E+00
5.906194E-05	8.143057E-04	9.861651E-03	9.214179E-02	-2.015647E-02	2.910225E-03	-4.341622E-02	1.608933E+00
5.906194E-05	7.440626E-04	1.397204E-02	9.167699E-02	-2.152220E-02	2.152516E-03	-6.826122E-02	1.536023E+00
5.906194E-05	6.738195E-04	1.707487E-02	9.107998E-02	-2.303953E-02	1.428060E-03	-1.096914E-01	1.464498E+00
5.906194E-05	6.035764E-04	1.941540E-02	9.063221E-02	-2.476358E-02	4.725899E-05	-1.240600E-01	1.390299E+00
5.906194E-05	5.333333E-04	1.907396E-02	9.065496E-02	-2.663273E-02	-2.473499E-03	-1.204069E-01	1.315318E+00
5.906194E-05	1.016094E-03	-3.746546E-03	9.257405E-02	-1.667103E-02	5.393030E-03	6.348351E-03	1.860947E+00
5.906194E-05	1.077396E-03	-4.513384E-03	9.261006E-02	-1.822268E-02	5.135110E-03	2.413837E-03	2.018902E+00
5.906194E-05	1.138698E-03	-4.981912E-03	9.260533E-02	-2.156997E-02	4.334027E-03	1.588023E-03	2.016862E+00

5.906194E-05	1.200000E-03	-6.928587E-03	9.256262E-02	-2.331978E-02	4.409274E-03	2.454926E-03	2.019119E+00
6.201503E-05	8.954854E-04	-1.019834E-03	9.025317E-02	-1.890931E-02	2.213393E-03	-9.264165E-04	1.682092E+00
6.201503E-05	8.230550E-04	-3.195546E-03	9.018977E-02	-2.015367E-02	2.645739E-04	9.241407E-03	1.616204E+00
6.201503E-05	7.506246E-04	-4.316671E-03	9.012910E-02	-2.139107E-02	-1.532335E-03	1.416761E-02	1.556692E+00
6.201503E-05	6.781942E-04	-6.994502E-03	8.998513E-02	-2.268943E-02	-3.441240E-03	2.524868E-02	1.506602E+00
6.201503E-05	6.057638E-04	-1.203427E-02	8.952482E-02	-2.398751E-02	-6.024807E-03	5.807758E-02	1.473168E+00
6.201503E-05	5.333333E-04	-1.567267E-02	8.910678E-02	-2.544489E-02	-8.570011E-03	6.922953E-02	1.430579E+00
6.201503E-05	1.025937E-03	-3.142940E-03	9.029552E-02	-1.653703E-02	5.359274E-03	6.143702E-04	1.859273E+00
6.201503E-05	1.083958E-03	-8.734708E-03	9.031198E-02	-1.519635E-02	7.184136E-03	-1.310733E-02	2.013352E+00
6.201503E-05	1.141979E-03	-1.389594E-02	9.031247E-02	-1.858405E-02	3.999760E-03	-2.524615E-02	2.043226E+00
6.201503E-05	1.200000E-03	-1.145402E-02	9.044392E-02	-2.187048E-02	4.442855E-03	-3.244165E-02	2.053749E+00
6.511579E-05	9.067710E-04	3.253546E-03	8.796615E-02	-1.862973E-02	3.012238E-03	-1.585135E-02	1.682743E+00
6.511579E-05	8.320834E-04	5.638626E-03	8.785732E-02	-1.992061E-02	1.788844E-03	-2.115740E-02	1.602644E+00
6.511579E-05	7.573959E-04	8.837440E-03	8.759231E-02	-2.127655E-02	7.582143E-04	-4.741804E-02	1.527149E+00
6.511579E-05	6.827084E-04	4.780581E-03	8.788324E-02	-2.268935E-02	-2.247497E-03	3.265574E-03	1.467392E+00
6.511579E-05	6.080209E-04	2.921584E-03	8.786303E-02	-2.420393E-02	-4.302068E-03	2.278499E-03	1.406364E+00
6.511579E-05	5.333333E-04	3.022126E-03	8.781305E-02	-2.587985E-02	-6.640484E-03	-5.709157E-04	1.338684E+00
6.511579E-05	1.036094E-03	-4.692480E-03	8.803178E-02	-1.651388E-02	4.890117E-03	-1.510312E-03	1.847590E+00
6.511579E-05	1.090729E-03	-6.822403E-03	8.800477E-02	-1.528893E-02	6.657494E-03	2.858174E-03	1.974242E+00
6.511579E-05	1.145365E-03	-6.359857E-03	8.804451E-02	-1.926950E-02	4.457555E-03	-1.569532E-04	2.037784E+00
6.511579E-05	1.200000E-03	-6.725786E-03	8.803808E-02	-2.159879E-02	4.416454E-03	-2.484587E-04	2.040369E+00
6.837158E-05	9.184184E-04	2.656427E-03	8.579162E-02	-1.847505E-02	2.745970E-03	-8.265984E-03	1.682405E+00
6.837158E-05	8.414014E-04	4.327896E-03	8.571139E-02	-1.977031E-02	1.429194E-03	-1.558558E-02	1.601905E+00
6.837158E-05	7.643844E-04	5.036143E-03	8.564378E-02	-2.113882E-02	-1.562638E-04	-2.085353E-02	1.528420E+00
6.837158E-05	6.873674E-04	7.269097E-03	8.548099E-02	-2.261904E-02	-1.611414E-03	-3.379821E-02	1.454441E+00
6.837158E-05	6.103504E-04	5.506108E-03	8.559012E-02	-2.416308E-02	-4.025944E-03	-7.489838E-03	1.392572E+00
6.837158E-05	5.333333E-04	3.604620E-03	8.562043E-02	-2.582247E-02	-6.781328E-03	6.228538E-03	1.330587E+00
6.837158E-05	1.046577E-03	-1.591317E-03	8.587374E-02	-1.623702E-02	5.364687E-03	-2.554792E-03	1.856171E+00
6.837158E-05	1.097718E-03	-6.484867E-03	8.591138E-02	-1.519719E-02	6.611301E-03	-1.604252E-02	1.961382E+00
6.837158E-05	1.148859E-03	-1.257118E-02	8.600417E-02	-1.645732E-02	5.552708E-03	-3.458399E-02	2.087494E+00
6.837158E-05	1.200000E-03	-1.585643E-02	8.603795E-02	-1.884251E-02	4.268973E-03	-4.269290E-02	2.090329E+00
7.179015E-05	9.304416E-04	-2.248081E-03	8.364309E-02	-1.838629E-02	1.735415E-03	1.534281E-02	1.686076E+00
7.179015E-05	8.510199E-04	-4.699488E-03	8.352381E-02	-1.963772E-02	-3.038292E-04	3.135849E-02	1.621430E+00
7.179015E-05	7.715983E-04	-7.463902E-03	8.333885E-02	-2.089731E-02	-2.165457E-03	4.293789E-02	1.567351E+00
7.179015E-05	6.921766E-04	-8.215925E-03	8.327302E-02	-2.221947E-02	-4.109667E-03	3.986747E-02	1.505985E+00
7.179015E-05	6.127550E-04	-9.784109E-03	8.312642E-02	-2.364985E-02	-6.340046E-03	4.417661E-02	1.449029E+00
7.179015E-05	5.333333E-04	-9.646353E-03	8.314295E-02	-2.528779E-02	-8.729211E-03	3.296348E-02	1.378003E+00
7.179015E-05	1.057397E-03	-2.443652E-03	8.371714E-02	-1.621033E-02	4.899831E-03	6.979280E-04	1.844750E+00
7.179015E-05	1.104932E-03	-5.432795E-03	8.361660E-02	-1.545085E-02	5.576203E-03	2.578097E-02	1.919945E+00
7.179015E-05	1.152466E-03	-4.019041E-03	8.370037E-02	-1.738015E-02	4.942440E-03	2.363562E-02	2.044108E+00
7.179015E-05	1.200000E-03	-4.023741E-03	8.369786E-02	-1.987248E-02	4.387698E-03	2.620822E-02	2.046705E+00
7.537966E-05	9.428548E-04	3.254418E-03	8.153502E-02	-1.806823E-02	2.889978E-03	-1.528337E-02	1.684061E+00
7.537966E-05	8.609505E-04	6.232939E-03	8.133769E-02	-1.938238E-02	1.958915E-03	-4.548517E-02	1.598892E+00
7.537966E-05	7.790462E-04	8.914532E-03	8.108208E-02	-2.082848E-02	7.644868E-04	-6.374845E-02	1.517584E+00
7.537966E-05	6.971419E-04	1.137845E-02	8.083254E-02	-2.241850E-02	-7.544149E-04	-6.475771E-02	1.438329E+00
7.537966E-05	6.152376E-04	1.345410E-02	8.068312E-02	-2.417271E-02	-2.685196E-03	-5.055021E-02	1.360158E+00
7.537966E-05	5.333333E-04	1.246917E-02	8.084020E-02	-2.601885E-02	-5.659379E-03	-3.010406E-02	1.286017E+00
7.537966E-05	1.068569E-03	-1.186560E-03	8.163711E-02	-1.601542E-02	5.047188E-03	-1.600888E-03	1.845981E+00
7.537966E-05	1.112380E-03	-4.235068E-03	8.161700E-02	-1.533923E-02	5.627909E-03	2.930369E-03	1.918681E+00
7.537966E-05	1.156190E-03	-6.504317E-03	8.158450E-02	-1.587216E-02	4.984107E-03	4.503642E-03	2.010693E+00
7.537966E-05	1.200000E-03	-6.325643E-03	8.159942E-02	-1.832843E-02	4.412754E-03	4.608062E-03	2.055859E+00
7.914864E-05	9.556731E-04	3.059853E-03	7.948710E-02	-1.787335E-02	2.868613E-03	-2.186441E-02	1.684778E+00
7.914864E-05	8.712051E-04	5.605810E-03	7.934470E-02	-1.922663E-02	1.592094E-03	-3.193001E-02	1.597415E+00
7.914864E-05	7.867372E-04	5.542629E-03	7.933493E-02	-2.066131E-02	-4.263909E-04	-1.957657E-02	1.519346E+00
7.914864E-05	7.022692E-04	6.837404E-03	7.919998E-02	-2.219737E-02	-2.033641E-03	-3.221033E-02	1.441607E+00
7.914864E-05	6.178013E-04	5.773711E-03	7.923163E-02	-2.386676E-02	-4.241117E-03	-1.998811E-02	1.371529E+00
7.914864E-05	5.333333E-04	7.376348E-03	7.913946E-02	-2.567669E-02	-7.017235E-03	-1.363305E-02	1.291536E+00
7.914864E-05	1.080106E-03	-3.075701E-03	7.957931E-02	-1.600725E-02	4.455200E-03	-3.337077E-03	1.833751E+00
7.914864E-05	1.120071E-03	-3.471688E-03	7.965166E-02	-1.509884E-02	5.987005E-03	-1.920764E-02	1.917819E+00
7.914864E-05	1.160035E-03	-6.402425E-03	7.963510E-02	-1.424675E-02	7.021352E-03	-1.417549E-02	2.019607E+00
7.914864E-05	1.200000E-03	-7.540543E-03	7.960964E-02	-1.715554E-02	4.125870E-03	-1.233672E-02	2.031063E+00
8.310608E-05	9.689124E-04	1.963393E-03	7.751722E-02	-1.771118E-02	2.551594E-03	-1.441149E-02	1.684543E+00
8.310608E-05	8.817966E-04	5.966057E-03	7.730537E-02	-1.903237E-02	1.742324E-03	-4.270256E-02	1.596268E+00
8.310608E-05	7.946808E-04	8.710730E-03	7.704767E-02	-2.047103E-02	5.284901E-04	-5.949141E-02	1.512074E+00
8.310608E-05	7.075650E-04	1.109078E-02	7.674389E-02	-2.214772E-02	-8.871079E-04	-7.514909E-02	1.428190E+00
8.310608E-05	6.204491E-04	1.124916E-02	7.667780E-02	-2.393413E-02	-3.145027E-03	-7.337240E-02	1.347336E+00
8.310608E-05	5.333333E-04	8.646197E-03	7.706477E-02	-2.578764E-02	-6.519430E-03	-1.548077E-02	1.278931E+00
8.310608E-05	1.092021E-03	-1.360128E-03	7.758575E-02	-1.582686E-02	4.537601E-03	5.413732E-03	1.833358E+00
8.310608E-05	1.128014E-03	-4.521035E-03	7.753117E-02	-1.516450E-02	5.693743E-03	1.256660E-02	1.879456E+00
8.310608E-05	1.164007E-03	-4.177726E-03	7.755143E-02	-1.462563E-02	5.914249E-03	1.890947E-02	1.962733E+00
8.310608E-05	1.200000E-03	-3.650662E-03	7.758242E-02	-1.641427E-02	4.688980E-03	2.103992E-02	2.035509E+00

Appendix B: Time-fixed optimal transfers

Table 35 presents the details of the transfers presented in chapter six. The parameters listed define optimal BLCTs in the time-fixed problem.

Table 35. Parameters of optimal time-fixed BLCTs

$\Delta v1_x$ (km/s)	$\Delta v1_y$ (km/s)	f (rad)	t_k (sec)	$\Delta v2_x$ (km/s)	$\Delta v2_y$ (km/s)	$\lambda_r(t_0)_x$	$\lambda_r(t_0)_y$	$\lambda_v(t_0)_x$	$\lambda_v(t_0)_y$	JED (days)	t_1 (sec)	t_f (sec)
-2.8459E+00	-1.1128E+00	1.9435E+00	3.0291E+06	6.7801E-02	5.1360E-02	-2.6798E-04	6.8526E-04	9.3134E-01	3.6417E-01	2453948	1.5204E+05	6.4823E+06
-2.8714E+00	-1.0258E+00	1.9127E+00	2.7043E+06	1.1658E-01	9.3300E-02	-2.4651E-04	6.9444E-04	9.4171E-01	3.3642E-01	2453948	8.9784E+05	6.4823E+06
-2.6791E+00	-1.4483E+00	2.0663E+00	2.3115E+06	1.6739E-01	1.7589E-01	-3.5071E-04	6.4887E-04	8.7969E-01	4.7555E-01	2453948	1.7518E+06	6.4823E+06
-2.1560E+00	-2.1489E+00	2.3566E+00	1.7665E+06	1.9459E-01	2.7918E-01	-5.2311E-04	5.2111E-04	7.0828E-01	7.0593E-01	2453948	2.4868E+06	6.4823E+06
-1.7299E+00	-2.4788E+00	2.5141E+00	8.9752E+05	2.5654E-01	4.5222E-01	-5.9553E-04	4.4359E-04	5.7228E-01	8.2006E-01	2453948	4.0218E+06	6.4823E+06
8.6842E-01	-2.8817E+00	3.4363E+00	6.9917E+05	-2.3153E-01	6.3239E-01	-7.1287E-04	-2.1748E-04	-2.8853E-01	9.5746E-01	2453948	5.0918E+06	6.4823E+06

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